



Inverse Functions



REVISE THIS
TOPIC

ANSWERS

1 $f(x) = 2x + 9$ $g(x) = \sqrt{x-3}$ $h(x) = x^3 + 4$

1 (a) Work out $f^{-1}(x)$ [2 marks]



$$y = 2x + 9$$

$$\frac{x-9}{2} = y$$

$$x = 2y + 9$$

$$x - 9 = 2y$$

$$f^{-1}(x) = \frac{x-9}{2}$$

1 (b) Work out $g^{-1}(x)$ [2 marks]



$$y = \sqrt{x-3}$$

$$x^2 + 3 = y$$

$$x = \sqrt{y-3}$$

$$x^2 = y - 3$$

$$g^{-1}(x) = x^2 + 3$$

1 (c) Work out $h^{-1}(31)$ [2 marks]



$$y = x^3 + 4$$

$$\sqrt[3]{x-4} = y$$

$$x = y^3 + 4$$

$$h^{-1}(x) = \sqrt[3]{x-4}$$

$$x - 4 = y^3$$

$$h^{-1}(31) = \sqrt[3]{31-4}$$

Answer 3





2 $f(x) = \frac{2x+3}{4}$ $g(x) = x^2 - 6$

2 (a) Work out $f^{-1}(x)$ [2 marks]



$$y = \frac{2x+3}{4}$$

$$4x = 2y+3 \quad \frac{4x-3}{2} = y$$

$$4x-3 = 2y$$

$$x = \frac{2y+3}{4}$$

$$\frac{4x-3}{2}$$

$$f^{-1}(x) = \frac{4x-3}{2}$$

2 (b) Work out $g^{-1}(x)$ [2 marks]



$$y = x^2 - 6$$

$$\sqrt{x+6} = y$$

$$x = y^2 - 6$$

$$x+6 = y^2$$

$$g^{-1}(x) = \sqrt{x+6}$$

3 $f(x) = 50 - x^2$ $g(x) = 4x^2 - 1$

3 (a) Work out $f^{-1}(1)$ [2 marks]



$$y = 50 - x^2$$

$$y^2 = 50 - x$$

$$x = 50 - y^2$$

$$y = \sqrt{50-x}$$

$$x + y^2 = 50$$

$$f^{-1}(1) = \sqrt{50-1}$$

$$7$$

Answer 7

3 (b) Work out $g^{-1}(0)$ [2 marks]



$$y = 4x^2 - 1$$

$$\frac{x+1}{4} = y^2$$

$$g^{-1}(x) = \sqrt{\frac{x+1}{4}}$$

$$x = 4y^2 - 1$$

$$\sqrt{\frac{x+1}{4}} = y$$

$$g^{-1}(0) = \sqrt{\frac{0+1}{4}}$$

$$x+1 = 4y^2$$

$$\frac{1}{2}$$

Answer $\frac{1}{2}$





4 $f(x) = \frac{2x^2}{5}$

$g(x) = \frac{x}{4} - 3$

4 (a) Work out $f^{-1}(x)$

[2 marks]



$y = \frac{2x^2}{5}$

$5x = 2y^2$

$y = \sqrt{\frac{5x}{2}}$

$x = \frac{2y^2}{5}$

$\frac{5x}{2} = y^2$

$\sqrt{\frac{5x}{2}}$

$f^{-1}(x) =$

4 (b) Work out $g^{-1}(x)$

[2 marks]



$y = \frac{x}{4} - 3$

$x + 3 = \frac{y}{4}$

$x = \frac{y}{4} - 3$

$4(x + 3) = y$

$g^{-1}(x) = 4(x + 3)$

5 $f(x) = \sqrt[3]{100 - x}$

$g(x) = 2(x + 14)$

5 (a) Work out $f^{-1}(4)$

[2 marks]



$y = \sqrt[3]{100 - x}$

$x^3 + y = 100$

$x = \sqrt[3]{100 - y}$

$y = 100 - x^3$

$x^3 = 100 - y$

$f^{-1}(4) = 100 - 4^3$

36

Answer

5 (b) Work out $g^{-1}(26)$

[2 marks]



$y = 2(x + 14)$

$\frac{x}{2} - 14 = y$

$x = 2(y + 14)$

$g^{-1}(26) = \frac{26}{2} - 14$

$\frac{x}{2} = y + 14$

-1

Answer





- 6 $f(x) = \frac{5}{x+10}$ $g(x) = \sqrt{2x^3-3}$
- 6 (a) Work out $f^{-1}(x)$ [2 marks]



$$y = \frac{5}{x+10}$$

$$x(y+10) = 5$$

$$y+10 = \frac{5}{x}$$

$$x = \frac{5}{y+10}$$

$$y = \frac{5}{x} - 10$$

$$f^{-1}(x) = \frac{5}{x} - 10$$

- 6 (b) Work out $g^{-1}(x)$ [2 marks]



$$y = \sqrt{2x^3-3}$$

$$x^2 + 3 = 2y^3$$

$$y = \sqrt[3]{\frac{x^2+3}{2}}$$

$$x = \sqrt{2y^3-3}$$

$$\frac{x^2+3}{2} = y^3$$

$$x^2 = 2y^3 - 3$$

$$3\sqrt{\frac{x^2+3}{2}}$$

$$g^{-1}(x) =$$

- 7 $f(x) = 3 - \frac{2}{x}$ $g(x) = (x-5)^3$

- 7 (a) Work out $f^{-1}(2.5)$ [2 marks]



$$y = 3 - \frac{2}{x}$$

$$x + \frac{2}{y} = 3$$

$$\frac{2}{3-x} = y$$

$$x = 3 - \frac{2}{y}$$

$$\frac{2}{y} = 3 - x$$

$$f^{-1}(2.5) = \frac{2}{3-2.5}$$

$$\text{Answer } 4$$

- 7 (b) Work out $g^{-1}(-27)$ [2 marks]



$$y = (x-5)^3$$

$$\sqrt[3]{x} + 5 = y$$

$$x = (y-5)^3$$

$$g^{-1}(27) = \sqrt[3]{27} + 5$$

$$\sqrt[3]{x} = y - 5$$

$$\text{Answer } 8$$





8

$$f(x) = \frac{x+4}{x-3}$$

$$g(x) = \sqrt{3x}$$

$$h(x) = 2x + 1$$

8 (a) Work out $f^{-1}(x)$

[3 marks]



$$y = \frac{x+4}{x-3}$$

$$x(y-3) = y+4$$

$$y = \frac{4+3x}{x-1}$$

$$xy - 3x = y + 4$$

$$x = \frac{y+4}{y-3}$$

$$xy - y = 4 + 3x$$

$$y(x-1) = 4 + 3x$$

$$f^{-1}(x) = \frac{4+3x}{x-1}$$

8 (b) Work out $g^{-1}(9)$

[2 marks]



$$y = \sqrt{3x}$$

$$\frac{x^2}{3} = y$$

$$x = \sqrt{3y}$$

$$x^2 = 3y$$

$$g^{-1}(9) = \frac{9^2}{3}$$

$$\text{Answer } 27$$

8 (c) $k(x) = gh(x)$ Work out $k^{-1}(x)$

[4 marks]



$$gh(x) = \sqrt{3(2x+1)}$$

$$x = \sqrt{6y+3}$$

$$gh(x) = \sqrt{6x+3}$$

$$x^2 = 6y+3$$

$$x^2 - 3 = 6y$$

$$k(x) = \sqrt{6x+3}$$

$$\frac{x^2 - 3}{6} = y$$

$$y = \sqrt{6x+3}$$

$$k^{-1}(x) = \frac{x^2 - 3}{6}$$

