



Forces as Vectors



REVISE THIS
TOPIC

1 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors due east and due north respectively.]

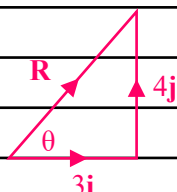
Two horizontal forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a particle P .

Given that $\mathbf{F}_1 = (4\mathbf{i} - 3\mathbf{j})$ N and $\mathbf{F}_2 = (-\mathbf{i} + 7\mathbf{j})$ N

- find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the **resultant force** acting on P . (1)
- find the **magnitude** of the **resultant force** acting on P . (2)
- find, to the nearest degree, the bearing of the **resultant force** acting on P . (3)

$$\begin{aligned} \text{(a) } \mathbf{R} &= (4\mathbf{i} - 3\mathbf{j}) + (-\mathbf{i} + 7\mathbf{j}) \\ &= (3\mathbf{i} + 4\mathbf{j}) \text{ N} \end{aligned}$$

(c)



$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\theta = 53.13\dots^\circ$$

$$\text{(b) } |\mathbf{R}| = \sqrt{3^2 + 4^2} = 5 \text{ N}$$

$$\text{Bearing} = 90 - 53.13\dots = \mathbf{037^\circ} \text{ (nearest degree)}$$

(Total for Question 1 is 6 marks)

2 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

A particle P is in equilibrium under the action of three horizontal forces.

$\mathbf{F}_1 = (5\mathbf{i} - 7\mathbf{j})$ N, $\mathbf{F}_2 = (-3\mathbf{i} + 3\mathbf{j})$ N and $\mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})$ N, where a and b are constants.

- Find the values of a and b . (2)

The force $\mathbf{F}_3$ is removed. Given that forces $\mathbf{F}_1$ and $\mathbf{F}_2$ continue to act on P ,

- find the **magnitude** of the **resultant force**, $\mathbf{R}$, acting on P , (2)
- find, to the nearest degree, the angle between the line of action of $\mathbf{R}$ and the vector $\mathbf{i}$. (2)

$$\text{(a) } (5\mathbf{i} - 7\mathbf{j}) + (-3\mathbf{i} + 3\mathbf{j}) + (a\mathbf{i} + b\mathbf{j}) = \mathbf{0}$$

$$\text{(b) } \mathbf{R} = (5\mathbf{i} - 7\mathbf{j}) + (-3\mathbf{i} + 3\mathbf{j}) = (2\mathbf{i} - 4\mathbf{j})$$

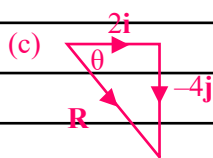
$$5 - 3 + a = 0$$

$$-7 + 3 + b = 0$$

$$|\mathbf{R}| = \sqrt{2^2 + (-4)^2} = \sqrt{20} \text{ N [OR } 2\sqrt{5} \text{ N OR } 4.47 \text{ N]}$$

$$a = -2$$

$$b = 4$$



$$\theta = \tan^{-1}\left(\frac{4}{2}\right)$$

$$\theta = 63.4\dots^\circ$$

$$\text{Angle} = \mathbf{63^\circ} \text{ (nearest degree)}$$

(Total for Question 2 is 6 marks)





3 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

Two horizontal forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a particle P .

$$\mathbf{F}_1 = (10\mathbf{i} + 5\mathbf{j}) \text{ N and } \mathbf{F}_2 = (-5\mathbf{i} + 7\mathbf{j}) \text{ N}$$

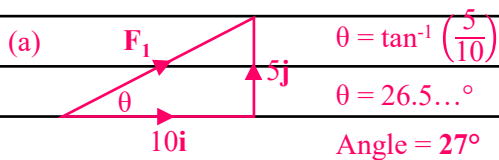
(a) Find, to the nearest degree, the angle between the line of action of $\mathbf{F}_1$ and the vector $\mathbf{i}$. (2)

(b) Find the **magnitude** of the **resultant force** acting on P . (3)

Two horizontal forces, $\mathbf{F}_3$ and $\mathbf{F}_4$, act on a particle Q .

$$\mathbf{F}_3 = (3\mathbf{i} - 3\mathbf{j}) \text{ N and } \mathbf{F}_4 = (a\mathbf{i} + 8\mathbf{j}) \text{ N, where } a \text{ is a constant.}$$

(c) Given that the magnitude of the resultant force acting on Q is equal to the magnitude of the resultant force acting on P , find the two possible values of a . (3)



(b) $\mathbf{R} = (10\mathbf{i} + 5\mathbf{j}) + (-5\mathbf{i} + 7\mathbf{j}) = (5\mathbf{i} + 12\mathbf{j}) \text{ N}$

$$|\mathbf{R}| = \sqrt{5^2 + 12^2} = 13 \text{ N}$$

(c) $\mathbf{R}_2 = (3\mathbf{i} - 3\mathbf{j}) + (a\mathbf{i} + 8\mathbf{j}) = (3 + a)\mathbf{i} + 5\mathbf{j}$

$$|\mathbf{R}_2| = \sqrt{(3 + a)^2 + 5^2}$$

$$|\mathbf{R}_2| = 13 \text{ (same as for } P)$$

$$\sqrt{(3 + a)^2 + 5^2} = 13$$

$$(3 + a)^2 + 5^2 = 169$$

$$(3 + a)^2 = 169 - 5^2$$

$$(3 + a)^2 = 144$$

$$3 + a = \pm 12$$

$$a = -3 \pm 12$$

$$a = 9 \quad a = -15$$

(Total for Question 3 is 8 marks)



4 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

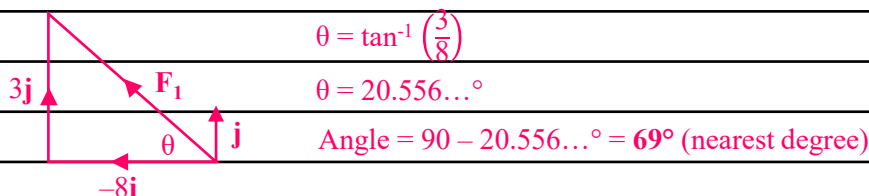
A particle P is in equilibrium under the action of three horizontal forces.

$\mathbf{F}_1 = (-8\mathbf{i} + 3\mathbf{j})$ N, $\mathbf{F}_2 = (b\mathbf{i} - a\mathbf{j})$ N and $\mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})$ N, where a and b are constants.

- Find $|\mathbf{F}_1|$ (2)
- Find, to the nearest degree, the acute angle between the line of action of force $\mathbf{F}_1$ and the vector $\mathbf{j}$. (3)
- Find the values of a and b . (3)

(a) $|\mathbf{F}_1| = \sqrt{(-8)^2 + 3^2} = \sqrt{73}$ N [OR 8.54 N]

(b)



(c) $(-8\mathbf{i} + 3\mathbf{j}) + (b\mathbf{i} - a\mathbf{j}) + (a\mathbf{i} + b\mathbf{j}) = 0$

$-8 + b + a = 0$ $3 - a + b = 0$

$a + b = 8$ $a - b = 3$

$\left. \begin{array}{l} a + b = 8 \\ a - b = 3 \end{array} \right\}$ Solving simultaneous equations on calculator gives $a = 5.5$
 $b = 2.5$

(Total for Question 4 is 8 marks)



5 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

A particle P of mass 4 kg moves on a smooth horizontal plane under the action of two horizontal forces $\mathbf{F}_1$ and $\mathbf{F}_2$.

$$\mathbf{F}_1 = (5\mathbf{i} + 2\mathbf{j}) \text{ N and } \mathbf{F}_2 = (-\mathbf{i} + 4\mathbf{j}) \text{ N}$$

- Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the **resultant force** acting on P . (1)
- Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the acceleration of P . (2)
- Find the **magnitude** of the acceleration of P . (2)
- Given that P starts from rest, calculate the distance travelled by P in the first 9 seconds. (2)

$$\begin{aligned} \text{(a)} \quad \mathbf{R} &= (5\mathbf{i} + 2\mathbf{j}) + (-\mathbf{i} + 4\mathbf{j}) \\ &= (4\mathbf{i} + 6\mathbf{j}) \text{ N} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \mathbf{F} &= m\mathbf{a} \\ (4\mathbf{i} + 6\mathbf{j}) &= 4\mathbf{a} \\ (\mathbf{i} + \frac{3}{2}\mathbf{j}) &= \mathbf{a} \\ \mathbf{a} &= (\mathbf{i} + \frac{3}{2}\mathbf{j}) \text{ ms}^{-2} \end{aligned}$$

$$\text{(c)} \quad |\mathbf{a}| = \sqrt{(1)^2 + \left(\frac{3}{2}\right)^2} = \frac{\sqrt{13}}{2} \text{ ms}^{-2} \text{ [OR } 1.80 \text{ ms}^{-2}]$$

$$\begin{aligned} \text{(d)} \quad s &= s & s &= ut + \frac{1}{2}at^2 \\ u &= 0 & s &= (0)(9) + \frac{1}{2}(1.80\ldots)(9)^2 \\ v &= v & s &= \mathbf{73.0 \text{ m}} \\ a &= 1.80\ldots \\ t &= 9 \end{aligned}$$

(Total for Question 5 is 7 marks)



6 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

A particle P of mass 3 kg moves on a smooth horizontal plane under the action of two horizontal forces $(7\mathbf{i} - 18\mathbf{j})$ N and $(c\mathbf{i} - 27\mathbf{j})$ N, where c is a constant.

Given that the acceleration of P is $(8\mathbf{i} - 15\mathbf{j}) \text{ ms}^{-2}$

- Find the **magnitude** of the acceleration of P . (2)
- Find the magnitude of the **resultant force** acting on P . (2)
- Find the value of c . (2)
- Given that P starts from rest, find the speed of P after 5 seconds. (2)

$$(a) \quad |a| = \sqrt{(8)^2 + (-15)^2} = 17 \text{ ms}^{-2}$$

$$(b) \quad |\mathbf{R}| = m|a|$$

$$|\mathbf{R}| = 3 \times 17$$

$$|\mathbf{R}| = 51 \text{ N}$$

$$(c) \quad \mathbf{F} = m\mathbf{a}$$

$$(7\mathbf{i} - 18\mathbf{j}) + (c\mathbf{i} - 27\mathbf{j}) = 3(8\mathbf{i} - 15\mathbf{j})$$

$$(7 + c)\mathbf{i} - 45\mathbf{j} = 24\mathbf{i} - 45\mathbf{j}$$

$$7 + c = 24$$

$$c = 17$$

$$(d) \quad s = s$$

$$v = u + at$$

$$u = 0$$

$$v = 0 + 17 \times 5$$

$$v = v$$

$$v = 85 \text{ ms}^{-1}$$

$$a = 17$$

$$t = 5$$

(Total for Question 6 is 8 marks)





7 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors due east and due north respectively.]

A particle P of mass 5 kg moves on a smooth horizontal plane under the action of two horizontal forces $\mathbf{F}_1 = (-8\mathbf{i} + 13\mathbf{j})$ N and $\mathbf{F}_2 = (18\mathbf{i} + 7\mathbf{j})$ N

(a) Find, to the nearest degree, the bearing of force $\mathbf{F}_1$. (2)

(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the acceleration of P . (2)

(c) Find the **magnitude** of the acceleration of P . (2)

Two more horizontal forces, $\mathbf{F}_3$ and $\mathbf{F}_4$, are now applied to P .

Under the action of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ and $\mathbf{F}_4$, the acceleration of P remains unchanged.

(d) Given that $\mathbf{F}_3 = (6\mathbf{i} - 3\mathbf{j})$ N, find $\mathbf{F}_4$ in terms of $\mathbf{i}$ and $\mathbf{j}$ (1)

(a)		$\theta = \tan^{-1}\left(\frac{13}{8}\right)$ $\theta = 58.39\dots^\circ$ Bearing = $270 + 58.39\dots = 328^\circ$ (nearest degree)
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(b) $\mathbf{R} = (-8\mathbf{i} + 13\mathbf{j}) + (18\mathbf{i} + 7\mathbf{j}) = (10\mathbf{i} + 20\mathbf{j})$ N

$$\mathbf{F} = m\mathbf{a}$$

$$(10\mathbf{i} + 20\mathbf{j}) = 5\mathbf{a}$$

$$(2\mathbf{i} + 4\mathbf{j}) = \mathbf{a}$$

$$\mathbf{a} = (2\mathbf{i} + 4\mathbf{j}) \text{ ms}^{-2}$$

(c) $|\mathbf{a}| = \sqrt{(2)^2 + (4)^2} = \sqrt{20} \text{ ms}^{-2}$ [OR $2\sqrt{5} \text{ ms}^{-2}$ OR 4.47 ms^{-2}]

(d) If acceleration is unchanged then resultant force is unchanged.

$$\text{So } \mathbf{F}_3 + \mathbf{F}_4 = \mathbf{0}$$

$$\mathbf{F}_4 = -\mathbf{F}_3$$

$$\mathbf{F}_4 = -(6\mathbf{i} - 3\mathbf{j}) \text{ N}$$

$$\mathbf{F}_4 = (-6\mathbf{i} + 3\mathbf{j}) \text{ N}$$

(Total for Question 7 is 7 marks)



8 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

Three horizontal forces, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, act on a particle P .

$$\mathbf{F}_1 = (5\mathbf{i} - 6\mathbf{j}) \text{ N}, \quad \mathbf{F}_2 = (4\mathbf{i} - 2\mathbf{j}) \text{ N} \text{ and } \mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})$$

Given that $\mathbf{R}$, the resultant force acting on P , is parallel to $\mathbf{i}$, and $|\mathbf{F}_3| = 17 \text{ N}$, find

(a) the value of b . (2)

(b) two possible values for a . (3)

(c) two possible values for the magnitude of $\mathbf{R}$. (2)

$$\begin{aligned} \text{(a)} \quad \mathbf{R} &= (5\mathbf{i} - 6\mathbf{j}) + (4\mathbf{i} - 2\mathbf{j}) + (a\mathbf{i} + b\mathbf{j}) \\ &= (9 + a)\mathbf{i} + (b - 8)\mathbf{j} \end{aligned}$$

$$\mathbf{R} = k\mathbf{i} \text{ for some constant } k.$$

$$\text{So } b - 8 = 0$$

$$b = 8$$

$$\begin{aligned} \text{(b)} \quad \mathbf{F}_3 &= (a\mathbf{i} + 8\mathbf{j}) \\ |\mathbf{F}_3| &= \sqrt{a^2 + 8^2} \quad |\mathbf{F}_3| = 17 \text{ (given in question)} \end{aligned}$$

$$\sqrt{a^2 + 8^2} = 17$$

$$a^2 + 8^2 = 17^2$$

$$a^2 = 17^2 - 8^2$$

$$a^2 = 225$$

$$a = 15 \quad a = -15$$

$$\begin{aligned} \text{(c)} \quad \mathbf{R} &= (9 + a)\mathbf{i} \\ \text{When } a = 15, \quad \mathbf{R} &= (9 + 15)\mathbf{i} = 24\mathbf{i}, \quad |\mathbf{R}| = 24 \text{ N} \\ \text{When } a = -15, \quad \mathbf{R} &= (9 - 15)\mathbf{i} = -6\mathbf{i}, \quad |\mathbf{R}| = 6 \text{ N} \end{aligned}$$

(Total for Question 8 is 7 marks)





9 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors due east and due north respectively.]

A single horizontal force, $\mathbf{F}$, acts on a particle P , of mass $\frac{2}{3}$ kg.

The magnitude of $\mathbf{F} = \sqrt{12}$ N

The magnitude of the acceleration of P is $\sqrt{k} \text{ ms}^{-2}$, where k is an integer.

(a) Find the value of k . (3)

(b) Given that P starts from rest, find, to 3 significant figures the time taken for P to reach a speed of 10 ms^{-1} (2)

(c) Given that $\mathbf{F} = (a\mathbf{i} + b\mathbf{j})$ N and the bearing of $\mathbf{F}$ is 060° , find the values of a and b . (4)

(a) $|\mathbf{F}| = m|a|$

(b) $s = s$

$v = u + at$

$$\sqrt{12} = \frac{2}{3} \times \sqrt{k}$$

$$u = 0$$

$$10 = 0 + \sqrt{27} \times t$$

$$v = 10$$

$$t = \frac{10}{\sqrt{27}}$$

$$2\sqrt{3} = \frac{2}{3} \times \sqrt{k}$$

$$a = \sqrt{27}$$

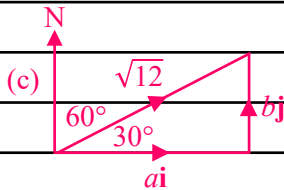
$$3\sqrt{3} = \sqrt{k}$$

$$t = t$$

$$t = 1.92 \text{ seconds}$$

$$\sqrt{27} = \sqrt{k}$$

$$k = 27$$



$$\sin(30^\circ) = \frac{b}{\sqrt{12}}$$

$$\cos(30^\circ) = \frac{a}{\sqrt{12}}$$

$$b = \sqrt{12} \sin(30^\circ)$$

$$a = \sqrt{12} \cos(30^\circ)$$

$$b = 2\sqrt{3} \times 0.5$$

$$a = 2\sqrt{3} \times \frac{\sqrt{3}}{2}$$

$$b = \sqrt{3}$$

$$a = 3$$



10 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

A particle P of mass m kg moves on a smooth horizontal plane under the action of two horizontal forces $\mathbf{F}_1 = (-8\mathbf{i} + 10\mathbf{j})$ N and $\mathbf{F}_2 = (-10\mathbf{i} + 14\mathbf{j})$ N

(a) Given that the magnitude of the acceleration of P is 40 ms^{-2} , find m . (4)

(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the acceleration of P . (2)

A third horizontal force, $\mathbf{F}_3$, is now applied to P .

Under the action of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$, the particle moves with **constant velocity**.

(c) Find $\mathbf{F}_3$, giving your answer in terms of $\mathbf{i}$ and $\mathbf{j}$. (1)

$$(a) \quad \mathbf{R} = (-8\mathbf{i} + 10\mathbf{j}) + (-10\mathbf{i} + 14\mathbf{j})$$

$$= (-18\mathbf{i} + 24\mathbf{j}) \text{ N}$$

$$|\mathbf{R}| = \sqrt{(-18)^2 + 24^2} = 30 \text{ N}$$

$$|\mathbf{R}| = m|a|$$

$$30 = m \times 40$$

$$m = 0.75 \text{ kg}$$

$$(b) \quad \mathbf{F} = m\mathbf{a}$$

$$-18\mathbf{i} + 24\mathbf{j} = 0.75\mathbf{a}$$

$$\mathbf{a} = (-24\mathbf{i} + 32\mathbf{j}) \text{ ms}^{-2}$$

$$(c) \quad \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$$

$$(-8\mathbf{i} + 10\mathbf{j}) + (-10\mathbf{i} + 14\mathbf{j}) + \mathbf{F}_3 = 0$$

$$-18\mathbf{i} + 24\mathbf{j} + \mathbf{F}_3 = 0$$

$$\mathbf{F}_3 = 18\mathbf{i} - 24\mathbf{j}$$

(Total for Question 10 is 7 marks)





11 [In this question, $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in a horizontal plane.]

A particle P of mass 0.5 kg moves on a smooth horizontal plane under the action of two horizontal forces $\mathbf{F}_1$ and $\mathbf{F}_2$

$$\mathbf{F}_1 = (2\mathbf{i} - 4\mathbf{j}) \text{ N}$$

$\mathbf{F}_2$ acts in a direction that is parallel to the vector $(\mathbf{i} + \mathbf{j})$

Given that the magnitude of the acceleration of P is $10\sqrt{2} \text{ ms}^{-2}$, find the two possible values of the magnitude of $\mathbf{F}_2$.

$$\mathbf{F}_2 = k\mathbf{i} + k\mathbf{j}$$

$$\mathbf{R} = (2\mathbf{i} - 4\mathbf{j}) + (k\mathbf{i} + k\mathbf{j})$$

$$\mathbf{R} = (2 + k)\mathbf{i} + (k - 4)\mathbf{j}$$

$$|\mathbf{R}| = \sqrt{(2 + k)^2 + (k - 4)^2}$$

$$|\mathbf{R}| = m|\mathbf{a}|$$

$$\sqrt{(2 + k)^2 + (k - 4)^2} = 0.5 \times 10\sqrt{2}$$

$$\sqrt{(2 + k)^2 + (k - 4)^2} = 5\sqrt{2}$$

$$\sqrt{(2 + k)^2 + (k - 4)^2} = \sqrt{25} \times \sqrt{2}$$

$$\sqrt{(2 + k)^2 + (k - 4)^2} = \sqrt{50}$$

$$(2 + k)^2 + (k - 4)^2 = 50$$

$$k^2 + 4k + 4 + k^2 - 8k + 16 = 50$$

$$2k^2 - 4k + 20 = 50$$

$$2k^2 - 4k - 30 = 0$$

$$k^2 - 2k - 15 = 0$$

$$(k - 5)(k + 3) = 0$$

$$k = 5 \quad k = -3$$

$$\mathbf{F}_2 = 5\mathbf{i} + 5\mathbf{j} \quad \text{or} \quad \mathbf{F}_2 = -3\mathbf{i} - 3\mathbf{j}$$

$$|\mathbf{F}_2| = \sqrt{5^2 + 5^2}$$

$$|\mathbf{F}_2| = \sqrt{(-3)^2 + (-3)^2}$$

$$|\mathbf{F}_2| = \sqrt{50} \text{ N}$$

$$|\mathbf{F}_2| = \sqrt{18} \text{ N}$$

$$|\mathbf{F}_2| = 5\sqrt{2} \text{ N}$$

$$|\mathbf{F}_2| = 3\sqrt{2} \text{ N}$$

