Equation of a Tangent

SCAN ME

REVISE THIS TOPIC

1 A circle has equation $x^2 + y^2 = 20$

The point P lies on the circle.
The coordinates of P are (2, 4)

The line **L** is tangent to the circle at point P.

Find an equation of **L**.
Give your answer in the form $y = mx + c$

gradient of OP $= \frac{4}{2}$
$= 2$

gradient of tangent $= -\frac{1}{2}$

$y = -\frac{1}{2}x + c$
$4 = -\frac{1}{2}(2) + c$
$4 = -1 + c$
$c = 5$

$y = -\frac{1}{2}x + 5$

(Total for Question 1 is 4 marks)

2 A circle has equation $x^2 + y^2 = 90$

The point P lies on the circle.
The coordinates of P are (9, 3)

The line **L** is tangent to the circle at point P.

Find an equation of **L**.
Give your answer in the form $y = mx + c$

gradient of OP $= \frac{3}{9}$
$= \frac{1}{3}$

gradient of tangent $= -3$

$y = -3x + c$
$3 = -3(9) + c$
$3 = -27 + c$
$c = 30$

$y = -3x + 30$

(Total for Question 2 is 4 marks)

3 A circle has equation $x^2 + y^2 = 29$

The point P lies on the circle.
The coordinates of P are (2, 5)

The line **L** is tangent to the circle at point P.

Find an equation of **L**.
Give your answer in the form $y = mx + c$

gradient of OP $= \frac{5}{2}$

gradient of tangent $= -\frac{2}{5}$

$y = -\frac{2}{5}x + c$

$5 = -\frac{2}{5}(2) + c$

$5 = -\frac{4}{5} + c$

$c = 5 + \frac{4}{5}$

$c = \frac{25}{5} + \frac{4}{5}$

$c = \frac{29}{5}$

$y = -\frac{2}{5}x + \frac{29}{5}$

(Total for Question 3 is 4 marks)

4 A circle has equation $x^2 + y^2 = 17$

The point P lies on the circle.
The coordinates of P are (1, -4)

The line **L** is tangent to the circle at point P.

Find an equation of **L**.
Give your answer in the form $y = mx + c$

gradient of OP $= \frac{-4}{1}$
$= -4$

gradient of tangent $= \frac{1}{4}$

$y = \frac{1}{4}x + c$

$-4 = \frac{1}{4}(1) + c$
$-4 = \frac{1}{4} + c$
$c = -4 - \frac{1}{4}$
$c = -\frac{16}{4} - \frac{1}{4}$
$c = -\frac{17}{4}$

$y = \frac{1}{4}x - \frac{17}{4}$

(Total for Question 4 is 4 marks)

5 A circle has equation $x^2 + y^2 = 34$

The point P lies on the circle.
The coordinates of P are (-3, 5)

The line **L** is tangent to the circle at point P.

Find an equation of **L**.
Give your answer in the form $y = mx + c$

gradient of OP $= -\frac{5}{3}$

gradient of tangent $= \frac{3}{5}$

$y = \frac{3}{5}x + c$

$5 = \frac{3}{5}(-3) + c$

$5 = \frac{-9}{5} + c$

$c = 5 + \frac{9}{5}$

$c = \frac{25}{5} + \frac{9}{5}$

$c = \frac{34}{5}$

$y = \frac{3}{5}x + \frac{34}{5}$

(Total for Question 5 is 4 marks)

6 A circle has equation $x^2 + y^2 = 65$

The point P lies on the circle.
The coordinates of P are $(7, k)$, where $k < 0$

The line **L** is tangent to the circle at point P.

Find an equation of **L**.
Give your answer in the form $y = mx + c$

$7^2 + k^2 = 65$

$49 + k^2 = 65$

$k^2 = 16$

$k = \pm 4$

as $k < 0$, $\underline{\underline{k = 4}}$

gradient of OP $= -\frac{4}{7}$

gradient of tangent $= 7/4$

$y = \frac{7}{4}x + c$

$-4 = \frac{7}{4}(7) + c$

$-4 = \frac{49}{4} + c$

$c = -4 - \frac{49}{4}$

$c = \frac{-16}{4} - \frac{49}{4}$

$c = -\frac{65}{4}$

$y = \frac{7}{4}x - \frac{65}{4}$

(Total for Question 6 is 5 marks)

7 A circle has equation $x^2 + y^2 = 117$

The point P lies on the circle.
The coordinates of P are $(9, k)$, where $k > 0$

The line **L** is tangent to the circle at point P.

Find an equation of **L**.
Give your answer in the form $y = mx + c$

$9^2 + k^2 = 117$
$81 + k^2 = 117$
$k^2 = 36$
$k = \pm 6$
as $k > 0$, $\underline{\underline{k = 6}}$

gradient of OP $= \frac{6}{9}$
$= \frac{2}{3}$

gradient of tangent $= -\frac{3}{2}$

$y = -\frac{3}{2}x + c$

$6 = -\frac{3}{2}(9) + c$
$6 = \frac{-27}{2} + c$
$c = 6 + \frac{27}{2}$
$c = \frac{12}{2} + \frac{27}{2}$
$c = \frac{39}{2}$

$y = -\frac{3}{2}x + \frac{39}{2}$

(Total for Question 7 is 5 marks)

8 A circle has equation $x^2 + y^2 = 22.25$

The point P lies on the circle.
The coordinates of P are $(-4, k)$, where $k < 0$

The line **L** is tangent to the circle at point P.

Find an equation of **L**.
Give your answer in the form $ay + bx + c = 0$ where a, b and c are integers.

$(-4)^2 + k^2 = 22.25$

$16 + k^2 = 22.25$

$k^2 = 6.25$

$k = \pm 2.5$

as $k < 0$, $\underline{\underline{k = -2.5}}$

gradient of OP $= \frac{-2.5}{-4}$

$= \frac{2.5}{4}$

$= \frac{5}{8}$

gradient of tangent $= -\frac{8}{5}$

$y = -\frac{8}{5}x + c$

$-\frac{5}{2} = -\frac{8}{5}(-4) + c$

$-\frac{5}{2} = \frac{32}{5} + c$

$c = -\frac{5}{2} - \frac{32}{5}$

$c = -\frac{25}{10} - \frac{64}{10}$

$c = -\frac{89}{10}$

$\times 10 \left(y = -\frac{8}{5}x - \frac{89}{10} \right) \times 10$

$10y = -16x - 89$

$10y + 16x + 89 = 0$

$10y + 16x + 89 = 0$

(Total for Question 8 is 6 marks)

9 A circle has equation $x^2 + y^2 = 13$

The point P lies on the circle.
The coordinates of P are (2, 3)

The line **L** is tangent to the circle at point P.
The line **L** crosses the x-axis at the point Q.

Work out the coordinates of the point Q.

gradient of OP $= \frac{3}{2}$

gradient of tangent $= -\frac{2}{3}$

$y = -\frac{2}{3}x + c$

$3 = -\frac{2}{3}(2) + c$

$3 = -\frac{4}{3} + c$

$c = 3 + \frac{4}{3}$

$c = \frac{9}{3} + \frac{4}{3}$

$c = \frac{13}{3}$

$y = -\frac{2}{3}x + \frac{13}{3}$

At Q, $y = 0$

$0 = -\frac{2}{3}x + \frac{13}{3}$

$\frac{2}{3}x = \frac{13}{3}$

$2x = 13$

$x = \frac{13}{2}$

$x = 6.5$

(6.5 , 0)

(Total for Question 9 is 5 marks)

10 A circle has equation $x^2 + y^2 = 212$

The point P lies on the circle.
The coordinates of P are (14, -4)

The line **L** is tangent to the circle at point P.
The line **L** crosses the y-axis at the point A.

Work out the coordinates of the point A.

gradient of OP $= -\frac{4}{14}$
$= -\frac{2}{7}$

gradient of tangent $= \frac{7}{2}$

$y = \frac{7}{2}x + c$
$-4 = \frac{7}{2}(14) + c$
$-4 = 49 + c$
$c = -4 - 49$
$c = -53$

$y = \frac{7}{2}x - 53$

At A, $x = 0$
$y = \frac{7}{2}(0) - 53$
$y = -53$

(0, -53)

(Total for Question 10 is 5 marks)

11 A circle has equation $x^2 + y^2 = 90$

The point P lies on the circle.
The coordinates of P are (3, 9)

The line **L** is tangent to the circle at point P.
The line **L** crosses the y-axis at the point A and the x-axis at the point B.

Work out the area of triangle AOB.

gradient of OP $= \frac{9}{3}$
$= 3$

gradient of tangent $= -\frac{1}{3}$

$y = -\frac{1}{3}x + c$
$9 = -\frac{1}{3}(3) + c$
$9 = -1 + c$
$c = 10$
$y = -\frac{1}{3}x + 10$

At A, $x = 0$
$y = -\frac{1}{3}(0) + 10$
$y = 10$
$A = (0, 10)$

At B, $y = 0$

$0 = -\frac{1}{3}x + 10$
$\frac{1}{3}x = 10$
$x = 30$
$B = (30, 0)$

Area $= \frac{1}{2} \times 30 \times 10$
$= 150$

150 units^2

(Total for Question 11 is 6 marks)

12 A circle has equation $x^2 + y^2 = 320$

The point P lies on the circle.
The coordinates of P are (-8, 16)

The line **L** is tangent to the circle at point P.
The line **L** crosses the x-axis at the point A and the y-axis at the point B.

Work out the length of AB.
Give your answer in the form $a\sqrt{5}$ where a is an integer.

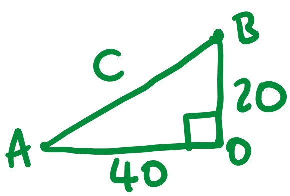
gradient of OP $= -\frac{16}{8}$
$= -2$

gradient of tangent $= \frac{1}{2}$

$y = \frac{1}{2}x + c$
$16 = \frac{1}{2}(-8) + c$
$16 = -4 + c$
$c = 16 + 4$
$c = 20$
$y = \frac{1}{2}x + 20$

At A, $y = 0$
$0 = \frac{1}{2}x + 20$
$-\frac{1}{2}x = 20$
$x = -40$
$A = (-40, 0)$

At B, $x = 0$
$y = \frac{1}{2}(0) + 20$
$y = 20$
$B = (0, 20)$



$c^2 = 20^2 + 40^2$
$c^2 = 2000$
$c = \sqrt{2000}$
$c = \sqrt{400} \times \sqrt{5}$

$20\sqrt{5}$ units

(Total for Question 12 is 6 marks)

13 A circle has equation $x^2 + y^2 = 29$

The point P lies on the circle.
The coordinates of P are (5, 2)

The line **L** is tangent to the circle at point P.
The line **L** crosses the y-axis at the point A and the x-axis at the point B.

Work out the length of AB.
Give your answer to 4 significant figures.

gradient of OP $= \frac{2}{5}$

gradient of tangent $= -\frac{5}{2}$

$$y = -\frac{5}{2}x + c$$
$$2 = -\frac{5}{2}(5) + c$$
$$2 = -\frac{25}{2} + c$$
$$c = 2 + \frac{25}{2}$$
$$c = \frac{4}{2} + \frac{25}{2}$$
$$c = \frac{29}{2}$$
$$y = -\frac{5}{2}x + \frac{29}{2}$$

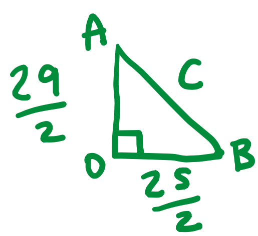
At A, $x = 0$

$$y = -\frac{5}{2}(0) + \frac{29}{2}$$
$$A = \left(0, \frac{29}{2}\right)$$

At B, $y = 0$

$$0 = -\frac{5}{2}x + \frac{29}{2}$$
$$\frac{5}{2}x = \frac{29}{2}$$
$$x = \frac{29}{5}$$

$$B = \left(\frac{29}{5}, 0\right)$$



$$c^2 = \left(\frac{29}{2}\right)^2 + \left(\frac{29}{5}\right)^2$$
$$c^2 = 243.89$$
$$c = \sqrt{243.89}$$
$$c = 15.6169...$$

15.62 units

(Total for Question 13 is 6 marks)

14 A circle has equation $x^2 + y^2 = 48$

The point *P* lies on the circle.
The coordinates of *P* are $(\sqrt{12}, 6)$

The line **L** is tangent to the circle at point *P*.
The line **L** crosses the *y*-axis at the point *A*.

Show that the length of *AP* is an integer.

gradient of OP $= \frac{6}{\sqrt{12}}$

gradient of tangent $= -\frac{\sqrt{12}}{6}$

$y = -\frac{\sqrt{12}}{6}x + c$

$6 = -\frac{\sqrt{12}}{6}(\sqrt{12}) + c$

$6 = -\frac{12}{6} + c$

$6 = -2 + c$

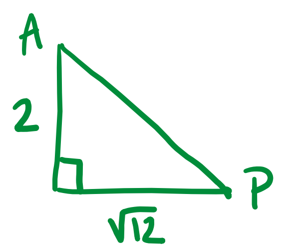
$c = 8$

$y = \frac{\sqrt{12}}{6}x + 8$

At A, $x = 0$

$y = \frac{\sqrt{12}}{6}(0) + 8$

$A = (0, 8)$



$AP^2 = 2^2 + (\sqrt{12})^2$

$AP^2 = 4 + 12$

$AP^2 = 16$

$AP = \sqrt{16}$

$AP = 4$

4 is an integer

(Total for Question 14 is 6 marks)

15 A circle has equation $x^2 + y^2 = 25$

The point P lies on the circle.
The coordinates of P are $(\sqrt{5}, \sqrt{20})$

The line **L** is tangent to the circle at point P.
The line **L** crosses the x-axis at the point A.

Work out the area of triangle AOP.

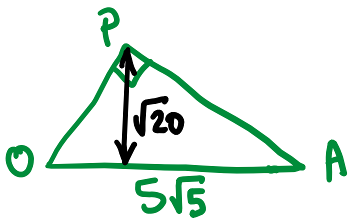
gradient of $OP = \frac{\sqrt{20}}{\sqrt{5}}$
$= \sqrt{4}$
$= 2$

gradient of tangent $= -\frac{1}{2}$

$y = -\frac{1}{2}x + c$
$\sqrt{20} = -\frac{1}{2}(\sqrt{5}) + c$
$c = \sqrt{20} + \frac{\sqrt{5}}{2}$
$c = 2\sqrt{5} + \frac{\sqrt{5}}{2}$
$c = \frac{5\sqrt{5}}{2}$
$y = -\frac{1}{2}x + \frac{5\sqrt{5}}{2}$

At A, $y = 0$

$0 = -\frac{1}{2}x + \frac{5\sqrt{5}}{2}$
$\frac{1}{2}x = \frac{5\sqrt{5}}{2}$
$x = 5\sqrt{5}$
$A = (5\sqrt{5}, 0)$



Area $= \frac{1}{2} \times 5\sqrt{5} \times \sqrt{20}$
$= \frac{1}{2} \times 5\sqrt{100}$
$= \frac{1}{2} \times 5 \times 10$
$= \frac{1}{2} \times 50$

25 units^2

(Total for Question 15 is 6 marks)