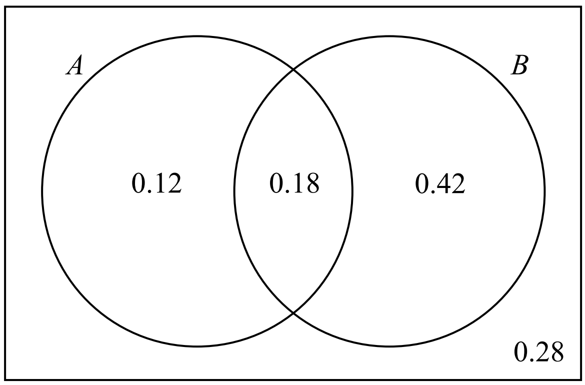
Probability

1 The Venn diagram shows the events *A* and *B* and their associated probabilities.



(a) Find

(i) P(*A*) **(1)**

(ii) P(*B*) **(1)**

(iii) P(*A* or *B* or both) **(1)**

(b) State, giving a reason, whether events *A* and *B* are mutually exclusive. **(1)**

(c) State, giving a reason, whether events *A* and *B* are independent. **(1)**

(a) (i) P(A) = 0.12 + 0.18

= 0.3

(ii) P(B) = 0.18 + 0.42

= 0.6

(iii) P(B) = 0.12 + 0.18 + 0.42

= 0.72

(b) Events A and B are not mutually exclusive as P(A and B) $\neq$ 0

(c) P(A) × P(B) = 0.3 × 0.6 = 0.18

P(A and B) = 0.18

P(A) × P(B) = P(A and B), therefore the events A and B are independent.

(Total for Question 1 is 5 marks)

2 Each time Finn goes fishing, he catches one fish and then stops fishing.

The probabilities for each fish caught are:

P(large fish) = 0.24
P(medium fish) = 0.72
P(small fish) = 0.04

Finn goes fishing four times in May, catching one fish each time.

For the four fish caught by Finn in May, find, to 4 decimal places, the probability that:

(a) at least one fish is large. **(2)**

(b) all fish are the same size. **(2)**

(c) exactly one fish is large and the rest are medium. **(3)**

(a) P(at least one large fish) = 1 – P(no large fish) [P(not large) = 1 – 0.24 = 0.76]

$= 1 - (0.76 \times 0.76 \times 0.76 \times 0.76)$

$= 0.6664$

(b) P(all fish the same size) = P(all large) + P(all medium) + P(all small)

$= (0.24^4 + 0.72^4 + 0.04^4)$

$= 0.3032$

(c) P(one large and three small) = P(LMMM) + P(MLMM) + P(MMLM) + P(MMML)

$= (0.24 \times 0.74 \times 0.74 \times 0.74) \times 4$

$= 0.3890$

(Total for Question 2 is 7 marks)

3 A college has 160 students. Of these students

60 study maths
50 study physics
64 were late to school
24 of the students who were late to school also study maths.
20 of the students who were late to school also study physics.
91 study maths or physics or both

One student from the college is selected at random.

M is the event that the student selected studies maths
P is the event that the student selected studies physics
L is the event that the student selected was late to school

(a) Determine, by calculation, if the events *M* and *L* are independent. **(2)**

(b) Determine, by calculation, if the events *P* and *L* are independent. **(2)**

(c) Determine, by calculation, if the events *M* and *P* are independent. **(3)**

(a) $P(M) = \frac{60}{160}$ $P(L) = \frac{64}{160}$

$P(M) \times P(L) = \frac{60}{160} \times \frac{64}{160} = \frac{3}{20}$

$P(M \text{ and } L) = \frac{24}{160} = \frac{3}{20}$ $P(M) \times P(L) = P(M \text{ and } L)$ therefore M and L are independent.

(b) $P(P) = \frac{50}{160}$ $P(L) = \frac{64}{160}$

$P(P) \times P(L) = \frac{50}{160} \times \frac{64}{160} = \frac{1}{8}$

$P(P \text{ and } L) = \frac{20}{160} = \frac{1}{8}$ $P(P) \times P(L) = P(P \text{ and } L)$ therefore P and L are independent.

(c) $P(M) = \frac{60}{160}$ $P(P) = \frac{50}{160}$ 60 study maths, 50 study physics, 91 study maths or physics or both.

Number who study both = 60 + 50 – 91 = 19

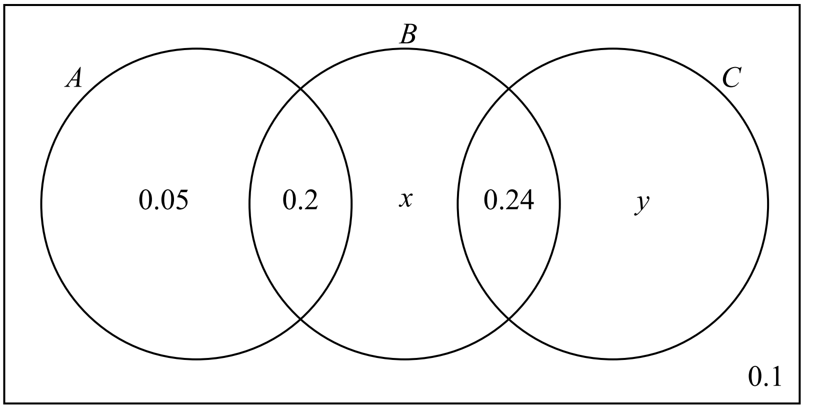
$P(M) \times P(P) = \frac{50}{160} \times \frac{60}{160} = \frac{15}{128}$

$P(M \text{ and } P) = \frac{19}{160}$

$\frac{15}{128} \neq \frac{19}{160}$ $P(M) \times P(P) \neq P(M \text{ and } P)$ therefore M and P are not independent.

(Total for Question 3 is 7 marks)

4 The Venn diagram shows the events A, B and C and their associated probabilities.



Events A and B are independent.

(a) Find the value of x **(2)**

(b) Find the value of y **(1)**

(c) Determine, by calculation, if the events B and C are independent. **(1)**

(a) P(A) = 0.05 + 0.2 = 0.25

P(B) = 0.2 + x + 0.24 = x + 0.44

P(A and B) = 0.2

(0.25) × (x + 0.44) = 0.2

x + 0.44 = 0.8

x = 0.36

(b) y = 1 – (0.05 + 0.2 + 0.36 + 0.24 + 0.1)

= 0.05

(c) P(B) = 0.2 + 0.36 + 0.24 = 0.8

P(C) = 0.24 + 0.05 = 0.29

P(B) × P(C) = 0.8 × 0.29 = 0.232

P(B and C) = 0.24

0.232 ≠ 0.24

P(B) × P(C) ≠ P(B and C) therefore B and C are not independent.

(Total for Question 4 is 4 marks)

5 Jeremy and Doug both play a computer game that has three levels.

If a player completes level 1 they progress to level 2.
If a player completes level 2 they progress to level 3.
If a player completes level 3 they complete the game.
If players fail to complete any of the levels the game is over and they stop playing.

The probabilities that Jeremy and Doug complete each of the levels of the game are shown below.

	Level 1	**Level 2**	**Level 3**
Jeremy	0.9	0.7	0.3
Doug	0.8	0.6	0.4

Both Jeremy and Doug make one attempt to complete the game.

(a) Work out who is more likely to complete the game. **(1)**

(b) Find, to 4 decimal places, the probability that exactly one of them completes the game. **(2)**

(c) Find, to 4 decimal places, the probability that at least one of them completes the game. **(2)**

(d) Find, to 4 decimal places, the probability that Jeremy fails on level 3 and Doug fails on level 2. **(2)**

(a) P(Jeremy completes the game) $= 0.9 \times 0.7 \times 0.3 = 0.189$

P(Doug completes the game) $= 0.8 \times 0.6 \times 0.4 = 0.192$

Doug is more likely to complete the game.

(b) P(exactly one completes) = P(J completes and D does not) + P(D completes and J does not)

$= (0.189 \times (1 - 0.192)) + (0.192 \times (1 - 0.189))$

$= 0.3084$

(c) P(at least one completes) = 1 – P(neither complete)

$= 1 - (1 - 0.189) \times (1 - 0.192)$

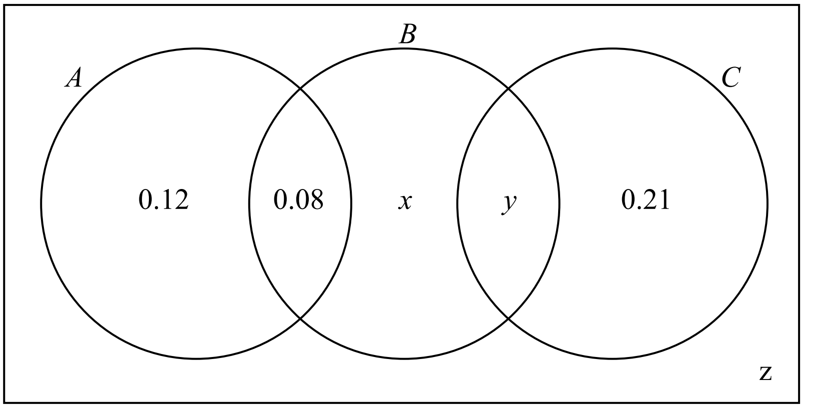
$= 0.3447$

(d) P(J fails on L3 and D fails on L2) $= (0.9 \times 0.7 \times (1 - 0.3)) \times (0.8 \times (1 - 0.6))$

$= 0.1411$

(Total for Question 5 is 7 marks)

6 The Venn diagram shows the events A, B and C and their associated probabilities.



Events A and B are independent.
Events B and C are independent.

Find the values of x, y and z **(5)**

$P(A) = 0.12 + 0.08 = 0.2$ $\quad$ $P(A \text{ and } B) = 0.08$

$P(B) = 0.08 + x + y$ $\quad$ $P(B \text{ and } C) = y$

$P(C) = 0.21 + y$

$P(A) \times P(B) = P(A \text{ and } B)$

$(0.2) \times (0.08 + x + y) = 0.08$

$(0.08 + x + y) = \frac{0.08}{0.2}$

$(0.08 + x + y) = 0.4$

$P(B) \times P(C) = P(B \text{ and } C)$

$(0.08 + x + y) \times (0.21 + y) = y$ Substitute

$0.4 \times (0.21 + y) = y$

$0.084 + 0.4y = y$

$0.084 = 0.6y$

$y = \frac{0.084}{0.6}$

$\mathbf{y = 0.14}$

$(0.2) \times (0.08 + x + y) = 0.08$

$(0.2) \times (0.08 + x + 0.14) = 0.08$

$(0.2) \times (x + 0.22) = 0.08$

$0.2x + 0.044 = 0.08$

$0.2x = 0.036$

$\mathbf{x = 0.18}$

$0.12 + 0.08 + x + y + 0.21 + z = 1$

$0.12 + 0.08 + 0.18 + 0.14 + 0.21 + z = 1$

$\mathbf{z = 0.27}$

(Total for Question 6 is 5 marks)

7 A bag contains 120 counters that are

either red, blue or green in colour
either large, medium or small in size

	Large	**Medium**	**Small**
Red	15	16	49
Blue	7	6	19
Green	1	2	5

Assume one counter is selected at random from the bag.

(a) Determine if the follow pairs of events are independent
(i) "The counter selected is green" and "the counter selected is small" **(2)**
(ii) "The counter selected is red" and "the counter selected is medium" **(2)**

Instead, assume that two counters are selected at random from the bag without replacement.

(b) Find
(i) the probability that both counters selected are the same colour. **(2)**
(ii) the probability that neither of the counters selected are large. **(1)**
(iii) the probability that at least one of the counters selected is red and small. **(2)**

(a) (i) P(green) = $\frac{8}{120}$ P(green and small) = $\frac{5}{120}$

P(small) = $\frac{73}{120}$

P(green) × P(small) = $\frac{8}{120} \times \frac{73}{120} = \frac{73}{1800}$ $\frac{73}{1800} \neq \frac{5}{120}$

P(green) × P(small) ≠ P(green and small) therefore they are not independent.

(ii) P(red) = $\frac{80}{120}$ P(red and medium) = $\frac{16}{120}$

P(medium) = $\frac{24}{120}$ $= \frac{2}{15}$

P(red) × P(medium) = $\frac{80}{120} \times \frac{24}{120} = \frac{2}{15}$

P(red) × P(medium) = P(red and medium) therefore they are independent.

(b) (i) P(same colour) = P(RR) + P(BB) + P(GG)

$= \left(\frac{80}{120} \times \frac{79}{120}\right) + \left(\frac{32}{120} \times \frac{31}{120}\right) + \left(\frac{8}{120} \times \frac{7}{120}\right) = \frac{307}{600}$ (or 0.5116… etc)

(ii) P(no large) = $\left(\frac{97}{120} \times \frac{96}{120}\right) = \frac{97}{150}$ (or 0.6466… etc)

(iii) P(at least one red and small) = 1 – P(no red and small)

$= 1 - \left(\frac{71}{120} \times \frac{70}{120}\right) = \frac{497}{1440}$ (or 0.3451… etc)

(Total for Question 7 is 9 marks)

8 Polly wanted to investigate if people in her town voted in a local election.
To collect data Polly stood in the town centre and sampled the first 600 people that she met.

(a) Name the type of sampling method used by Polly. **(1)**

The table below shows the data collected by Polly.

	Below 18	18 – 30	Over 30
Voted	0	64	176
Did not vote	78	96	186

(b) Work out, to 1 decimal place, the percentage of those aged over 30 that voted. **(1)**

Polly selects one person at random from her sample to conduct an interview with.

V is the event that the person selected voted.
D is the event that the person selected did not vote.
X is the event that the person selected is below 18 years old.
Y is the event that the person selected is between 18 and 30 years old.
Z is the event that the person selected is over 30 years old.

(c) Determine, by calculation, if the events V and Y are independent. **(2)**

(d) Write down all pairs of mutually exclusive events from V, D, X, Y and Z. **(3)**

(a) Opportunity sampling.

(b) $\frac{176}{362} \times 100 = 48.6\%$

(c) $P(V) = \frac{240}{600}$

$P(Y) = \frac{160}{600}$

$P(V) \times P(Y) = \frac{240}{600} \times \frac{160}{600} = \frac{8}{75}$

$P(V \text{ and } Y) = \frac{64}{600} = \frac{8}{75}$ $P(V) \times P(Y) = P(V \text{ and } Y)$ therefore P and L are independent.

(d) V and D

X and Y

Y and Z

X and Z

V and X

(Total for Question 8 is 7 marks)

9 A bag contains 16 red dice, 5 blue dice and 3 green dice.

Each red die is fair and 4-sided, with faces numbered 1, 2, 5 and 6
Each blue die is fair and 5-sided, with faces numbered 1, 2, 3, 4 and 5
Each green die is fair and 3-sided, with faces numbered 1, 2 and 3

One of the dice is selected at random and then rolled.

R is the event the dice selected is red.
X is the event the number rolled is 2.

(a) Show that events *R* and *X* are independent. **(3)**

The first die is not returned to the bag.
A second die is selected at random and then rolled.

(b) Work out the probability that the total of both rolls is 11. **(3)**
Give your answer as a fraction in its simplest form.

(a) $P(R) = \frac{16}{24}$

$P(X) = P(\text{Red and } 2) + P(\text{Blue and } 2) + P(\text{Green and } 2)$

$= \left(\frac{16}{24} \times \frac{1}{4}\right) + \left(\frac{5}{24} \times \frac{1}{5}\right) + \left(\frac{3}{24} \times \frac{1}{3}\right) = \frac{1}{4}$

$P(R) \times P(X) = \frac{16}{24} \times \frac{1}{4} = \frac{1}{6}$ $P(R \text{ and } X) = \frac{16}{24} \times \frac{1}{4} = \frac{1}{6}$

$P(R) \times P(X) = P(R \text{ and } X)$ therefore they are independent.

(b) P(roll is 11) = P(red 5 and red 6) + P(red 6 and red 5) + P(blue 5 and red 6) + P(red 6 and blue 5)

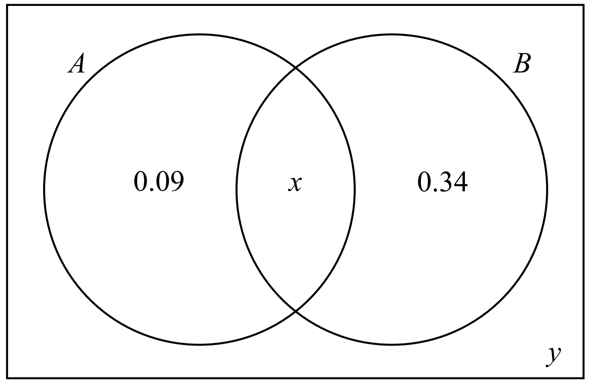
$= \left(\frac{16}{24} \times \frac{1}{4} \times \frac{15}{23} \times \frac{1}{4}\right) + \left(\frac{15}{24} \times \frac{1}{4} \times \frac{16}{23} \times \frac{1}{4}\right) + \left(\frac{5}{24} \times \frac{1}{5} \times \frac{16}{23} \times \frac{1}{4}\right) + \left(\frac{16}{24} \times \frac{1}{4} \times \frac{5}{23} \times \frac{1}{5}\right)$

$= \frac{5}{184} + \frac{5}{184} + \frac{1}{138} + \frac{1}{138}$

$= \frac{19}{276}$

(Total for Question 9 is 6 marks)

10 The Venn diagram shows the events A and B and their associated probabilities.



Events A and B are independent.

Give that $x > y$, find the value of x and y. **(4)**

$P(A) = 0.09 + x$

$P(B) = 0.34 + x$

$P(A \text{ and } B) = x$

$P(A) \times P(B) = P(A \text{ and } B)$

$(0.09 + x) \times (0.34 + x) = x$

$0.0306 + 0.43x + x^2 = x$

$x^2 - 0.57x + 0.0306 = 0$

$x = 0.51 \quad x = 0.06$ (from calculator)

$0.09 + x + 0.34 + y = 1$

$y = 1 - 0.09 - 0.34 - x$

$y = 1 - 0.09 - 0.34 - 0.51$

$y = 0.06$

$y = 1 - 0.09 - 0.34 - x$

$y = 1 - 0.09 - 0.34 - 0.06$

$y = 0.51$

(Disregard as $y > x$)

Answer: $x = 0.51$ and $y = 0.06$

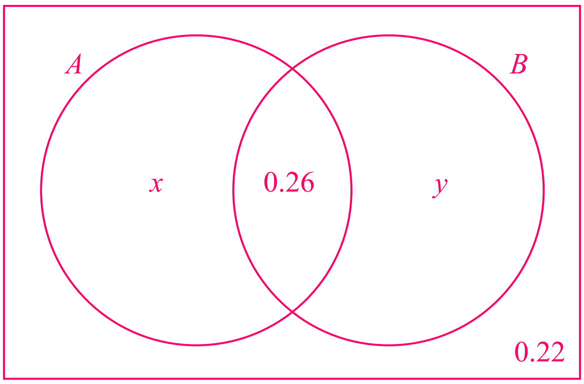
(Total for Question 10 is 4 marks)

11 P(A or B or both) = 3 × P(A and B)

P(A) = 0.6 × P(B)

P(not A and not B) = 0.22

Show that events A and B are not independent. **(5)**



P(A or B or both) = 1 – P(not A and not B)

= 1 – 0.22

= 0.78

P(A or B or both) = 3 × P(A and B)

0.78 = 3 × P(A and B)

P(A and B) = 0.26

$x + y = 1 – 0.26 – 0.22$ P(A) = x + 0.26

$x + y = 0.52$ P(B) = y + 0.26

P(A) = 0.6P(B)

$x + 0.26 = 0.6(y + 0.26)$

$x + 0.26 = 0.6y + 0.156$

$x – 0.6y = -0.104$

$\left.\begin{array}{l} x + y = 0.52 \\ x - 0.6y = -0.104 \end{array}\right\}$ $x = 0.13$ (from calculator), $y = 0.39$

P(A) = x + 0.26 P(B) = y + 0.26

= 0.13 + 0.26 = 0.39 + 0.26

= 0.39 = 0.65

P(A) × P(B) = 0.39 × 065

= 0.2535

P(A and B) = 0.26

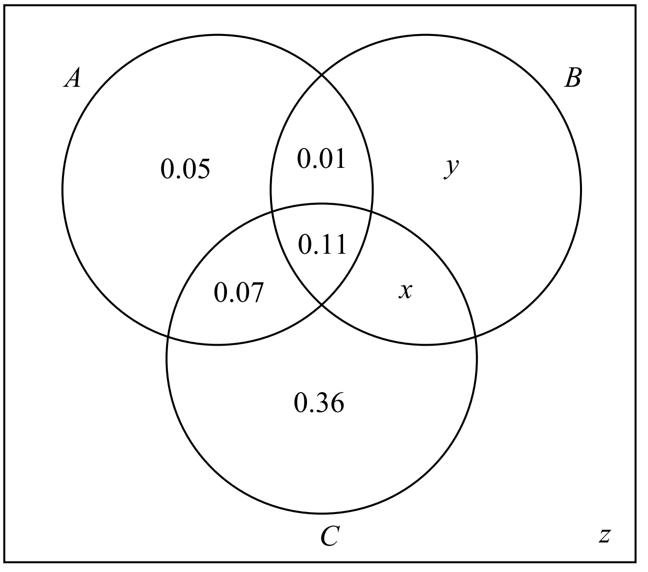
0.2535 ≠ 0.26

P(A) × P(B) ≠ P(A and B)

therefore A and B are not independent.

(Total for Question 11 is 5 marks)

12 The Venn diagram shows the events A, B and C and their associated probabilities.



Events A and B are independent.
Events B and C are independent.
Events A and C are independent.

Find the values of x, y and z **(5)**

$P(A) = 0.05 + 0.01 + 0.07 + 0.11$

$= 0.24$

$P(B) = 0.01 + 0.11 + x + y$

$= 0.12 + x + y$

$P(C) = 0.07 + 0.11 + 0.36 + x$

$= 0.54 + x$

$P(A \text{ and } B) = 0.11 + 0.01$

$= 0.12$

$P(A \text{ and } C) = 0.11 + 0.07$

$= 0.18$

$P(A) \times P(C) = P(A \text{ and } C)$

$0.24(0.54 + x) = 0.18$

$0.1296 + 0.24x = 0.18$

$0.24x = 0.0504$

$x = 0.21$

$P(A) \times P(B) = P(A \text{ and } B)$

$0.24(0.12 + x + y) = 0.12$

$0.24(0.12 + 0.21 + y) = 0.12$

$0.24(0.33 + y) = 0.12$

$0.0792 + 0.24y = 0.12$

$0.24y = 0.0408$

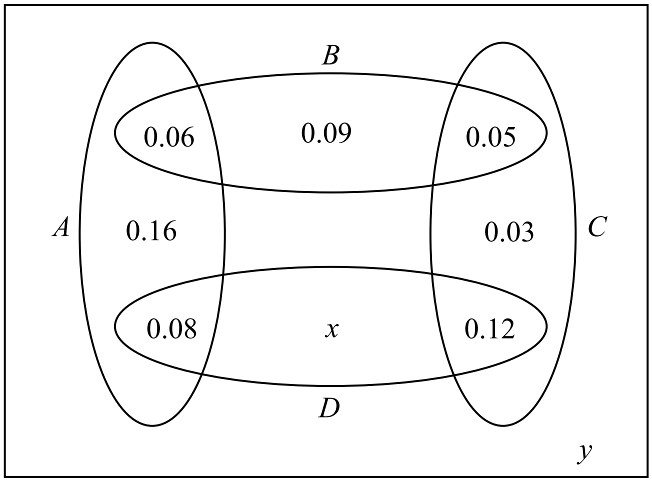
$y = 0.17$

$z = 1 - 0.05 - 0.07 - 0.01 - 0.11 - 0.36 - 0.21 - 0.17$

$z = 0.02$

(Total for Question 12 is 5 marks)

13 The Venn diagram shows the events A, B, C and D and their associated probabilities.



(a) Write down all pairs of mutually exclusive events from A, B, C and D. **(1)**

(b) Show that events A and B are independent. **(1)**

(c) Show that events B and C are not independent. **(1)**

Events C and D are independent.

(d) Work out the values of x and y. **(3)**

(a) (A and C) (B and D)	P(B) × P(C) = 0.2 × 0.2 = 0.04
	P(B and C) = 0.05
(b) P(A) = 0.06 + 0.16 + 0.08	P(B) × P(C) ≠ P(B and C)
= 0.3	therefore B and C are not independent.
P(B) = 0.06 + 0.09 + 0.05	
= 0.2	(d) P(C) = 0.2
	P(D) = 0.08 + 0.12 + x
P(A) × P(B) = 0.3 × 0.2 = 0.06	= 0.2 + x
P(A and B) = 0.06	P(C and D) = 0.12
P(A) × P(B) = P(A and B)	
therefore A and B are independent.	P(C) × P(D) = P(C and D)
	0.2 × (0.2 + x) = 0.12
(c) P(B) = 0.06 + 0.09 + 0.05	0.2 + x = 0.6
= 0.2	x = 0.4
P(C) = 0.05 + 0.03 + 0.12	y = 1 – 0.06 – 0.16 – 0.08 – 0.4 – 0.12 – 0.03 – 0.05 – 0.09
= 0.2	y = 0.01

(Total for Question 13 is 6 marks)