



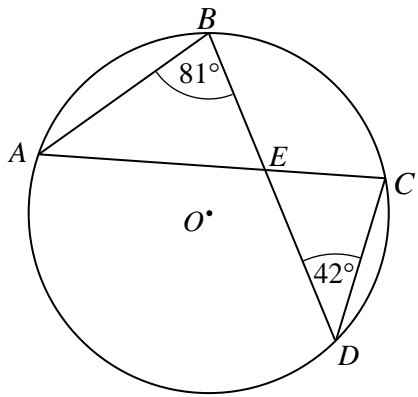
Circle Theorems



REVISE THIS TOPIC



1 A, B, C and D are points on the circumference of a circle with centre O.



Angle $ABD = 81^\circ$
 Angle $BDC = 42^\circ$

(a) Work out the size of angle CAB .

42

(1)

(b) Work out the size of angle ACD .

81

(1)

(c) Work out the size of angle AEB .

57

(1)

(d) Work out the size of angle BEC .

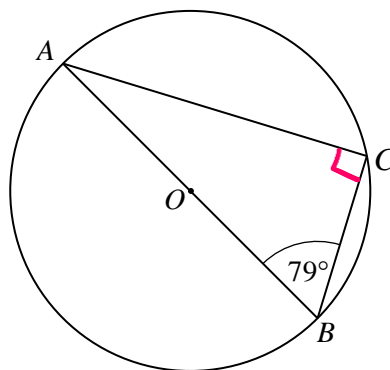
123

(1)

(Total for Question 1 is 4 marks)



2 A, B and C are points on the circumference of a circle with centre O.



Angle $ABC = 79^\circ$

(a) Work out the size of angle CAB .

11

 (1)

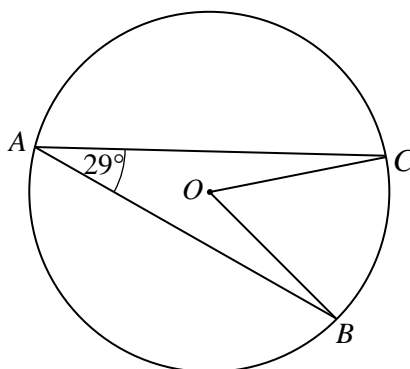
(b) Give a reason for your answer to part (a)

The angle in a semicircle is 90°
 Angles in a triangle add to 180°

(1)

(Total for Question 2 is 2 marks)

3 A, B and C are points on the circumference of a circle with centre O.



Angle $CAB = 29^\circ$

(a) Work out the size of angle COB .

58

 (1)

(b) Give a reason for your answer to part (a)

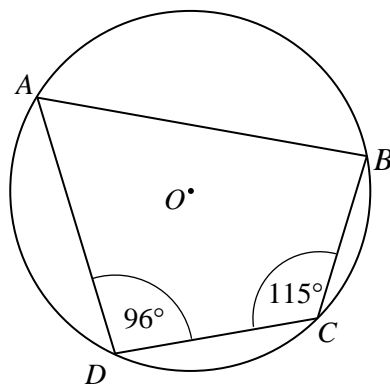
The angle at the centre is twice the angle at the circumference.

(1)

(Total for Question 3 is 2 marks)



4 A, B, C and D are points on the circumference of a circle with centre O .



Angle $ADC = 96^\circ$
 Angle $BCD = 115^\circ$

(a) Work out the size of angle ABC .

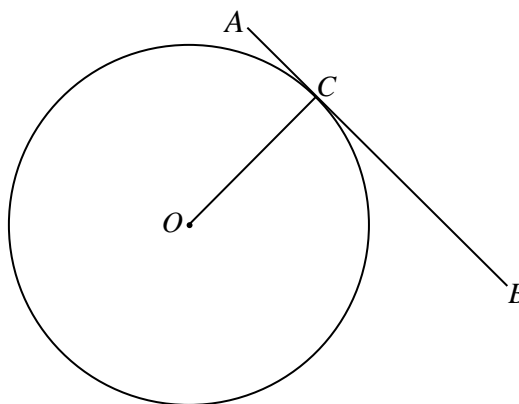
84

(b) Give a reason for your answer to part (a)

Opposite angles in a cyclic quadrilateral add to 180°

(1)
(Total for Question 4 is 2 marks)

5 $A, B,$ and C are points on the circumference of a circle with centre O .
 AB is the tangent to the circle at point C .



(a) Write down the size of angle OCB .

90

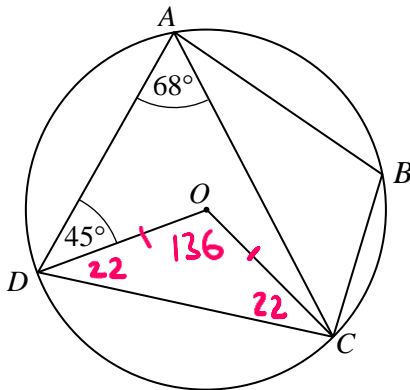
(b) Give a reason for your answer to part (a)

A tangent meets a radius at 90°

(1)
(Total for Question 5 is 2 marks)



6 A, B, C and D are points on the circumference of a circle with centre O .



Angle $DAC = 68^\circ$

Angle $ADO = 45^\circ$

Work out the size of angle ABC .

Give reasons for each stage of your working.

Angle $DOC = 136^\circ$

The angle at the centre is twice the angle at the circumference.

Angle $ODC = \text{Angle } OCD = 22^\circ$

Base angles in an isosceles triangle are equal

Angle $ADC = 22 + 45$
 $= 67$

Angle $ABC = 180 - 67$
 $= 113$

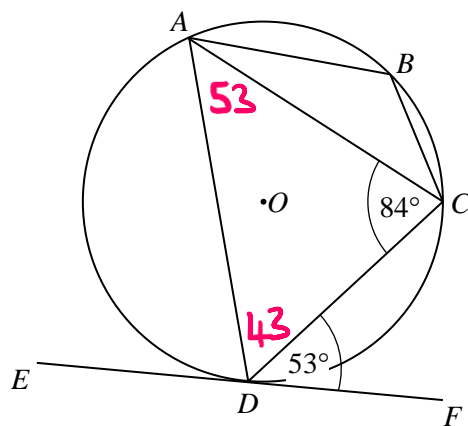
Opposite angles in a cyclic quadrilateral add to 180°

113

(Total for Question 6 is 4 marks)



- 7 A, B, C and D are points on the circumference of a circle with centre O .
 EF is the tangent to the circle at point D .



Angle $ACD = 84^\circ$

Angle $CDF = 53^\circ$

Work out the size of angle ABC .

Give reasons for each stage of your working.

Angle $DAC = 53^\circ$

Alternate segment theorem

Angle $ADC = 43^\circ$

Angles in a triangle add to 180°

Angle $ABC = 180 - 43$
 $= 137^\circ$

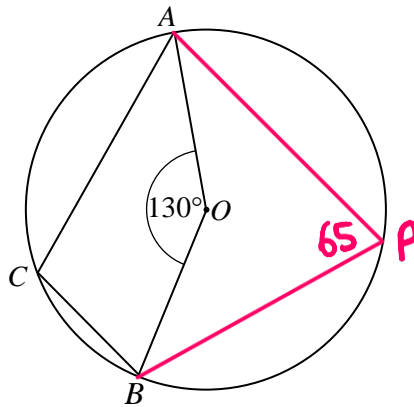
Opposite angles in a cyclic quadrilateral
 add to 180°

137

(Total for Question 7 is 4 marks)



8 $A, B,$ and C are points on the circumference of a circle with centre O .



Angle $AOB = 130^\circ$

Work out the size of angle ACB .
Give reasons for each stage of your working.

Angle $APB = 65^\circ$

The angle at the centre is twice the angle at the circumference.

Angle $ABC = 180 - 65$
 $= 115$

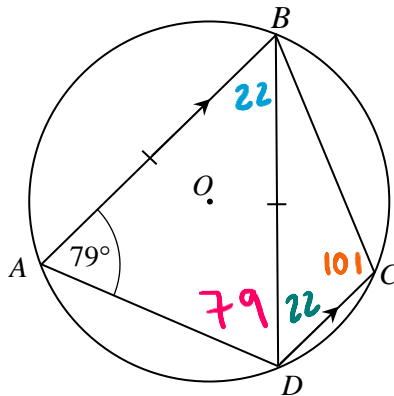
Opposite angles in a cyclic quadrilateral add to 180°

115

(Total for Question 8 is 3 marks)



9 A, B, C and D are points on the circumference of a circle with centre O.



Angle $BAD = 79^\circ$

$BA = BD$

Lines AB and DC are parallel.

Work out the size of angle DBC .

Give reasons for each stage of your working.

Angle $BAD = \text{Angle } BDA = 79^\circ$

Base angles in an isosceles triangle are equal

Angle $ABD = 22^\circ$

Angles in a triangle add to 180°

Angle $BDC = \text{Angle } ABD$

Alternate angles are equal

Angle $ABC = 180 - 43$
 $= 137^\circ$

Opposite angles in a cyclic quadrilateral add to 180°

Angle $DBC = 57^\circ$

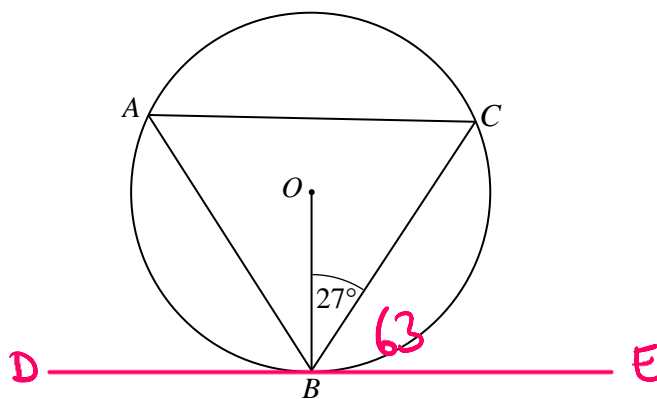
Angles in a triangle add to 180°

57

(Total for Question 9 is 5 marks)



10 A, B and C are points on the circumference of a circle with centre O .



Angle $CBO = 27^\circ$

Work out the size of angle BAC .
Give reasons for each stage of your working.

$$\begin{aligned} \text{Angle } CBE &= 90 - 27 \\ &= 63^\circ \end{aligned}$$

A tangent meets a radius at 90°

$$\text{Angle } BAC = 63^\circ$$

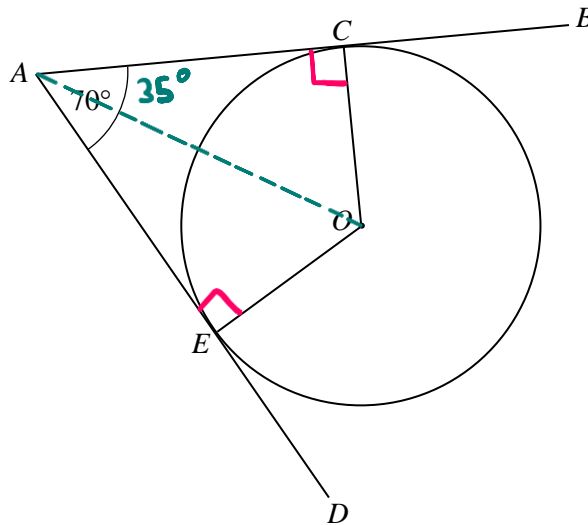
Alternate segment theorem

63

(Total for Question 10 is 3 marks)



11 C and E are points on the circumference of a circle with centre O .



AB and AD are tangents to the circle at C and E .
Angle $EAC = 70^\circ$

(a) Work out the size of angle COE .

Angle $AEO = \text{Angle } ACO = 90^\circ$

A tangent meets a radius at 90°

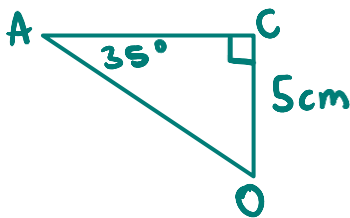
Angle $COE = 110^\circ$ angles in a quadrilateral add to 360°

$OC = 5 \text{ cm}$

..... 110
(2)

(b) Work out the length of CA .
Give your answer to 1 decimal place.

Angle $CAO = 35^\circ$



$\tan(35) = \frac{5}{CA}$

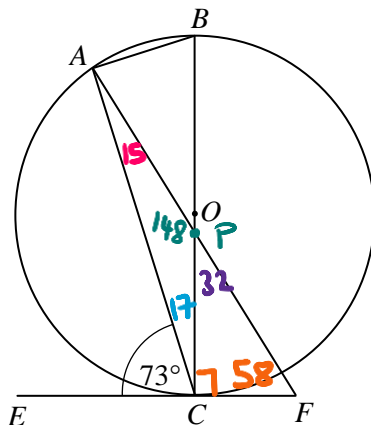
$CA = \frac{5}{\tan(35)}$

$CA = 7.1407\dots$ 7.1 cm
(2)

(Total for Question 11 is 4 marks)



12 A, B and C are points on the circumference of a circle with centre O .



EF is the tangent to the circle at C .

Angle $ACE = 73^\circ$

Angle $FAB = 5 \times$ Angle CAF

Work out the size of angle AFC .

[Reasons not required in this Q]

Angle $BAC = 90^\circ$ The angle in a semicircle is 90°

$90 \div 6 = 15$

Angle $CAF = 15^\circ$

Angle $ECB = 90^\circ$ Angle $ACB = 17^\circ$

A tangent meets a radius at 90°

Angle $APC = 148^\circ$ Angles in a triangle add to 180°

Angle $CPF = 32^\circ$ Angles on a straight line add to 180°

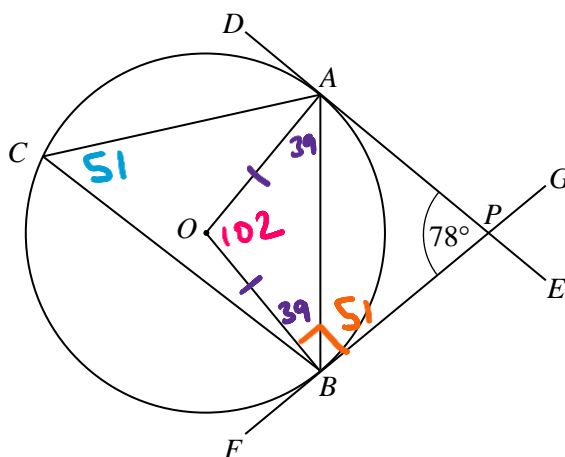
Angle $AFC = 58^\circ$ Angles in a triangle add to 180°

58

(Total for Question 12 is 4 marks)



13 A, B and C are points on the circumference of a circle with centre O.



DE and FG are tangents to the circle at A and B that intersect at the point P.
Angle APB = 78°

[Reasons not required in this Q]

(a) Work out the size of angle ACB.

Angle AOB = 102° A tangent meets a radius at 90°
and angles in a quadrilateral add to 360°

Angle ACB = 51° Angle at the circumference is
half the angle at the centre

51

(b) Work out the size of angle ABP.

Angle ABO = Angle BAO = 39°

Base angles in an isosceles triangle are equal

Angle ABP = $90 - 39$
= 51

A tangent meets a radius at 90°

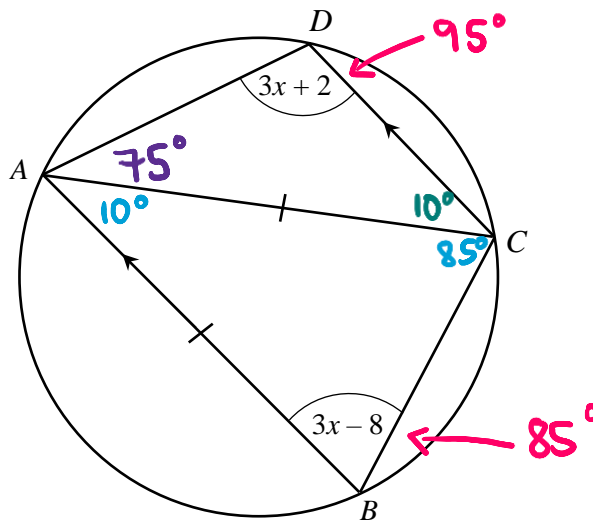
51

(2)

(Total for Question 13 is 4 marks)



14



A, B, C and D are points on the circumference of a circle.
 ABCD is a trapezium with AB parallel to DC.

$AB = AC$

[Reasons not required in this Q]

Work out the size of angle DAC.
 You must show all your working.

$3x + 2 + 3x - 8 = 180^\circ$

Opposite angles in a cyclic quadrilateral add to 180°

$6x - 6 = 180$	$3(31) + 2 = 95$
$6x = 186$	$3(31) - 8 = 85$
$x = 31^\circ$	

Angle ACB = Angle ABC = 85

Angle CAB = 10° Angles in a triangle add to 180°

Angle DCA = Angle CAB alternate angles are equal

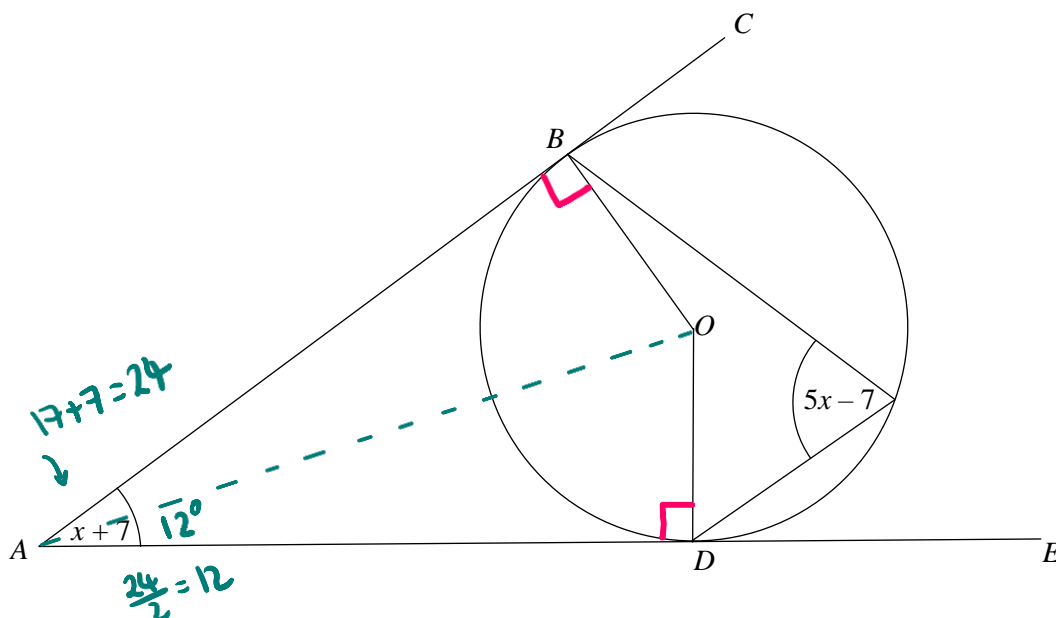
Angle DAC = 75° Angles in a triangle add to 180°

75

(Total for Question 14 is 5 marks)



15



B and D are points on the circumference of a circle, centre O .
 ABC and ADE are tangents to the circle.

[Reasons not required in this Q]

- (a) Work out value of x .
 You must show all your working.

$\text{Angle } BOD = 2(5x - 7)$
 (twice angle at circumference)

$10x - 14 = 180 - x - 7$
 $10x - 14 = 173 - x$
 $11x = 187$
 $x = 17$

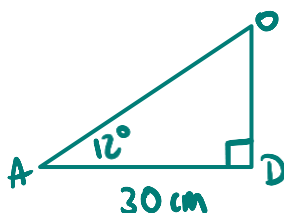
$\text{Angle } BOD = 180 - (x + 7)$
 (angles in quadrilateral add to 360°)

$x = \underline{\quad 17 \quad}$
 (3)

$AD = 30$ cm

- (b) Work out the radius of the circle.
 Give your answer to three significant figures.

$\angle BAD = 24^\circ$
 $\angle AOD = 12^\circ$



$\tan(12) = \frac{OD}{30}$
 $OD = 30 \tan(12)$

$\underline{\quad 6.38 \quad}$ cm
 (3)

(Total for Question 15 is 6 marks)

