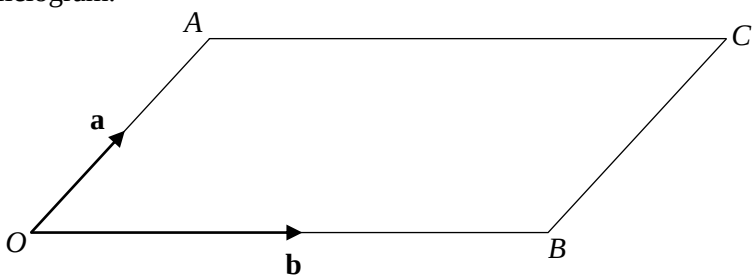
Vectors

SCAN ME

REVISE THIS TOPIC

1 *OACB* is a parallelogram.



$\overrightarrow{OA}$ = **a** $\overrightarrow{OB}$ = **b**

Work out the following vectors in terms of **a** and **b**.

(a) $\overrightarrow{AO}$

$-a$

(1)

(b) $\overrightarrow{BC}$

a

(1)

(c) $\overrightarrow{AB}$

$b-a$

(1)

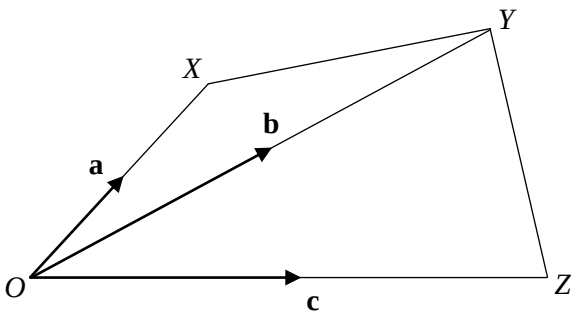
(d) $\overrightarrow{CO}$

$-a-b$

(1)

(Total for Question 1 is 4 marks)

2 *OXYZ* is a quadrilateral.



$\overrightarrow{OX} = \mathbf{a}$ $\overrightarrow{OY} = \mathbf{b}$ $\overrightarrow{OZ} = \mathbf{c}$

Work out the following vectors in terms of **a** and **b**.

(a) $\overrightarrow{ZO}$

$-c$

(1)

(b) $\overrightarrow{XY}$

$b - a$

(1)

(c) $\overrightarrow{ZY}$

$b - c$

(1)

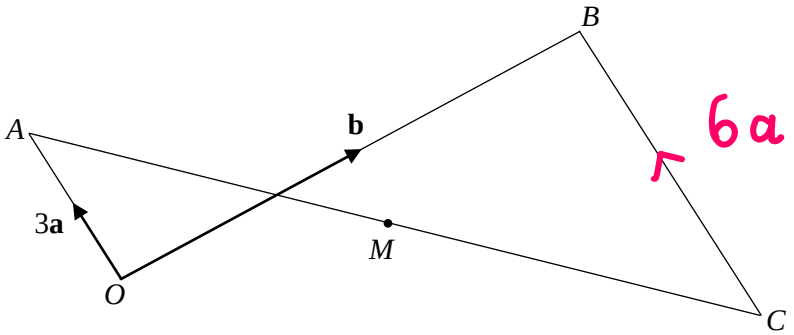
(d) $\overrightarrow{XZ}$

$c - a$

(1)

(Total for Question 2 is 4 marks)

3



$\overrightarrow{OA} = 3\mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$ $\overrightarrow{CB} = 2\overrightarrow{OA}$

Write the following vectors in terms of **a, b** and **c**.

Work out the following vectors in terms of **a** and **b**.

(a) $\overrightarrow{AB}$

$b - 3a$

(1)

(b) $\overrightarrow{CA}$

$\overrightarrow{CA} = \overrightarrow{CB} + \overrightarrow{BO} + \overrightarrow{OA}$

$= 6a - b + 3a$

$9a - b$

(2)

M is the midpoint of AC.

(c) Write $\overrightarrow{CM}$ in terms of **a** and **b**.

$\overrightarrow{CM} = \frac{1}{2}\overrightarrow{CA}$

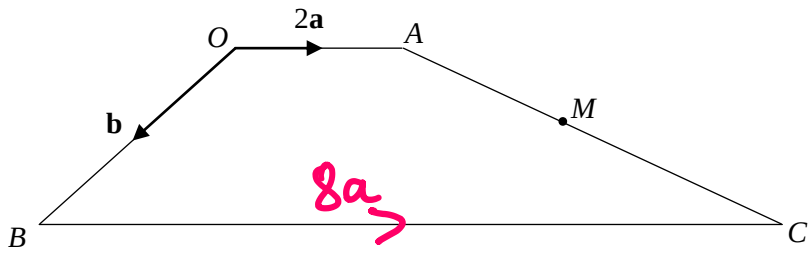
$= \frac{1}{2}(9a - b)$

$\frac{9}{2}a - \frac{1}{2}b$

(2)

(Total for Question 3 is 5 marks)

4 *OACB* is a trapezium



$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$ $\overrightarrow{BC} = 4\overrightarrow{OA}$

(a) Write $\overrightarrow{AC}$ in terms of **a** and **b**.

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{BC}$

$= -2\mathbf{a} + \mathbf{b} + 8\mathbf{a}$

$6\mathbf{a} + \mathbf{b}$

(2)

M is the midpoint of *AC*.

(b) Write $\overrightarrow{BM}$ in terms of **a** and **b**.

$\overrightarrow{BM} = \overrightarrow{BO} + \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$

$= -\mathbf{b} + 2\mathbf{a} + \frac{1}{2}(6\mathbf{a} + \mathbf{b})$

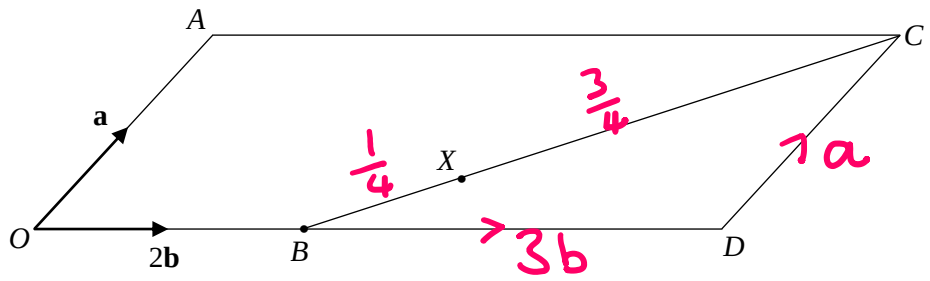
$= -\mathbf{b} + 2\mathbf{a} + 3\mathbf{a} + \frac{1}{2}\mathbf{b}$

$5\mathbf{a} - \frac{1}{2}\mathbf{b}$

(3)

(Total for Question 4 is 5 marks)

5 *OACD* is a parallelogram.



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = 2\mathbf{b}$ $\overrightarrow{OD} = 2.5\ \overrightarrow{OB}$

(a) Write $\overrightarrow{AD}$ in terms of **a** and **b**.

$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$

$= -a + 5b$

$5b - a$

(2)

(b) Write $\overrightarrow{BC}$ in terms of **a** and **b**.

$\overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{DC}$

$= 3b + a$

$3b + a$

(2)

$BX : XC = 1 : 3$

(c) Write $\overrightarrow{OX}$ in terms of **a** and **b**.

$\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{BX}$

$= 2b + \frac{1}{4}\overrightarrow{BC}$

$= 2b + \frac{1}{4}(3b + a)$

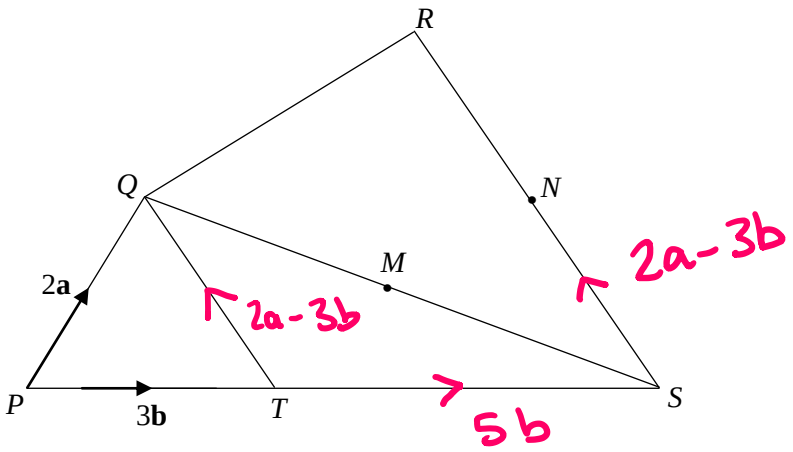
$= 2b + \frac{3}{4}b + \frac{1}{4}a$

$\frac{11}{4}b + \frac{1}{4}a$

(2)

(Total for Question 5 is 6 marks)

6 *PQRS* is a quadrilateral



$\overrightarrow{PQ} = 2\mathbf{a}$ $\overrightarrow{PT} = 3\mathbf{b}$ $\overrightarrow{RS} = 2\overrightarrow{QT}$

PTS is a straight line with $PT : TS = 3 : 5$

$\uparrow\ TS = 5b$

M is the midpoint of *QS*.
N is the midpoint of *RS*.

Write $\overrightarrow{MN}$ in term of **a** and **b**.

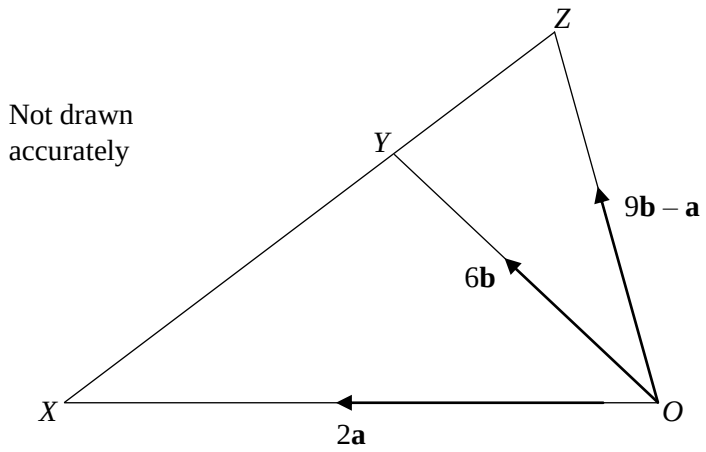
$\overrightarrow{QT} = 2a - 3b$ $\overrightarrow{RS} = 2(2a - 3b)$
$\overrightarrow{SN} = 2a - 3b$ $= 4a - 6b$

$\overrightarrow{QS} = \overrightarrow{QT} + \overrightarrow{TS}$
$= 3b - 2a + 5b$
$= 8b - 2a$
$\overrightarrow{MS} = \frac{1}{2}(8b - 2a)$
$= 4b - a$
$\overrightarrow{MN} = \overrightarrow{MS} + \overrightarrow{SN}$
$= 4b - a + 2a - 3b$

$a + b$

(Total for Question 6 is 4 marks)

7



Prove, using vectors, that *XYZ* is a straight line.

$\overrightarrow{XY} = 6b - 2a$

$\overrightarrow{XZ} = 9b - a - 2a$

$= 9b - 3a$

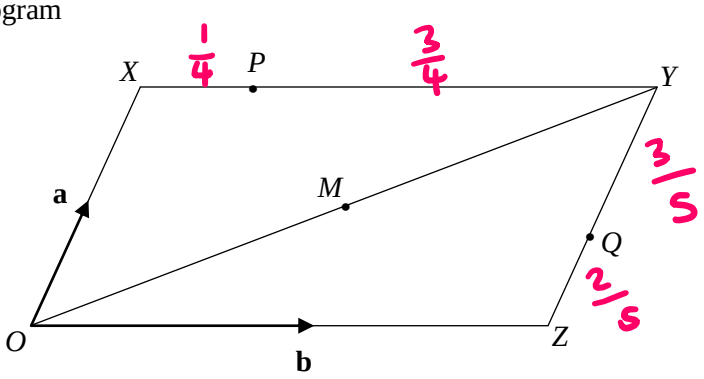
$= \frac{3}{2}(6b - 2a)$

$\overrightarrow{XZ} = \frac{3}{2}XY$

therefore XYZ is a straight line

(Total for Question 7 is 3 marks)

8 *OXYZ* is a parallelogram



$\overrightarrow{OX} = \mathbf{a}$ $\overrightarrow{OZ} = \mathbf{b}$

$XP : PY = 1 : 3$
$ZQ : QY = 2 : 3$

M is the midpoint of OY

(a) Write $\overrightarrow{PQ}$ in terms of **a** and **b**.

$\overrightarrow{PQ} = \overrightarrow{PY} + \overrightarrow{YQ}$
$= \frac{3}{4}b - \frac{3}{5}a$

$\frac{3}{4}b - \frac{3}{5}a$

(2)

(b) Write $\overrightarrow{MQ}$ in terms of **a** and **b**.

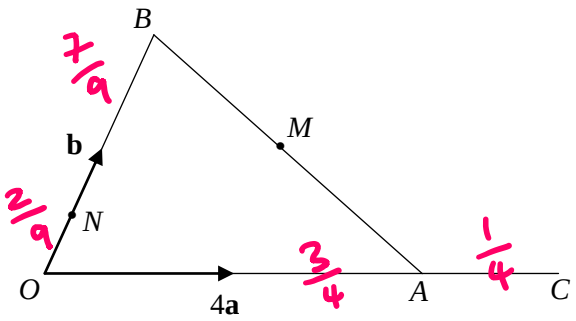
$\overrightarrow{MQ} = \overrightarrow{MY} + \overrightarrow{YQ}$
$= \frac{1}{2}(OY) + \overrightarrow{YQ}$
$= \frac{1}{2}(a+b) - \frac{3}{5}a$
$= \frac{1}{2}a + \frac{1}{2}b - \frac{3}{5}a$

$\frac{1}{2}b - \frac{1}{10}a$

(3)

(Total for Question 8 is 5 marks)

9



$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$OA : OC = 3 : 4$
$ON : OB = 2 : 9$

M is the midpoint of AB

(a) Write $\overrightarrow{MC}$ in terms of **a** and **b**.

$\overrightarrow{MC} = \overrightarrow{MA} + \overrightarrow{AC}$
$= \frac{1}{2}\overrightarrow{BA} + \overrightarrow{AC}$
$= \frac{1}{2}(4\mathbf{a} - \mathbf{b}) + \frac{4}{3}\mathbf{a}$
$= 2\mathbf{a} - \frac{1}{2}\mathbf{b} + \frac{4}{3}\mathbf{a}$

$\frac{10}{3}\mathbf{a} - \frac{1}{2}\mathbf{b}$

(3)

(b) Write $\overrightarrow{NM}$ in terms of **a** and **b**.

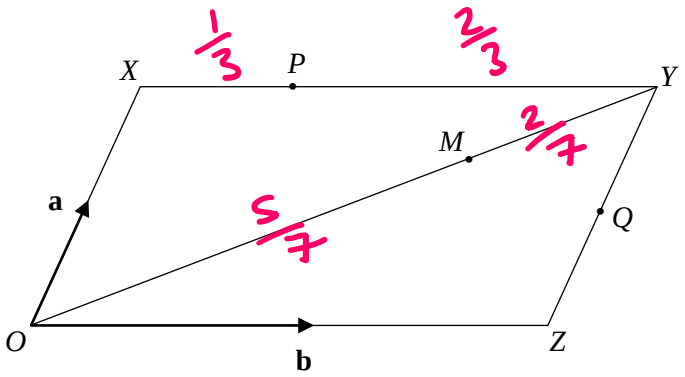
$\overrightarrow{NM} = \overrightarrow{NB} + \overrightarrow{BM}$
$= \frac{7}{9}\mathbf{b} + 2\mathbf{a} - \frac{1}{2}\mathbf{b}$

$\frac{5}{18}\mathbf{b} + 2\mathbf{a}$

(2)

(Total for Question 9 is 5 marks)

10 *OXYZ* is a parallelogram



$\overrightarrow{OX} = \mathbf{a}$ $\overrightarrow{OZ} = \mathbf{b}$

$ZQ = QY$
$XP : PY = 1 : 2$
$OM : MY = 5 : 2$

Prove, using vectors, that PMQ is a straight line.

$\overrightarrow{PM} = \overrightarrow{PY} + \overrightarrow{YM}$
$= \overrightarrow{PY} + \frac{2}{7}(\overrightarrow{YO})$
$= \frac{2}{3}b + \frac{2}{7}(-b-a)$
$= \frac{2}{3}b - \frac{2}{7}b - \frac{2}{7}a$
$= \frac{8}{21}b - \frac{2}{7}a$

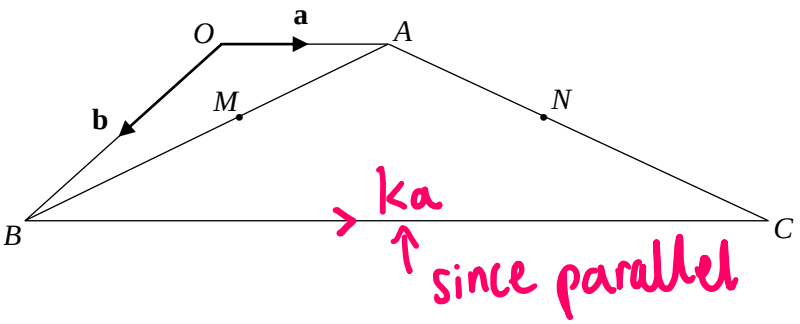
$\overrightarrow{PQ} = \overrightarrow{PY} + \overrightarrow{YQ}$
$= \frac{2}{3}b - \frac{1}{2}a$

$\overrightarrow{PQ} = \frac{7}{4}\overrightarrow{PM}$

therefore PMQ is a straight line

(Total for Question 10 is 4 marks)

11 *OACB* is a trapezium



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

M and *N* are the midpoints of *AB* and *AC*.

Prove, using vectors, that *MN* is parallel to *OA*.

$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN}$

$= \frac{1}{2}\overrightarrow{BA} + \frac{1}{2}\overrightarrow{AC}$

$= \frac{1}{2}(a - b) + \frac{1}{2}(-a + b + ka)$

$= \frac{1}{2}a - \frac{1}{2}b - \frac{1}{2}a + \frac{1}{2}b + \frac{k}{2}a$

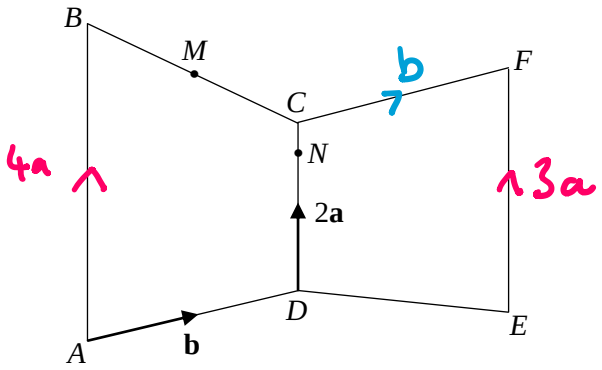
$= \frac{k}{2}a$

$\overrightarrow{MN} = \frac{k}{2}\overrightarrow{OA}$

therefore they are parallel

(Total for Question 11 is 4 marks)

12 *ABCD* and *CDEF* are trapeziums



$\overrightarrow{CD} = 2\mathbf{a}$ $\overrightarrow{AD} = \overrightarrow{CF} = \mathbf{b}$

$AB : DC : EF = 4 : 2 : 3$
M is the midpoint of BC.
N is on the line CD.

MNE is a straight line.

$DN : NC = k : 1$, where k is an integer.

Work out the value of k.

$\overrightarrow{MN} = \overrightarrow{MC} + \overrightarrow{CN}$
$= \frac{1}{2}(\overrightarrow{BC}) + \overrightarrow{CN}$
$= \frac{1}{2}(-4\mathbf{a} + \mathbf{b} + 2\mathbf{a}) - \alpha(2\mathbf{a})$
$= \frac{1}{2}\mathbf{b} - \mathbf{a} - 2\alpha\mathbf{a}$
$= \frac{1}{2}\mathbf{b} - (2\alpha + 1)\mathbf{a}$

$\overrightarrow{NE} = \overrightarrow{NC} + \overrightarrow{CF} + \overrightarrow{FE}$
$= \alpha(2\mathbf{a}) + \mathbf{b} - 3\mathbf{a}$
$= \mathbf{b} - (3 - 2\alpha)\mathbf{a}$

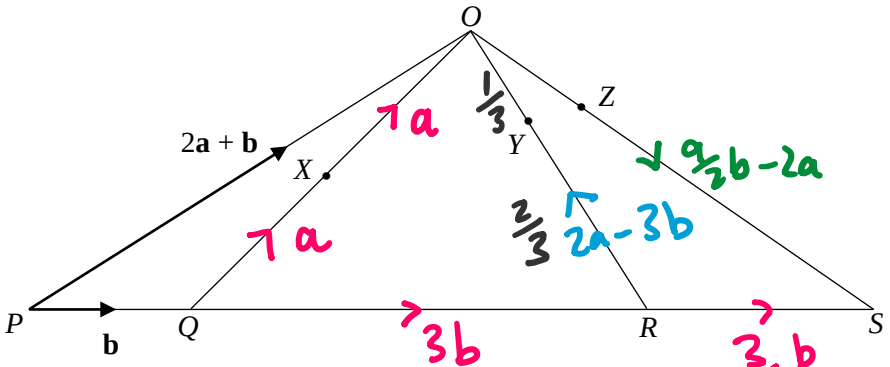
$\overrightarrow{NE} = 2\overrightarrow{MN}$
$3 - 2\alpha = 2(2\alpha + 1)$
$3 - 2\alpha = 4\alpha + 2$
$\alpha = \frac{1}{6}$

so $NC = \frac{1}{6}DC$
$DC : NC = 6 : 1$
$DN : NC = 5 : 1$

$k =$ 5

(Total for Question 12 is 5 marks)

13 *POS* is a triangle.



$\overrightarrow{PQ} = \mathbf{b}$ $\overrightarrow{PO} = 2\mathbf{a} + \mathbf{b}$

X is the midpoint of QO

$OY : YR = 1 : 2$

$PQ : QR : RS = 2 : 6 : 3$

XYZ is a straight line.

$OZ : OS = 1 : k$

Work out the value of k.

$\overrightarrow{RO} = \overrightarrow{RQ} + \overrightarrow{QO}$
$= -3b + 2a$
$= 2a - 3b$

$\overrightarrow{OS} = \overrightarrow{OR} + \overrightarrow{RS}$
$= 3b - 2a + \frac{3}{2}b$
$= \frac{9}{2}b - 2a$

$\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY}$
$= a - \frac{1}{3}(2a - 3b)$
$= a - \frac{2}{3}a + b$
$= b + \frac{1}{3}a$

$\overrightarrow{XZ} = n(\overrightarrow{XY})$
$= n(b + \frac{1}{3}a)$
$= nb + \frac{n}{3}a$

$\overrightarrow{XZ} = \overrightarrow{XO} + \overrightarrow{OZ}$
$= a + \frac{1}{k}(\frac{9}{2}b - 2a)$
$= a + \frac{9}{2k}b - \frac{2}{k}a$
$= (1 - \frac{2}{k})a + \frac{9}{2k}b$

equate

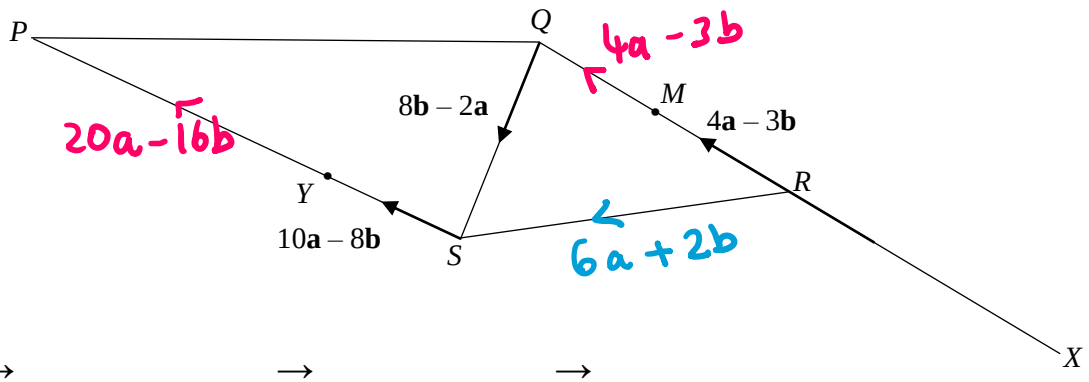
$1 - \frac{2}{k} = \frac{n}{3}$
$3 - \frac{6}{k} = n$
and $\frac{9}{2k} = n$

$3 - \frac{6}{k} = \frac{9}{2k}$
$3k - 6 = \frac{9}{2}$
$6k - 12 = 9$
$k = \frac{21}{6} = \frac{7}{2}$

$k =$ 3.5

(Total for Question 13 is 6 marks)

14 $PQRS$ is a quadrilateral.



$\overrightarrow{SY} = 10\mathbf{a} - 8\mathbf{b}$ $\overrightarrow{QS} = 8\mathbf{b} - 2\mathbf{a}$ $\overrightarrow{RM} = 4\mathbf{a} - 3\mathbf{b}$

$RM = MQ$

$SY : YP = 1 : 2$

QRX is a straight line.

XS is parallel to RP.

Work out $XS : RP$

Give your answer in the form $n : 1$

$\overrightarrow{RP} = \overrightarrow{RQ} + \overrightarrow{QS} + \overrightarrow{SP}$

$= 8a - 6b + 8b - 2a + 30a - 24b$

$= 36a - 22b$

$\overrightarrow{XS} = \overrightarrow{XR} + \overrightarrow{RS}$

$= k(4a - 3b) + 6a + 2b$

$= (4k + 6)a - (3k - 2)b$

$\overrightarrow{XS} = n\overrightarrow{RP}$

$= 36n\,a - 22n\,b$

$4k + 6 = 36n \quad (\times 3)$

$3k - 2 = 22n \quad (\times 4)$

$12k + 18 = 108n$

$12k - 8 = 88n$

$26 = 20n$

$n = \frac{26}{20}$

$n = \frac{13}{10}$

$1.3 : 1$

(Total for Question 14 is 6 marks)