

Non-Linear Simultaneous Equations



REVISE THIS TOPIC

Solve algebraically the simultaneous equations

$$y = x^2 - 3x - 4$$
$$y = 2x - 10$$

$$x^{2}-3x-4=2x-10$$

 $x^{2}-5x-4=-10$
 $x^{2}-5x+6=0$
 $(x-2)(x-3)=0$
 $x=2$

when
$$x=2$$

$$y = 2(2) - 10$$

 $y = -6$
when $x = 3$
 $y = 2(3) - 10$
 $y = -4$

$$x=2 \quad y=-6, x=3 \quad y=-4$$
(Total for Question 1 is 5 marks)

Solve algebraically the simultaneous equations

$$y = x^2 + 5x - 8$$

 $y - 4x = 4$ \iff $y = 4 + 4x$

$$x^{2}+5x-8=4+4x$$
 $x^{2}+x-8=4$
 $x^{2}+x-12=0$
 $(x+4)(x-3)=0$
 $x=-4$ $x=3$

when
$$x = -4$$

 $y = 4 + 4(-4)$
 $y = -12$

when
$$x=3$$



$$x=-4$$
 $y=-12$ $x=3$ $y=16$ (Total for Question 2 is 5 marks)





3 Solve algebraically the simultaneous equations



$$y = 3x^2 + 2x - 8$$
$$y = 9x - 10$$

$$3x^{2}+2x-8=9x-10$$

$$3x^{2}-7x-8=-10$$

$$3x^{2}-7x+1=0$$

$$(3x-1)(x-1)=0$$

$$3x-1=0 x-2=0$$

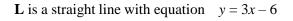
$$x=\frac{1}{3} x=1$$

when
$$x=\frac{1}{3}$$

 $y=9(\frac{1}{3})-10$
 $y=-7$
when $x=2$
 $y=9(2)-10$
 $y=8$

$$x=\frac{1}{3}y=-7$$
 $x=2$ $y=8$
(Total for Question 3 is 5 marks)

4 C is a graph with equation $y = x^2 - 5x + 10$





Using algebra, find the coordinates of the point of intersection of ${\bf C}$ and ${\bf L}$. You must show all your working.

$$x^{2}-5x+10 = 3x-6$$

$$x^{2}-8x+10 = -6$$

$$x^{2}-8x+16 = 0$$

$$(x-4)(x-4) = 0$$

$$x=4$$

when
$$x=4$$

 $y=3(4)-6$
 $y=12-6$
 $y=6$



4 , 6

(Total for Question 4 is 4 marks)

5 C is a graph with equation $y = 5x^2 - x - 15$



L is a straight line with equation y = 10x - 3

Using algebra, find the coordinates of the points of intersection of C and L.

You must show all your working.

$$5x^{2}-x-15 = 10x-3$$

$$5x^{2}-11x-15 = -3$$

$$5x^{2}-11x-12 = 0$$

$$(5x+4)(x-3) = 0$$

$$5x+4=0 \quad x-3=0$$

$$0x=-\frac{4}{5} \quad x=3$$

when
$$x = -\frac{4}{5}$$

 $y = 10(-\frac{4}{5}) - 3$
 $y = -8 - 3$
 $y = -11$
when $x = 3$
 $y = 10(3) - 3$ $-\frac{4}{5}$ -11
 $y = 27$
 3 27

- (Total for Question 5 is 5 marks)
- Solve algebraically the simultaneous equations



$$y = x^2 + x + 1$$
$$y = x + 3$$

Give your answers as exact values.

$$x^{2}+x+1=x+3$$

$$x^{2}+1=3$$

$$x^{2}-2=0$$

$$x^{2}=2$$

$$x=\pm\sqrt{2}$$

when
$$x = \sqrt{2}$$

 $y = \sqrt{2} + 3$
when $x = -\sqrt{2}$
 $y = -\sqrt{2} + 3$



 $x=\sqrt{2}$ y=3+\frac{12}{2}, x=-\frac{12}{2} y=3-\frac{12}{2} (Total for Question 6 is 5 marks)

7 Solve algebraically the simultaneous equations



$$x^2 + y^2 = 100$$
$$y = x - 2$$

$$x^{2} + (x-2)^{2} = 100$$

$$x^{2} + x^{2} - 2x - 2x + 4 = 100$$

$$2x^{2} - 4x + 4 = 100$$

$$2x^{2} - 4x - 96 = 0$$

$$x^{2} - 2x - 48 = 0$$

$$(x-8)(x+6) = 0$$

$$x = 8 \quad x = -6$$

when
$$x = 8$$
 $y = 8 - 2$
 $y = 6$
when $x = -6$
 $y = -6 - 2$
 $y = -8$

$$\chi = 8$$
 $y = 6$ $\chi = -6$ $y = -8$ (Total for Question 7 is 5 marks)

8 Solve algebraically the simultaneous equations



$$x^2 + y^2 = 200$$
$$y = 2x - 10$$

$$\chi^{2} + (2x-10)^{2} = 200$$

$$\chi^{2} + 4x^{2} - 20x - 20x + 100 = 200$$

$$5x^{2} - 40x + 100 = 200$$

$$5x^{2} - 40x - 100 = 0$$

$$\chi^{2} - 8x - 20 = 0$$

$$(x-10)(x+2) = 0$$

$$\chi = 10 \quad x = -2$$

when
$$x = 10$$

 $y = 2(10) - 10$
 $y = 10$

when
$$x = -2$$

 $y = 2(-2) - 10$
 $y = -14$



$$x = 10 y = 10, x = -2 y = -14$$

(Total for Question 8 is 5 marks)

9 C is a graph with equation $x^2 + y^2 = 65$



L is a straight line with equation 2y = 10 - x

Using algebra, find the coordinates of the point of intersection of C and L.

You must show all your working.

$$(10-2y)^{2} + y^{2} = 65$$

$$(00-20y-20y+4y^{2}+y^{2}=65)$$

$$5y^{2}-40y+100=65$$

$$5y^{2}-40y+35=0$$

$$y^{2}-8y+7=0$$

$$(y-7)(y-1)=0$$

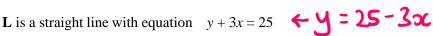
$$y=7 y=1$$

when
$$y=7$$

 $x=10-2(7)$
 $x=-4$
when $y=1$
 $x=10-2(1)$
 $x=8$
 $(-4, 7)$

(Total for Question 9 is 5 marks)

10 C is a graph with equation $x^2 + y^2 = 85$





Using algebra, find the coordinates of the points of intersection of **C** and **L**. You must show all your working.

$$x^{2} + (2s - 3x)^{2} = 85$$

$$x^{2} + 625 - 75x - 75x + 9x^{2} = 85$$

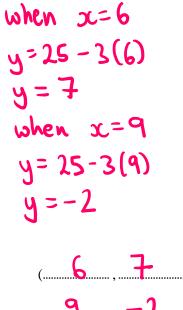
$$|0x^{2} - 150x + 625 = 85$$

$$|0x^{2} - 150x + 540 = 0$$

$$x^{2} - 15x + 54 = 0$$

$$(x - 6)(x - 9) = 0$$

$$x = 6 \quad x = 9$$





(Total for Question 10 is 5 marks)

11 Solve algebraically the simultaneous equations



$$x^2 - 2y^2 = 7$$

 $2y = x + 1$ $x = 2y - 1$

$$(2y-1)^{2}-2y^{2}=7$$

$$4y^{2}-2y-2y+1-2y^{2}=7$$

$$2y^{2}-4y+1=7$$

$$2y^{2}-4y-6=0$$

$$y^{2}-2y-3=0$$

$$(y-3)(y+1)=0$$

$$y=3 \quad y=-1$$

when
$$y=3$$

 $x=2(3)-1$
 $x=5$
when $y=-1$
 $x=2(-1)-1$
 $x=-3$

$$x=5 y=3, x=-3 y=-1$$

(Total for Question 11 is 5 marks)

12 C is a graph with equation $2x^2 - 3y^2 = 15$



L is a straight line with equation y = x - 2

Using algebra, find the coordinates of the points of intersection of ${\bf C}$ and ${\bf L}$. You must show all your working.

$$2x^{2} - 3(x-2)^{2} = 15$$

$$2x^{2} - 3(x^{2} - 4x + 4) = 15$$

$$2x^{2} - 3x^{2} + 12x - 12 = 15$$

$$-x^{2} + 12x - 12 = 15$$

$$-x^{2} + 12x - 14 = 0$$

$$x^{2} - 12x + 14 = 0$$

$$(x - 3)(x - 9) = 0$$

$$x = 3 \quad x = 9$$

when
$$x = 3$$

 $y = 3 - 2$
 $y = 1$
when $x = 9$
 $y = 9 - 2$
 $y = 7$

(Total for Question 12 is 5 marks)

13 C is a graph with equation $x^2 - 8y^2 = k$ where k is a positive integer.



L is a straight line with equation 4y = x - 1

L and C intersect at points A and B.

The *x*-coordinate of point A is -7.

(a) Work out the value of k.

At A
$$x=-7$$

 $4y=x-1$
 $4y=-7-1$
 $4y=-8$
 $y=-2$
 $A=(-7,-2)$

$$32^{2} - 8y^{2} = k$$

$$(-7)^{2} - 8(-2)^{2} = k$$

$$49 - 8(4) = k$$

$$49 - 32 = k$$

$$17 = k$$

k = ______(3)

(b) Work out the coordinates of point B.

$$x = 4y + 1$$

$$(4y+1)^{2} - 8y^{2} = 17$$

$$16y^{2} + 8y + 1 - 8y^{2} = 17$$

$$8y^{2} + 8y + 1 = 17$$

$$8y^{2} + 8y - 16 = 0$$

$$y^{2} + y - 2 = 0$$

$$(y+2)(y-1) = 0$$

$$y = -2$$

$$y = 1$$

$$A$$

when
$$y=1$$

 $x=4(1)+1$
 $x=5$

(Total for Question 13 is 7 marks)