# Forces as Vectors

SCAN ME

REVISE THIS TOPIC

**1** [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

Two horizontal forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a particle $P$.

Given that $\mathbf{F}_1 = (4\mathbf{i} - 3\mathbf{j})$ N and $\mathbf{F}_2 = (-\mathbf{i} + 7\mathbf{j})$ N

(a) find, in terms of **i** and **j**, the **resultant force** acting on $P$. **(1)**

(b) find the **magnitude** of the **resultant force** acting on $P$. **(2)**

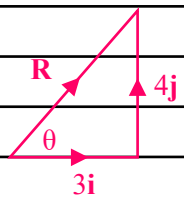
(c) find, to the nearest degree, the bearing of the **resultant force** acting on $P$. **(3)**

(a) $\mathbf{R} = (4\mathbf{i} - 3\mathbf{j}) + (-\mathbf{i} + 7\mathbf{j})$

$= (3\mathbf{i} + 4\mathbf{j})$ N

(b) $|\mathbf{R}| = \sqrt{3^2 + 4^2} = \mathbf{5}$ **N**

(c)




$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\theta = 53.13\ldots°$

Bearing $= 90 - 53.13\ldots = \mathbf{037°}$ (nearest degree)

**(Total for Question 1 is 6 marks)**

**2** [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle $P$ is in equilibrium under the action of three horizontal forces.

$\mathbf{F}_1 = (5\mathbf{i} - 7\mathbf{j})$ N, $\mathbf{F}_2 = (-3\mathbf{i} + 3\mathbf{j})$ N and $\mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})$ N, where $a$ and $b$ are constants.

(a) Find the values of $a$ and $b$. **(2)**

The force $\mathbf{F}_3$ is removed. Given that forces $\mathbf{F}_1$ and $\mathbf{F}_2$ continue to act on $P$,

(b) find the **magnitude** of the **resultant force, R,** acting on $P$, **(2)**

(c) find, to the nearest degree, the angle between the line of action of **R** and the vector **i**. **(2)**

(a) $(5\mathbf{i} - 7\mathbf{j}) + (-3\mathbf{i} + 3\mathbf{j}) + (a\mathbf{i} + b\mathbf{j}) = 0$

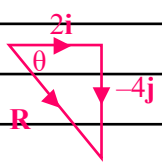
$5 - 3 + a = 0$ $\quad -7 + 3 + b = 0$

$\boldsymbol{a = -2}$ $\quad \boldsymbol{b = 4}$

(b) $\mathbf{R} = (5\mathbf{i} - 7\mathbf{j}) + (-3\mathbf{i} + 3\mathbf{j}) = (2\mathbf{i} - 4\mathbf{j})$

$|\mathbf{R}| = \sqrt{2^2 + (-4)^2} = \sqrt{\mathbf{20}}$ **N** [OR $2\sqrt{5}$ N OR 4.47 N]

(c)




$\theta = \tan^{-1}\left(\frac{4}{2}\right)$

$\theta = 63.4\ldots°$

Angle $= \mathbf{63°}$ (nearest degree)

**(Total for Question 2 is 6 marks)**



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**3** [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

Two horizontal forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a particle $P$.

$\mathbf{F}_1 = (10\mathbf{i} + 5\mathbf{j})$ N and $\mathbf{F}_2 = (-5\mathbf{i} + 7\mathbf{j})$ N

(a) Find, to the nearest degree, the angle between the line of action of $\mathbf{F}_1$ and the vector **i**. **(2)**

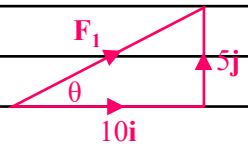
(b) Find the **magnitude** of the **resultant force** acting on $P$. **(3)**

Two horizontal forces, $\mathbf{F}_3$ and $\mathbf{F}_4$, act on a particle $Q$.

$\mathbf{F}_3 = (3\mathbf{i} - 3\mathbf{j})$ N and $\mathbf{F}_4 = (a\mathbf{i} + 8\mathbf{j})$ N, where $a$ is a constant.

(c) Given that the magnitude of the resultant force acting on $Q$ is equal to the magnitude of the resultant force acting on $P$, find the two possible values of $a$. **(3)**

(a)




$\theta = \tan^{-1}\left(\frac{5}{10}\right)$

$\theta = 26.5\ldots°$

Angle = **27°**

(b) $\mathbf{R} = (10\mathbf{i} + 5\mathbf{j}) + (-5\mathbf{i} + 7\mathbf{j}) = (5\mathbf{i} + 12\mathbf{j})$ N

$|\mathbf{R}| = \sqrt{5^2 + 12^2} = \mathbf{13\ N}$

(c) $\mathbf{R}_2 = (3\mathbf{i} - 3\mathbf{j}) + (a\mathbf{i} + 8\mathbf{j}) = (3 + a)\mathbf{i} + 5\mathbf{j}$

$|\mathbf{R}_2| = \sqrt{(3 + a)^2 + 5^2}$

$|\mathbf{R}_2| = 13$ (same as for $P$)

$\sqrt{(3 + a)^2 + 5^2} = 13$

$(3 + a)^2 + 5^2 = 169$

$(3 + a)^2 = 169 - 5^2$

$(3 + a)^2 = 144$

$3 + a = \pm 12$

$a = -3 \pm 12$

$a = 9$ $\quad$ $a = -15$

**(Total for Question 3 is 8 marks)**



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**4** [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle $P$ is in equilibrium under the action of three horizontal forces.

$\mathbf{F}_1 = (-8\mathbf{i} + 3\mathbf{j})$ N, $\mathbf{F}_2 = (b\mathbf{i} - a\mathbf{j})$ N and $\mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})$ N, where $a$ and $b$ are constants.

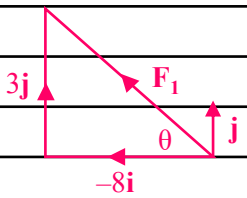
(a) Find $|\mathbf{F}_1|$ **(2)**

(b) Find, to the nearest degree, the acute angle between the line of action of force $\mathbf{F}_1$ and the vector $\mathbf{j}$. **(3)**

(c) Find the values of $a$ and $b$. **(3)**

(a) $|\mathbf{F}_1| = \sqrt{(-8)^2 + 3^2} = \sqrt{73}$ **N** [OR 8.54 N]

(b)




$\theta = \tan^{-1}\left(\frac{3}{8}\right)$

$\theta = 20.556\ldots°$

Angle $= 90 - 20.556\ldots° =$ **69°** (nearest degree)

(c) $(-8\mathbf{i} + 3\mathbf{j}) + (b\mathbf{i} - a\mathbf{j}) + (a\mathbf{i} + b\mathbf{j}) = 0$

$-8 + b + a = 0$ $\quad 3 - a + b = 0$

$a + b = 8$ $\quad a - b = 3$

$a + b = 8$
$a - b = 3$
} Solving simultaneous equations on calculator gives $a = 5.5$, $b = 2.5$

**(Total for Question 4 is 8 marks)**



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**5** [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle $P$ of mass 4 kg moves on a smooth horizontal plane under the action of two horizontal forces $\mathbf{F}_1$ and $\mathbf{F}_2$.

$\mathbf{F}_1 = (5\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (-\mathbf{i} + 4\mathbf{j})$ N

(a) Find, in terms of **i** and **j**, the **resultant force** acting on $P$. **(1)**

(b) Find, in terms of **i** and **j**, the acceleration of $P$. **(2)**

(c) Find the **magnitude** of the acceleration of $P$. **(2)**

(d) Given that $P$ starts from rest, calculate the distance travelled by $P$ in the first 9 seconds. **(2)**

(a) $\mathbf{R} = (5\mathbf{i} + 2\mathbf{j}) + (-\mathbf{i} + 4\mathbf{j})$

$= (4\mathbf{i} + 6\mathbf{j})$ N

(b) $\mathbf{F} = m\boldsymbol{a}$

$(4\mathbf{i} + 6\mathbf{j}) = 4\boldsymbol{a}$

$(\mathbf{i} + \frac{3}{2}\mathbf{j}) = \boldsymbol{a}$

$\boldsymbol{a} = (\mathbf{i} + \frac{3}{2}\mathbf{j})$ **ms$^{-2}$**

(c) $|\boldsymbol{a}| = \sqrt{(1)^2 + \left(\frac{3}{2}\right)^2} = \frac{\sqrt{13}}{2}$ **ms$^{-2}$** [OR 1.80 ms$^{-2}$]

(d)

| | |
|---|---|
| $s = s$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 0$ | $s = (0)(9) + \frac{1}{2}(1.80\ldots)(9)^2$ |
| $v = v$ | $s =$ **73.0 m** |
| $a = 1.80\ldots$ | |
| $t = 9$ | |

**(Total for Question 5 is 7 marks)**



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**6** [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle $P$ of mass 3 kg moves on a smooth horizontal plane under the action of two horizontal forces $(7\mathbf{i} - 18\mathbf{j})$ N and $(c\mathbf{i} - 27\mathbf{j})$ N, where $c$ is a constant.

Given that the acceleration of $P$ is $(8\mathbf{i} - 15\mathbf{j})$ ms$^{-2}$

(a) Find the **magnitude** of the acceleration of $P$. **(2)**

(b) Find the magnitude of the **resultant force** acting on $P$. **(2)**

(c) Find the value of $c$. **(2)**

(d) Given that $P$ starts from rest, find the speed of $P$ after 5 seconds. **(2)**

(a) $|\boldsymbol{a}| = \sqrt{(8)^2 + (-15)^2} = \mathbf{17\ ms^{-2}}$

(b) $|\mathbf{R}| = m|\boldsymbol{a}|$

$|\mathbf{R}| = 3 \times 17$

$|\mathbf{R}| = \mathbf{51\ N}$

(c) $\mathbf{F} = m\boldsymbol{a}$

$(7\mathbf{i} - 18\mathbf{j}) + (c\mathbf{i} - 27\mathbf{j}) = 3(8\mathbf{i} - 15\mathbf{j})$

$(7 + c)\mathbf{i} - 45\mathbf{j} = 24\mathbf{i} - 45\mathbf{j}$

$7 + c = 24$

$c = 17$

(d) $s = s$ $\quad v = u + at$

$u = 0$ $\quad v = 0 + 17 \times 5$

$v = v$ $\quad v = 85\ \text{ms}^{-1}$

$a = 17$

$t = 5$

**(Total for Question 6 is 8 marks)**



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**7** [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A particle $P$ of mass 5 kg moves on a smooth horizontal plane under the action of two horizontal forces $\mathbf{F}_1 = (-8\mathbf{i} + 13\mathbf{j})$ N and $\mathbf{F}_2 = (18\mathbf{i} + 7\mathbf{j})$ N

(a) Find, to the nearest degree, the bearing of force $\mathbf{F}_1$. **(2)**

(b) Find, in terms of **i** and **j**, the acceleration of $P$. **(2)**

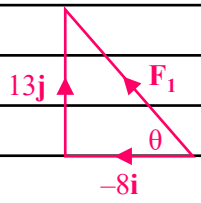
(c) Find the **magnitude** of the acceleration of $P$. **(2)**

Two more horizontal forces, $\mathbf{F}_3$ and $\mathbf{F}_4$, are now applied to $P$.

Under the action of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ and $\mathbf{F}_4$, the acceleration of $P$ remains unchanged.

(d) Given that $\mathbf{F}_3 = (6\mathbf{i} - 3\mathbf{j})$ N, find $\mathbf{F}_4$ in terms of **i** and **j** **(1)**

(a)




$\theta = \tan^{-1}\left(\frac{13}{8}\right)$

$\theta = 58.39\ldots°$

Bearing $= 270 + 58.39\ldots = \mathbf{328°}$ (nearest degree)

(b) $\mathbf{R} = (-8\mathbf{i} + 13\mathbf{j}) + (18\mathbf{i} + 7\mathbf{j}) = (10\mathbf{i} + 20\mathbf{j})$ N

$\mathbf{F} = m\boldsymbol{a}$

$(10\mathbf{i} + 20\mathbf{j}) = 5\boldsymbol{a}$

$(2\mathbf{i} + 4\mathbf{j}) = \boldsymbol{a}$

$\boldsymbol{a} = (2\mathbf{i} + 4\mathbf{j})$ **ms$^{-2}$**

(c) $|\boldsymbol{a}| = \sqrt{(2)^2 + (4)^2} = \sqrt{\mathbf{20}}$ **ms$^{-2}$** [OR $2\sqrt{5}$ **ms$^{-2}$** OR 4.47 **ms$^{-2}$**]

(d) If acceleration is unchanged then resultant force is unchanged.

So $\mathbf{F}_3 + \mathbf{F}_4 = 0$

$\mathbf{F}_4 = -\mathbf{F}_3$

$\mathbf{F}_4 = -(6\mathbf{i} - 3\mathbf{j})$ N

$\mathbf{F}_4 = (-6\mathbf{i} + 3\mathbf{j})$ **N**

**(Total for Question 7 is 7 marks)**



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**8** [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

Three horizontal forces, $\mathbf{F}_1$ , $\mathbf{F}_2$ and $\mathbf{F}_3$, act on a particle $P$.

$\mathbf{F}_1 = (5\mathbf{i} - 6\mathbf{j})$ N, $\mathbf{F}_2 = (4\mathbf{i} - 2\mathbf{j})$ N and $\mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})$ N

Given that **R**, the resultant force acting on $P$, is parallel to **i**, and $|\mathbf{F}_3| = 17$ N, find

(a) the value of $b$. **(2)**

(b) two possible values for $a$. **(3)**

(c) two possible values for the magnitude of **R**. **(2)**

(a) $\mathbf{R} = (5\mathbf{i} - 6\mathbf{j}) + (4\mathbf{i} - 2\mathbf{j}) + (a\mathbf{i} + b\mathbf{j})$

$= (9 + a)\mathbf{i} + (b - 8)\mathbf{j}$

$\mathbf{R} = k\mathbf{i}$ for some constant $k$.

So $b - 8 = 0$

$\boldsymbol{b = 8}$

(b) $\mathbf{F}_3 = (a\mathbf{i} + 8\mathbf{j})$

$|\mathbf{F}_3| = \sqrt{a^2 + 8^2}$ $\quad |\mathbf{F}_3| = 17$ (given in question)

$\sqrt{a^2 + 8^2} = 17$

$a^2 + 8^2 = 17^2$

$a^2 = 17^2 - 8^2$

$a^2 = 225$

$\boldsymbol{a = 15 \quad a = -15}$

(c) $\mathbf{R} = (9 + a)\mathbf{i}$

When $a = 15$, $\mathbf{R} = (9 + 15)\mathbf{i} = 24\mathbf{i}$, $|\mathbf{R}| = \mathbf{24}$ **N**

When $a = -15$, $\mathbf{R} = (9 - 15)\mathbf{i} = -6\mathbf{i}$, $|\mathbf{R}| = \mathbf{6}$ **N**

**(Total for Question 8 is 7 marks)**



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**9** [*In this question,* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A single horizontal force, **F**, acts on a particle $P$, of mass $\frac{2}{3}$ kg.

The magnitude of $\mathbf{F} = \sqrt{12}$ N

The magnitude of the acceleration of $P$ is $\sqrt{k}$ ms$^{-2}$, where $k$ is an integer.

(a) Find the value of $k$. **(3)**

(b) Given that $P$ starts from rest, find, to 3 significant figures the time taken for $P$ to reach a speed of 10 ms$^{-1}$ **(2)**

(c) Given that $\mathbf{F} = (a\mathbf{i} + b\mathbf{j})$ N and the bearing of **F** is 060°, find the values of $a$ and $b$. **(4)**

(a) $|\mathbf{F}| = m|\boldsymbol{a}|$

$\sqrt{12} = \frac{2}{3} \times \sqrt{k}$

$2\sqrt{3} = \frac{2}{3} \times \sqrt{k}$

$3\sqrt{3} = \sqrt{k}$

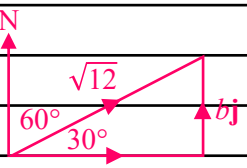
$\sqrt{27} = \sqrt{k}$

$\boldsymbol{k = 27}$

(b)

| | |
|---|---|
| $s = s$ | $v = u + at$ |
| $u = 0$ | $10 = 0 + \sqrt{27} \times t$ |
| $v = 10$ | $t = \frac{10}{\sqrt{27}}$ |
| $a = \sqrt{27}$ | |
| $t = t$ | $t =$ **1.92 seconds** |

(c)




$\sin(30°) = \frac{b}{\sqrt{12}}$ $\cos(30°) = \frac{a}{\sqrt{12}}$

| | |
|---|---|
| $b = \sqrt{12}\sin(30°)$ | $a = \sqrt{12}\cos(30°)$ |
| $b = 2\sqrt{3} \times 0.5$ | $a = 2\sqrt{3} \times \frac{\sqrt{3}}{2}$ |
| $\boldsymbol{b = \sqrt{3}}$ | $\boldsymbol{a = 3}$ |

**(Total for Question 9 is 9 marks)**



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**10** [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle $P$ of mass $m$ kg moves on a smooth horizontal plane under the action of two horizontal forces $\mathbf{F}_1 = (-8\mathbf{i} + 10\mathbf{j})$ N and $\mathbf{F}_2 = (-10\mathbf{i} + 14\mathbf{j})$ N

(a) Given that the magnitude of the acceleration of $P$ is 40 ms$^{-2}$, find $m$. **(4)**

(b) Find, in terms of **i** and **j**, the acceleration of $P$. **(2)**

A third horizontal force, $\mathbf{F}_3$ , is now applied to $P$.

Under the action of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, the particle moves with **constant velocity**.

(c) Find $\mathbf{F}_3$, giving your answer in terms of **i** and **j**. **(1)**

(a) $\mathbf{R} = (-8\mathbf{i} + 10\mathbf{j}) + (-10\mathbf{i} + 14\mathbf{j})$

$= (-18\mathbf{i} + 24\mathbf{j})$ N

$|\mathbf{R}| = \sqrt{(-18)^2 + 24^2} = 30$ N

$|\mathbf{R}| = m|\boldsymbol{a}|$

$30 = m \times 40$

$\boldsymbol{m = 0.75}$ **kg**

(b) $\mathbf{F} = m\boldsymbol{a}$

$-18\mathbf{i} + 24\mathbf{j} = 0.75\boldsymbol{a}$

$\boldsymbol{a} = (-24\mathbf{i} + 32\mathbf{j})$ **ms$^{-2}$**

(c) $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

$(-8\mathbf{i} + 10\mathbf{j}) + (-10\mathbf{i} + 14\mathbf{j}) + \mathbf{F}_3 = 0$

$-18\mathbf{i} + 24\mathbf{j} + \mathbf{F}_3 = 0$

$\mathbf{F}_3 = 18\mathbf{i} - 24\mathbf{j}$

**(Total for Question 10 is 7 marks)**



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**11** [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle $P$ of mass 0.5 kg moves on a smooth horizontal plane under the action of two horizontal forces $\mathbf{F}_1$ and $\mathbf{F}_2$

$\mathbf{F}_1 = (2\mathbf{i} - 4\mathbf{j})$ N
$\mathbf{F}_2$ acts in a direction that is parallel to the vector $(\mathbf{i} + \mathbf{j})$

Given that the magnitude of the acceleration of $P$ is $10\sqrt{2}$ ms$^{-2}$, find the two possible values of the magnitude of $\mathbf{F}_2$.

$\mathbf{F}_2 = k\mathbf{i} + k\mathbf{j}$

$\mathbf{R} = (2\mathbf{i} - 4\mathbf{j}) + (k\mathbf{i} + k\mathbf{j})$

$\mathbf{R} = (2 + k)\mathbf{i} + (k - 4)\mathbf{j}$

$|\mathbf{R}| = \sqrt{(2 + k)^2 + (k - 4)^2}$

$|\mathbf{R}| = m|\boldsymbol{a}|$

$\sqrt{(2 + k)^2 + (k - 4)^2} = 0.5 \times 10\sqrt{2}$

$\sqrt{(2 + k)^2 + (k - 4)^2} = 5\sqrt{2}$

$\sqrt{(2 + k)^2 + (k - 4)^2} = \sqrt{25} \times \sqrt{2}$

$\sqrt{(2 + k)^2 + (k - 4)^2} = \sqrt{50}$

$(2 + k)^2 + (k - 4)^2 = 50$

$k^2 + 4k + 4 + k^2 - 8k + 16 = 50$

$2k^2 - 4k + 20 = 50$

$2k^2 - 4k - 30 = 0$

$k^2 - 2k - 15 = 0$

$(k - 5)(k + 3) = 0$

$k = 5 \quad k = -3$

$\mathbf{F}_2 = 5\mathbf{i} + 5\mathbf{j}$ **or** $\mathbf{F}_2 = -3\mathbf{i} + -3\mathbf{j}$

$|\mathbf{F}_2| = \sqrt{5^2 + 5^2}$ $\quad$ $|\mathbf{F}_2| = \sqrt{(-3)^2 + (-3)^2}$

$|\mathbf{F}_2| = \sqrt{50}$ N $\quad$ $|\mathbf{F}_2| = \sqrt{18}$ N

$|\mathbf{F}_2| = \mathbf{5\sqrt{2}}$ **N** $\quad$ $|\mathbf{F}_2| = \mathbf{3\sqrt{2}}$ **N**

**(Total for Question 11 is 5 marks)**



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