



Composite Functions



REVISE THIS TOPIC

1 $f(x) = 3x + 4$

$g(x) = x + 10$

$h(x) = x^2$



(a) Work out $fg(x)$

Give your answer in the form $ax + b$ where a and b are integers

$$\begin{aligned}
 fg(x) &= 3(x+10) + 4 \\
 &= 3x + 30 + 4 \\
 &= 3x + 34
 \end{aligned}$$

$$fg(x) = \frac{3x + 34}{(2)}$$

(b) Work out $gf(x)$

Give your answer in the form $ax + b$ where a and b are integers

$$\begin{aligned}
 gf(x) &= 3x + 4 + 10 \\
 &= 3x + 14
 \end{aligned}$$

$$gf(x) = \frac{3x + 14}{(2)}$$

(a) Work out $gh(x)$

$$gh(x) = x^2 + 10$$

$$gh(x) = \frac{x^2 + 10}{(1)}$$

(Total for Question 1 is 5 marks)



2 $f(x) = x - 3$

$g(x) = x^2 + 1$

$h(x) = 10x$



- (a) Work out $fg(x)$
Fully simplify your answer.

$$\begin{aligned}
 fg(x) &= x^2 + 1 - 3 \\
 &= x^2 - 2
 \end{aligned}$$

$$fg(x) = \frac{x^2 - 2}{(2)}$$

- (b) Work out $hg(x)$
Fully simplify your answer.

$$\begin{aligned}
 hg(x) &= 10(x^2 + 1) \\
 &= 10x^2 + 10
 \end{aligned}$$

$$hg(x) = \frac{10x^2 + 10}{(2)}$$

- (c) Work out $gh(x)$
Fully simplify your answer.

$$\begin{aligned}
 gh(x) &= (10x)^2 + 1 \\
 &= 100x^2 + 1
 \end{aligned}$$

$$gh(x) = \frac{100x^2 + 1}{(2)}$$

(Total for Question 2 is 6 marks)



3 $f(x) = \frac{x}{4}$

$g(x) = 4x - 8$

$h(x) = \sqrt{x}$



- (a) Work out $fg(x)$
Fully simplify your answer.

$$fg(x) = \frac{4x-8}{4}$$

$$= 2x-2$$

$fg(x) = \underline{2x-2}$
(2)

- (b) Work out $gf(x)$
Fully simplify your answer.

$$gf(x) = 4\left(\frac{x}{4}\right) - 8$$

$$= x - 8$$

$gf(x) = \underline{x-8}$
(2)

- (c) Work out $hf(x)$
Fully simplify your answer.

$$hf(x) = \sqrt{\frac{x}{4}}$$

$$= \frac{\sqrt{x}}{\sqrt{4}}$$

$hf(x) = \underline{\frac{\sqrt{x}}{2}}$
(2)

(Total for Question 3 is 6 marks)





4 $f(x) = x - 5$

$g(x) = x^2 + 30$

(a) Work out $fg(x)$

Fully simplify your answer.

$$\begin{aligned}
 fg(x) &= x^2 + 30 - 5 \\
 &= x^2 + 25
 \end{aligned}$$

$$fg(x) = \frac{x^2 + 25}{(2)}$$

(b) Work out $fg(3)$

$$\begin{aligned}
 fg(x) &= x^2 + 25 \\
 fg(3) &= 3^2 + 25 \\
 &= 9 + 25
 \end{aligned}$$

$$\frac{34}{(2)}$$

(c) Work out $gf(x)$

Give your answer in the form $ax^2 + bx + c$ where a , b and c are integers.

$$\begin{aligned}
 gf(x) &= (x - 5)^2 + 30 \\
 &= (x - 5)(x - 5) + 30 \\
 &= x^2 - 5x - 5x + 25 + 30 \\
 &= x^2 - 10x + 55
 \end{aligned}$$

$$gf(x) = \frac{x^2 - 10x + 55}{(3)}$$

(Total for Question 4 is 7 marks)





5 $f(x) = 2x + 1$

$g(x) = \sqrt{x + 3}$

(a) Work out $g(13)$

$$g(13) = \sqrt{13 + 3}$$

$$= \sqrt{16}$$

4

(1)

(b) Work out $fg(13)$

$$fg(13) = f(4)$$

$$= 2(4) + 1$$

9

(1)

(c) Work out $gf(16)$

$$gf(x) = \sqrt{2x + 1 + 3}$$

$$= \sqrt{2x + 4}$$

$$gf(16) = \sqrt{2(16) + 4}$$

$$= \sqrt{36}$$

6

(2)

(Total for Question 5 is 4 marks)



6 $f(x) = x + 2$

$g(x) = x^3$

$h(x) = \sqrt{x}$



(a) Work out $gf(3)$

$$\begin{aligned}
 gf(x) &= (x+2)^3 \\
 gf(3) &= (3+2)^3 \\
 &= 5^3
 \end{aligned}$$

$$\begin{array}{r}
 125 \\
 \hline
 (2)
 \end{array}$$

(b) Work out $gh(x)$

Give your answer in the form x^k where k is a fraction.

$$\begin{aligned}
 gh(x) &= (\sqrt{x})^3 \\
 &= (x^{1/2})^3 \\
 &= x^{3/2}
 \end{aligned}$$

$$gh(x) = \frac{x^{3/2}}{(2)}$$

(c) Work out $gf(x)$

Give your answer in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

$$\begin{aligned}
 gf(x) &= (x+2)^3 \\
 &= (x+2)(x+2)(x+2) \\
 &= (x^2 + 2x + 2x + 4)(x+2) \\
 &= (x^2 + 4x + 4)(x+2) \\
 &= x^3 + 2x^2 + 4x^2 + 8x + 4x + 8
 \end{aligned}$$

$$gf(x) = \frac{x^3 + 6x^2 + 12x + 8}{(3)}$$

(Total for Question 6 is 7 marks)



7 $f(x) = 2^x$

$g(x) = 1 - x$

$h(x) = 2 + x$



(a) Work out $gf(-3)$

$$gf(x) = 1 - 2^x$$

$$gf(-3) = 1 - 2^{-3}$$

$$2^{-3} = \frac{1}{2^3}$$

$$1 - \frac{1}{8}$$

$$\frac{7}{8}$$

(2)

$hg(x) - gh(x) = k$ where k is an integer.

(b) Find the value of k .

$$hg(x) = 2 + (1 - x)$$

$$= 2 + 1 - x$$

$$= 3 - x$$

$$gh(x) = 1 - (2 + x)$$

$$= 1 - 2 - x$$

$$= -1 - x$$

$$hg(x) - gh(x) = (3 - x) - (-1 - x)$$

$$= 3 - x + 1 + x$$

$$k = 4$$

(4)

(c) Show that $\frac{fh(x)}{fg(x)} = 2^{ax+b}$ where a and b are integers.

$$fh(x) = 2^{2+x}$$

$$fg(x) = 2^{1-x}$$

$$\frac{fh(x)}{fg(x)} = \frac{2^{2+x}}{2^{1-x}}$$

$$= 2^{(2+x) - (1-x)}$$

$$= 2^{2+x-1+x}$$

$$= 2^{2x+1}$$

(3)

(Total for Question 7 is 9 marks)

