Upper and Lower Bounds

REVISE THIS
TOPIC

1 To the nearest pound, Eric has $£ 8.00$
To the nearest 10p, Nicks has £1.60

1 (a) Work out the maximum possible total amount of money.
[3 marks]

$$
\begin{aligned}
& 7.50 \leqslant E<8.50 \\
& 1.55 \leqslant N<1.65
\end{aligned}
$$

$28.49+21.64$

Answer £
$10 \cdot 13$

1 (b) Eric buys a new phone case.
The phone case costs £2.50 (to the nearest 50p).
Work out the maximum amount of money that Eric could have left after buying the phone case.

$$
2.25 \leqslant P<2.75
$$

$$
z 8.49-z 2.25=z 6.24
$$

2 To 2 significant figures, the capacity of a can of drink is 330 ml A multipack contains 24 cans of drink.

2 (a) Work out the upper bound for the capacity of the multipack of cans.

$$
325 \leqslant c<335
$$

$335 \times 24=8040$

Answer

2 (b) Work out the lower bound for the capacity of the multipack of cans.

$$
325 \times 24=7800
$$

Answer $\qquad$ ml

2 (c) Arya opens one of the cans of drink.
She drinks 72 ml (to the nearest ml ) of the drink.
Work out the lower bound for the amount of drink that could be left in the can.

$$
325 \leqslant c<335
$$

$$
71.5 \leqslant A<72.5
$$

$\qquad$
$325-72 \cdot 5=252 \cdot 5$
Answer $\quad 252.5 \mathrm{ml}$
$\qquad$ 252.5
$\qquad$
$325-72.5=252.5$

3 A stadium contains 32000 fans (to 2 significant figures).
On average, each fan spends $£ 3.50$ (to the nearest 50 p) at the stadium.

3 (a) Work out the upper bound for the total amount of money spent.

$$
\begin{aligned}
& 31500 \leqslant F<32500 \\
& 3.25 \leqslant s<3.75
\end{aligned}
$$

$$
32500 \times \approx 3.75=\approx 121875
$$

Answer £ $\qquad$ 121875

3 (b) Work out the lower bound for the total amount of money spent.
$\qquad$
$31500 \times ま 3.25$
$=\$ 102375$
$\qquad$

Answer £ $\qquad$

3 (c) At half time 30\% (to the nearest 10\%) of the fans leave the stadium.
Work out the lower bound for the number of fans that leave the stadium. [2 marks]
$\qquad$ $25 \% \leqslant P<35 \%$
$\qquad$
$31500 \times 0.25=7875$

Answer

$$
7875
$$

4 The dimensions of a rectangle are shown to the nearest metre.


4 (a) Work out the upper bound for the area of the rectangle.
$12.5 \leqslant l<13.5$ $3.5 \leqslant \omega<4.5$

$$
13.5 \times 4.5=60.75
$$

Answer $\qquad$ 60.75 $\mathrm{m}^{2}$

4 (b) Work out the lower bound for the perimeter of the rectangle.

$$
12 \cdot 5+3 \cdot 5+12 \cdot 5+3 \cdot 5
$$

$\qquad$
$\qquad$
$\qquad$

Answer $\qquad$ m

5 (a) Work out the lower bound for the area of the circle.

$$
6.45 \leqslant r<6.55
$$

$$
\pi \times 6.4 s^{2}=130.6981084
$$

Answer 130.7 $\mathrm{cm}^{2}$

5 (b) Work out the upper bound for the circumference of the circle.

$$
2 \times \pi \times 6.55=41.15486376
$$

$\qquad$
$\qquad$

Answer

6
$x=700$ (to 1 significant figure)
$y=84$ (to the nearest integer)
Work out the upper bound for $2 x+y$

$$
83.5 \leqslant y<84.5
$$

$2 \times 750+84.5=1584.5$

7 Jacob invest £600 (to 1 significant figure) in a bank for 4 years. The bank pays compound interest at $3.2 \%$ (to 1 decimal place).

Work out the upper and lower bound for the total amount of money Jacob has in his account after 4 years.
$650 \times 1.0325^{4}=738.71$
$550 \times 1.0315^{4}=622.64$
$\qquad$
$\qquad$
Upper Bound $\varepsilon \quad 738 \cdot 71$

Lower Bound £ 622.64

8 The interior angle of a regular polygon is $150^{\circ}$ (correct to 2 significant figures).
Work out the maximum and minimum number of sides of the regular polygon.
$145^{\circ} \leqslant$ interior $<155^{\circ}$ $25^{\circ}<$ exterior $\leqslant 35^{\circ}$
$360 \div 25=14 \cdot 4$
$360 \div 35=10.28571429$


Box A has a mass of 800 kg (to the nearest 100 kg )
Box B has a mass of 600 kg (to the nearest 100 kg )
Box $C$ has a mass of 1500 kg (to the nearest 100 kg )
A lorry can safely carry a load of 3 tonnes.
Can all three boxes be carried safely in the lorry?
Tick one box.

$\square$ No


Show working to support your answer.
[1 tonne $=1000 \mathrm{~kg}$ ]
$750 \leqslant A<850 \quad 750+550+1450$
$550 \leqslant B<650=2750 \mathrm{~kg}$
$1450 \leqslant C<1550 \quad 850+650+1550$
$=3050 \mathrm{~kg}$
The to tall mass may be under or over 3000 kg
10
$p=3.9$ (to 1 decimal place)
$q=0.33$ (to 2 decimal places)
Work out the lower bound for $\frac{p^{2}}{q}$ giving your answer to 6 significant figures.
$3.85 \leqslant P<3.95$
$0.325 \leqslant q<0.335$

$$
\frac{3.85^{2}}{0.335}=44.24626866
$$

Answer

11 The dimensions of a cuboid are shown to the nearest metre.


The outside surfaces of the cuboid are to be painted. Each tin of paint covers $28 \mathrm{~m}^{2}$ (to the nearest square metre).

Show clearly that 6 tins of paint may not be enough to paint the outside surfaces. $11.5 \leqslant 1<12.5 \quad 12.5 \times 2.5=31.2^{[4 \text { marks] }}$
$\qquad$ $1.5 \leqslant \omega<2.5 \quad 12.5 \times 3.5=43.75$ $2.5 \leqslant h<3.5 \quad 3.5 \times 2.5=\frac{8.75}{83.75}$
$27.5 \leqslant p \leqslant 28.5$ $83.75 \times 2=167.5 \quad 27.5 \times 6=165$

12 The dimensions of a triangle are shown to the nearest 0.1 m $167.5>16 S$


Show clearly that angle $A B C$ cannot be a right angle. $8.05 \leqslant B C<8.15 \quad A B^{2}+B C^{2}=A C^{2}$ $10.25 \leqslant A C<10.35$
$U B_{A C}=\sqrt{6.1 S^{2}+8.15^{2}}=10.21004407$ $10.21 \ldots<10.25$ so cannot betrue.

13 The dimensions of a cylinder are shown to the nearest metre.

$\square$

The cylinder exerts a force of $8 \times 10^{5}$ Newtons (to 1 significant figure) onto a floor.
Calculate the lower bound for the pressure between the cylinder and the floor. Give your answer to 5 significant figures.

$$
\text { pressure }=\frac{\text { force }}{\text { area }}
$$

$$
\begin{aligned}
750000 & \leqslant F<850000 \\
2.5 & \leqslant \text { Diameter }<3.5 \\
1.25 & \leqslant \text { Radius }<1.75
\end{aligned}
$$

$\qquad$

$$
=77953.44181
$$ Answer $\quad 77953 \mathrm{Nm} \mathrm{m}^{2}$

$\qquad$
$\qquad$
$\qquad$

14
$a=400$ (to 1 significant figure)
$b=320$ (to 2 significant figures)
$c=1.1$ (to 1 decimal place)
Work out the upper bound for $\sqrt{\frac{a-b}{c}}$
Give your answer to 3 decimal places.
$\qquad$
$350 \leqslant a<450$
$315 \leqslant b<325$ $1.05 \leqslant c<1.15$

$$
\begin{array}{r}
\frac{\frac{450-315}{1.05}}{}=11.3380 \\
\text { Answer_ } 11.339
\end{array}
$$

15
$m=3.8$ (to 1 decimal place)
$n=7$ (to the nearest integer)
$h=0.43$ (to 2 decimal places)
Work out the lower bound for $\frac{m+n}{6-h}$

Give your answer to 3 decimal places.

$$
\begin{aligned}
& 3.75 \leqslant m<3.85 \\
& 6.5 \leqslant n<7.5 \\
& 0.425 \leqslant h<0.435 \\
& \frac{3.75+6.5}{6-0.425}=1.838565022
\end{aligned}
$$

Answer $\qquad$ 1.839

16 A container is in the shape of a hemisphere The radius of the hemisphere is 26 cm (to the nearest centimetre).


Liquid fills the hemisphere at a constant rate.
The constant rate $=550 \mathrm{ml}$ (to the nearest 50 ml ) per minute.
Show that it takes at least 1 hour to fill the hemisphere.
$\qquad$

$$
525 \leqslant \text { rate }<575
$$

$$
\text { Volume }_{(L B)}=\frac{2}{3} \times \pi \times 25 \cdot 5^{3}
$$

$$
=34727.95059 \mathrm{~cm}^{3}
$$

$$
\text { Time }_{(L B)}=\frac{34727.95 \ldots}{575}
$$

$$
=60.396 \ldots \mathrm{mins}
$$

$60.396>60$ ming ( 1 hour)

