Similar Areas/Volumes

REVISE THIS
TOPIC

1 Quadrilaterals A and B are similar.


The area of quadrilateral $\mathbf{A}$ is $32 \mathrm{~cm}^{2}$
Work out the area of quadrilateral B.
$A \rightarrow B$ Length scale factor $=16 \div 8$

$$
=2
$$

Area scale factor $=2^{2}$

$$
=4
$$

$32 \times 4=128$

Prisms $\mathbf{A}$ and $\mathbf{B}$ are similar.


The volume of prism $\mathbf{A}$ is $7 \mathrm{~cm}^{3}$
Work out the volume of prism B.
Length scale factor $=7.8 \div 2.6$

$$
=3
$$

$$
\begin{array}{r}
\text { Volume scale factor }= \\
\\
=7 \times 27=189
\end{array}
$$

Answer
nd $\mathbf{Q}$ are similar.
$\mathbf{P}$ has a height of 10 cm and $\mathbf{Q}$ has a height of 8 cm .
The volume of $\mathbf{P}$ is $800 \mathrm{~cm}^{3}$
Work out the volume of $\mathbf{Q}$.
$P \rightarrow Q$ Length scale factor $=8 \div 10$

$$
=0.8
$$

Volume scale factor $=0.8^{3}$
$=0.512$
$800 \times 0.512=409.6$

Answer 409.6 $\mathrm{cm}^{3}$

4 Solids $\mathbf{M}$ and $\mathbf{N}$ are similar.
Height of $\mathbf{M}$ : Height of $\mathbf{N}=2: 3$
The surface area of $\mathbf{N}$ is $360 \mathrm{~cm}^{2}$
Work out the surface area of $\mathbf{M}$.
$N \rightarrow M$

$\qquad$
$\qquad$
Answer $\qquad$ $\mathrm{cm}^{2}$
$5 \quad$ Solids $\mathbf{X}$ and $\mathbf{Y}$ are similar.
$\mathbf{X}$ has a volume of $24 \mathrm{~cm}^{3}$ and $\mathbf{Y}$ has a volume of $81000 \mathrm{~cm}^{3}$.
The height of $\mathbf{X}$ is 4 cm

Work out the height of $\mathbf{Y}$.
$x \rightarrow 4$ Volume scale factor $=81000 \div 24$

$$
=3375
$$

Length scale factor $=\sqrt[3]{3375}$

$$
=15
$$

$4 \times 15=60$

Answer $\qquad$ cm
$6 \quad$ Here is some information about similar solids $\mathbf{X}, \mathbf{Y}$ and $\mathbf{Z}$.

|  | $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{Z}$ |
| :---: | :---: | :---: | :---: |
| Height | 6 cm | 15 cm | 18 |
| Volume | $240 \mathrm{~cm}^{3}$ | 3750 | $6480 \mathrm{~cm}^{3}$ |

6 (a) Complete the table.
$x \rightarrow 4$ Length scale factor $=15 \div 6$

$$
=2.5
$$

$$
\begin{aligned}
\text { Volume scale factor } & =2.5^{3} \\
& =15.625 \\
240 \times 15.625 & =3750 \mathrm{~cm}^{3}
\end{aligned}
$$

$$
\begin{aligned}
4 \rightarrow 2 \text { Volume scale factor } & =6480 \div 3750 \\
& =1.728 \\
\text { Length scale factor } & =\sqrt[3]{1.728} \\
& =1.2
\end{aligned}
$$

(b) Work out
surface area of $\mathbf{X}$ : surface area of $\mathbf{Y}$ : surface area of $\mathbf{Z}$
Give your answer in its simplest form.
Lengths $6: 15: 18=2: 5: 6$
Areas $2^{2}: 5^{2}: 6^{2}=4: 25: 36$

$$
\text { Answer } 4: 25: 36
$$

$7 \quad$ Here are triangle prisms $\mathbf{A}$ and $\mathbf{B}$.


Surface area $=960 \mathrm{~cm}^{2}$


Surface area $=1500 \mathrm{~cm}^{2}$

Show that prisms $\mathbf{A}$ and $\mathbf{B}$ are not similar.
$A \rightarrow B$ Length scale fatter $=20 \div 15$

$$
=1 \cdot 3
$$

Area scale factor $=1500 \div 960$

$$
=1.5625
$$

$$
\sqrt{1.5625}=1.25
$$

$1.25 \neq 1.3$
8 Solids $\mathbf{G}$ and $\mathbf{H}$ are similar.
$\mathbf{G}$ has a surface area of $3430 \mathrm{~cm}^{2}$ and $\mathbf{H}$ has a surface area of $280 \mathrm{~cm}^{2}$.
The height of $\mathbf{G}$ is 84 cm
Work out the height of $\mathbf{H}$.
$G \rightarrow H$ Area scale factor $=280 \div 3430$

$$
=\frac{4}{49}
$$

Length scale factor $=\sqrt{\frac{4}{49}}$

$$
84 \times 2 / 7=24
$$

Answer 24 cm
$9 \quad$ Solids $\mathbf{C}$ and $\mathbf{D}$ are similar.
C has a volume of $40 \mathrm{~cm}^{3}$ and $\mathbf{D}$ has a volume of $1080 \mathrm{~cm}^{3}$.
The surface area of $\mathbf{C}$ is $100 \mathrm{~cm}^{2}$

Work out the surface area of $\mathbf{D}$.
$C \rightarrow D$.
Volume scale factor $=1080 \div 40$

$$
=27
$$

Length scale feutor $=\sqrt[3]{27}=3$ Area scale factor $=3^{2}=9$

$$
100 \times 9=900
$$

Answer $\qquad$ $\mathrm{cm}^{2}$
$10 \quad$ Solids $\mathbf{U}$ and $\mathbf{V}$ are similar.
$\mathbf{U}$ has a surface area of $375 \mathrm{~cm}^{2}$ and $\mathbf{V}$ has a surface area of $540 \mathrm{~cm}^{2}$. The volume of $\mathbf{V}$ is $432 \mathrm{~cm}^{3}$

Work out the volume of $\mathbf{U}$.
$V \rightarrow U$
$\begin{aligned} \text { Area scale factor } & =375 \div 540 \\ & =\frac{25}{36}\end{aligned}$

$$
=\frac{25}{36}
$$

Length scale factor $=\sqrt{\frac{25}{36}}=\frac{5}{6}$
Volume scale factor $=\left(\frac{5}{6}\right)^{3}=\frac{125}{216}$ $432 \times \frac{125}{216}=250$
Answer $\quad 250$ $\mathrm{cm}^{3}$
$11 \quad$ Solids $\mathbf{M}$ and $\mathbf{N}$ are similar.
volume of $\mathbf{M}$ : volume of $\mathbf{N}=1000: 1$
The surface area of $\mathbf{M}$ is $80 \mathrm{~cm}^{2}$
Work out the surface area of $\mathbf{N}$.
$\qquad$

Answer $\qquad$ $\mathrm{cm}^{2}$

12 Solids A, B and C are similar.
surface area of Solid $\mathbf{A}$ : surface area of Solid $\mathbf{B}=4: 25$
volume of Solid $\mathbf{A}$ : volume of solid $\mathbf{C}=64: 729$
height of Solid $\mathbf{A}$ : height of Solid $\mathbf{B}$ : height of Solid $\mathbf{C}=p: q: r$
where $p, q$ and $r$ are integers in their simplest form.
Work out the values of $p, q$ and $r$.

13 Prisms $\mathbf{A}$ and $\mathbf{B}$ are similar.
The cross sections are shaded.

Prism A


Prism B


The area of the cross section of prism $\mathbf{A}$ is $32 \mathrm{~cm}^{2}$
The length of prism $\mathbf{B}$ is 18 cm .
volume of prism $\mathbf{A}$ : volume of prism $\mathbf{B}=8: 27$
Work out the volume of prism $\mathbf{B}$.
$A: B$
Volume 8:27
length 2:3
Area ${ }^{6} 6_{32: 72} 4: 9$
Cross section of $B=72 \mathrm{~cm}^{2}$
Volume of prism $=$ area of cross section $x$ length $=72 \times 18$

Answer 1296 $\mathrm{cm}^{3}$
$14 \quad$ Prisms $\mathbf{A}$ and $\mathbf{B}$ are similar.
The cross sections are shaded.

Prism A


Prism B
Volume $=1536 \mathrm{~cm}^{3}$


Here is some information about the prisms.

|  | Length | Height | Cross Section <br> Area | Volume |
| :---: | :---: | :---: | :---: | :---: |
| Prism A |  |  | $25 \mathrm{~cm}^{2}$ |  |
| Prism B | 24 cm | 4.8 cm |  | $1536 \mathrm{~cm}^{3}$ |

Work out the height of prism $\mathbf{A}$.

Area of cross section of $B=1536 \div 24$ $=64 \mathrm{~cm}^{2}$
$B \rightarrow A$ Area scale factor $=\frac{25}{64}$ Length scale factor
$4.8 \times \frac{5}{8}=3$


15 Solids $\mathbf{X}$ and $\mathbf{Y}$ are similar.
$\mathbf{X}$ has a height of 14 cm and $\mathbf{Y}$ has a height of 21 cm .
The volume of $\mathbf{Y}$ is $950 \mathrm{~cm}^{3}$ greater than the volume of $\mathbf{X}$.
Work out the volume of Solid $\mathbf{X}$.
Volume of $X=x$
Volume of $Y=x+950$

$$
\begin{aligned}
4 \rightarrow x \quad \text { Length scale factor } & =14 \div 21 \\
& =2 / 3 \\
\text { Volume scale factor } & =\left(\frac{2}{3}\right)^{3} \\
& =8 / 27
\end{aligned}
$$

also volume scale factor $=\frac{x}{x+950}$

$$
\begin{aligned}
\frac{x}{x+950} & =\frac{8}{27} \\
27 x & =8(x+950) \\
27 x & =8 x+7600 \\
19 x & =7600 \\
x & =7600 \div 19
\end{aligned}
$$

Answer $\qquad$ $\mathrm{cm}^{3}$

Solid $\mathbf{S}$ is shown below.


Two of the faces of Solid $\mathbf{S}$ are squares with areas of $36 \mathrm{~cm}^{2}$ and $225 \mathrm{~cm}^{2}$. Four of the faces of Solid $\mathbf{S}$ are trapeziums.

The vertical height of Solid $\mathbf{S}$ is 12 cm .
Solid $\mathbf{T}$ is similar to Solid $\mathbf{S}$.
The area of one of the square faces of Solid $\mathbf{T}$ is $100 \mathrm{~cm}^{2}$
Work out two possible values for the vertical height of Solid T.
[4 marks]

$\qquad$


$$
12 \times \frac{10}{15}=8 \mathrm{~cm}
$$

$\qquad$
$\qquad$
mneme 20
$\qquad$ cm and $\qquad$ cm
$17 \quad$ Solids $\mathbf{X}, \mathbf{Y}$ and $\mathbf{Z}$ are similar.
volume of $\mathbf{X}$ : volume of $\mathbf{Y}=1: 8$
surface area of $\mathbf{Y}$ : surface area of $\mathbf{Z}=9: 20$
height of $\mathbf{X}$ : height of $\mathbf{Y}$ : height of $\mathbf{Z}=a: b: c \sqrt{5}$
where $a, b$ and $c$ are integers.
Work out the values of $a, b$ and $c$.

$$
\begin{array}{rl}
x: 4 & 4: 2 \\
\text { Volume } 1: 8 & \text { Area } 9: 20 \\
\text { Length } 1: 2 & \text { length } 3: \sqrt{20} \\
\begin{aligned}
x: 4 & =1: 2
\end{aligned} \quad \begin{aligned}
& 4: 2=3: \sqrt{20} \\
&=3: 6
\end{aligned} \\
\begin{aligned}
x: 4: 2 & =3: 6: 2 \sqrt{20} \\
& =3: 6: 2 \times \sqrt{4} \times \sqrt{5} \\
& =3: 6: 4 \sqrt{5}
\end{aligned} \\
& =3 \quad b=6 \quad c=4
\end{array}
$$

