Expanding Triple Brackets

REVISE THIS TOPIC

2 Expand and simplify $(x+3)(x+4)(x+6)$

$$
(x+3)(x+4)=x^{2}+4 x+3 x+12
$$

$$
\left(x^{2}+7 x+12\right)(x+6)
$$

$$
=x^{2}+7 x+12
$$

$$
=x^{3}+6 x^{2}+7 x^{2}+42 x+12 x+72
$$

$$
\text { Answer } x^{3}+13 x^{2}+54 x+72
$$

3 Expand and simplify $(x+5)(x-2)(x+1)$

$$
\begin{aligned}
&(x+5)(x-2)=x^{2}-2 x+5 x-10 \\
&=x^{2}+3 x-10 \\
&\left(x^{2}+3 x-10\right)(x+1) \\
&= x^{3}+x^{2}+3 x^{2}+3 x-10 x-10
\end{aligned}
$$

$$
x^{3}+4 x^{2}-7 x-10
$$

$$
\begin{aligned}
& \text { Expand and simplify } \quad(x+1)(x+2)(x+5) \\
& (x+1)(x+2)=x^{2}+2 x+x+2 \\
& =x^{2}+3 x+2 \\
& \left(x^{2}+3 x+2\right)(x+5) \\
& =x^{3}+5 x^{2}+3 x^{2}+15 x+2 x+10 \\
& \text { Answer } x^{3}+8 x^{2}+17 x+10
\end{aligned}
$$

Expand and simplify $\quad(x-3)(x-4)(x+2)$

$$
\begin{aligned}
&(x-3)(x-4)=x^{2}-4 x-3 x+12 \\
&=x^{2}-7 x+12 \\
&\left(x^{2}-7 x+12\right)(x+2) \\
&=x^{3}+2 x^{2}-7 x^{2}-14 x+12 x+24 \\
& \text { Answer } x^{3}-5 x^{2}-2 x+24
\end{aligned}
$$

$5 \quad$ Expand and simplify $(y-2)(y-2)(y-4)$

$$
\begin{aligned}
& \text { Exxandand simply } \\
&(y-2)(y-2)=y^{2}-2 y-2 y+4 \\
&=y^{2}-4 y+4
\end{aligned}
$$

$$
\left(y^{2}-4 y+4\right)(y-4)
$$

$$
=y^{3}-4 y^{2}-4 y^{2}+16 y+4 y-16
$$

Answer $y^{3}-8 y^{2}+20 y-16$
6 Expand and simplify $(x+5)(x+3)^{2}$

$$
\begin{gathered}
\begin{array}{c}
(x+5)(x+3)=x^{2}+3 x+5 x+15^{\text {[3 mars] }} \\
\left.=x^{2}+8 x+15 x+15\right)(x+3) \\
=x^{3}+3 x^{2}+8 x^{2}+24 x+15 x+45 \\
\text { Answer } x^{3}+11 x^{2}+39 x+45
\end{array}
\end{gathered}
$$

Expand and simplify $(x+10)(x-6)^{2}$

$$
\begin{aligned}
&(x+10)(x-6)=x^{2}-6 x+10 x-60 \\
&=x^{2}+4 x-60 \\
&\left(x^{2}+4 x-60\right)(x-6) \\
&= x^{3}-6 x^{2}+4 x^{2}-24 x-60 x+360 \\
& \text { Answer } x^{3}-2 x^{2}-84 x+360
\end{aligned}
$$

$8 \quad$ Expand and simplify $(h-5)^{3}$

$$
\begin{aligned}
& (h-5)(h-5)=h^{2}-5 h-5 h+25 \\
& =h^{2}-10 h+25 \\
& \left(h^{2}-10 h+25\right)(h-5) \\
& =h^{3}-5 h^{2}-10 h^{2}+50 h+25 h-125 \\
& \text { Answer } h^{3}-15 h^{2}+75 h-125
\end{aligned}
$$

$9 \quad$ Expand and simplify $(x+12)(x-2)(x+2)$

$$
\begin{aligned}
(x+2)(x-2) & =x^{2}-2 x+2 x-4 \\
& =x^{2}-4 \\
\left(x^{2}-4\right)(x+12) & =x^{3}+12 x^{2}-4 x-48
\end{aligned}
$$

Answer $x^{3}+12 x^{2}-4 x-48$ $\square$

$$
\begin{aligned}
&(2 x+1)(x-3)=2 x^{2}-6 x+x-3 \\
&=2 x^{2}-5 x-3 \\
&\left(2 x^{2}-5 x-3\right)(x-1) \\
&= 2 x^{3}-2 x^{2}-5 x^{2}+5 x-3 x+3
\end{aligned}
$$

$$
\text { Answer } 2 x^{3}-7 x^{2}+2 x+3
$$

11 Expand and simplify $(3 p+2)(2 p+1)(p+5)$

$$
\begin{aligned}
(3 p+2)(2 p+1) & =6 p^{2}+3 p+4 p+2 \\
& =6 p^{2}+7 p+2
\end{aligned}
$$

$$
\begin{aligned}
& \left(6 p^{2}+7 p+2\right)(p+5) \\
& =6 p^{3}+30 p^{2}+7 p^{2}+35 p+2 p+10 \\
& \text { Answer } 6 p^{3}+37 p^{2}+37 p+10
\end{aligned}
$$

12 Expand and simplify $(3 x+1)(2 x-1)(4 x-1)$ [3 marks]

$$
\begin{aligned}
(3 x+1)(2 x-1) & =6 x^{2}-3 x+2 x-1 \\
& =6 x^{2}-x-1
\end{aligned}
$$

$$
\left(6 x^{2}-x-1\right)(4 x-1)
$$

$$
=24 x^{3}-6 x^{2}-4 x^{2}+x-4 x+1
$$

Answer $24 x^{3}-10 x^{2}-3 x+1$

Show that $(3 x+1)(3 x-1)(2 x+3)$ can be written in the form $a x+b x^{2}+c x+d$ where $a, b, c$ and $d$ are all integers.

$$
\begin{aligned}
(3 x+1)(3 x-1) & =9 x^{2}-3 x+3 x-1 \\
& =9 x^{2}-1
\end{aligned}
$$

$$
\left(9 x^{2}-1\right)(2 x+3)=18 x^{3}+27 x^{2}-2 x-3
$$

$$
\text { Answer } 18 x^{3}+27 x^{2}-2 x-3
$$

14 Show that $(5 x+1)(x-3)(x-2)-(x+2)^{2} \quad$ can be written in the form $a x+b x^{2}+c x+d$ where $a, b, c$ and $d$ are all integers. [6 marks]

$$
\begin{aligned}
(5 x+1)(x-3) & =5 x^{2}-15 x+x-3 \\
& =5 x^{2}-14 x-3
\end{aligned}
$$

$$
\begin{gathered}
\left(5 x^{2}-14 x-3\right)(x-2) \\
=5 x^{3}-10 x^{2}-14 x^{2}+28 x-3 x+6 \\
=5 x^{3}-24 x^{2}+25 x+6 \\
(x+2)(x+2)=x^{2}+2 x+2 x+4 \\
=x^{2}+4 x+4 \\
5 x^{3}-24 x^{2}+25 x+6-x^{2}-4 x-4 \\
\text { Answer } 5 x^{3}-25 x^{2}+21 x+2
\end{gathered}
$$

$15(x+4)(x+3)(x-1)-(x+2)(x-2)(x+5) \equiv(x+a)(x+b)$
Given that $a>b$, work out the values of $a$ and $b$.

$$
\begin{aligned}
&(x+4)(x+3)=x^{2}+3 x+4 x+12 \\
&=x^{2}+7 x+12 \\
&\left(x^{2}+7 x+12\right)(x-1)=x^{3}-x^{2}+7 x^{2}-7 x+12 x-12 \\
&=x^{3}+6 x^{2}+5 x-12 \\
&(x+2)(x-2)=x^{2}-2 x+2 x-4 \\
&=x^{2}-4 \\
&\left(x^{2}-4\right)(x+5)=x^{3}+5 x^{2}-4 x-20 \\
& x^{3}+6 x^{2}+5 x-12-\left(x^{3}+5 x^{2}-4 x-20\right) \\
&=x^{3}+6 x^{2}+5 x-12-x^{3}-5 x^{2}+4 x+20 \\
&=x^{2}+9 x+8 \\
&=(x+8)(x+1)
\end{aligned}
$$

