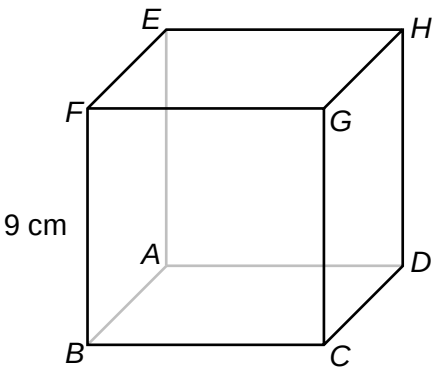
3D Trig/Pythagoras

SCAN ME

REVISE THIS TOPIC

1 Here is a cube.

BF = 9 cm



1 (a) Work out the length of AC giving your answer to 1 decimal place. **[2 marks]**

$AC^2 = AB^2 + BC^2$

$AC^2 = 9^2 + 9^2$

$AC^2 = 162$

$AC = \sqrt{162}$

$AC = 12.7279...$

Answer 12.7 cm

1 (b) Work out the length of CE giving your answer to 1 decimal place. **[2 marks]**

$CE^2 = AC^2 + CE^2$

$CE^2 = (\sqrt{162})^2 + 9^2$

$CE^2 = 243$

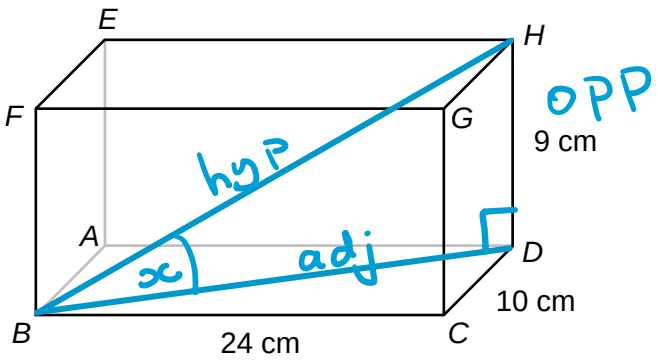
$CE = \sqrt{243}$

$CE = 15.5884...$

Answer 15.6 cm

2 Here is a cuboid.

BC = 24 cm CD = 10 cm DH = 9 cm



2 (a) Work out the length of BD. **[2 marks]**

$BD^2 = BC^2 + CD^2$

$BD^2 = 24^2 + 10^2$

$BD^2 = 676$

$BD = \sqrt{676}$

Answer 26 cm

2 (b) Work out the length of BH giving your answer to 1 decimal place. **[2 marks]**

$BH^2 = BD^2 + DH^2$

$BH^2 = 26^2 + 9^2$

$BH^2 = 757$

$BH = \sqrt{757}$

$BH = 27.5136\ldots$

Answer 27.5 cm

2 (c) Work out the size of angle DBH giving your answer to 1 decimal place. **[2 marks]**

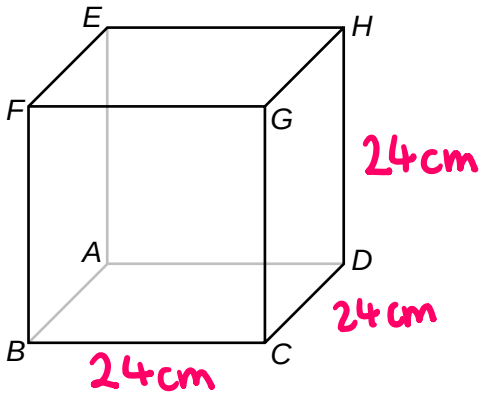
$\sin(x) = \frac{9}{27.5\ldots}$

$x = \sin^{-1}\left(\frac{9}{27.5\ldots}\right)$

$x = 19.093\ldots$

Answer 19.1 °

3 Here is a cube.
The surface area of the cube is 3456 cm^2



Work out the length of *EC* giving your answer to 1 decimal place. **[5 marks]**

$3456 \div 6 = 576\text{cm}^2$ (area of one face)

$\sqrt{576} = 24\text{cm}$ (length of one edge)

$AC^2 = AB^2 + BC^2$
$AC^2 = 24^2 + 24^2$
$AC^2 = 1152$
$AC = \sqrt{1152}$
$AC = 33.94...$

$EC^2 = AC^2 + AE^2$
$EC^2 = (\sqrt{1152})^2 + 24^2$
$EC^2 = 1728$
$EC = \sqrt{1728}$
$EC = 41.5692...$

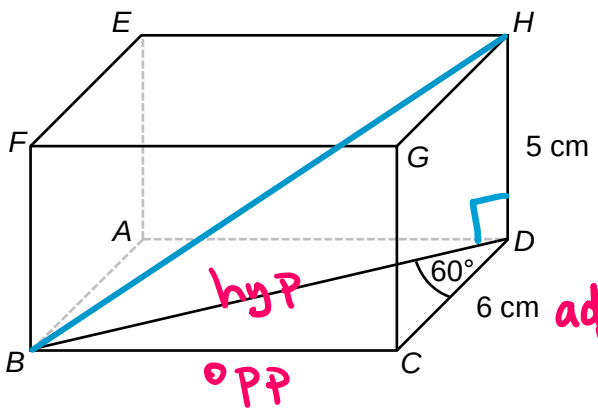
Answer 41.6 cm

11

Turn over ►

4 Here is a cuboid.

$CD = 6$ cm $DH = 5$ cm Angle $BDC = 60°$



Work out the perimeter of triangle *BDH*. **[4 marks]**

$\cos(60) = \frac{6}{BD}$

$BD = \frac{6}{\cos(60)}$

$BD = \frac{6}{0.5}$

$BD = 12$ cm

$BH^2 = BD^2 + DH^2$

$BH^2 = 12^2 + 5^2$

$BH^2 = 169$

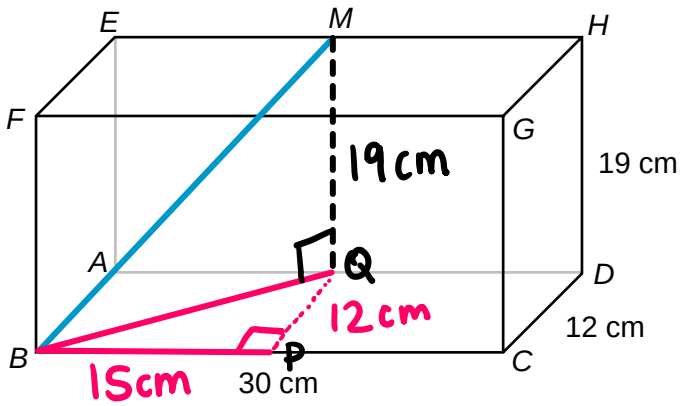
$BH = \sqrt{169}$

$BH = 13$ cm

Perimeter $= 5 + 12 + 13$

Answer 30 cm

5 Here is a cuboid.
M is the midpoint of line *EH*.
BC = 30 cm *CD* = 12 cm *DH* = 19 cm



Work out the length of *BM* giving your answer to 1 decimal place. **[4 marks]**

$BQ^2 = BP^2 + PQ^2$
$BQ^2 = 15^2 + 12^2$
$BQ^2 = 369$
$BQ = \sqrt{369}$
$BQ = 19.209...$

$BM^2 = BQ^2 + QM^2$
$BM^2 = (\sqrt{369})^2 + 19^2$
$BM^2 = 730$
$BM = \sqrt{730}$
$BM = 27.0185...$

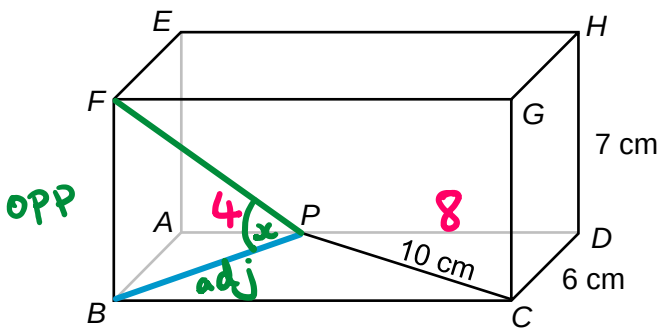
Answer 27.0 cm

8

6 Here is a cuboid.

P is the point on the line AD so that $AP : PD = 1 : 2$

$CD =$ 6cm $\quad DH =$ 7 cm $\quad PC =$ 10 cm



6 (a) Work out the length of BC giving your answer to 1 decimal place. **[3 marks]**

$PD^2 = PC^2 - CD^2$ $\quad AP = 8 \div 2 = 4$

$PD = \sqrt{10^2 - 6^2}$ $\quad BC = 4 + 8$

$PD = 8$

Answer 12 cm

6 (b) Work out the length of BP giving your answer to 1 decimal place. **[2 marks]**

$BP^2 = BA^2 + AP^2$ $\quad BP = \sqrt{52}$

$BP^2 = 6^2 + 4^2$ $\quad BP = 7.211\ldots$

$BP^2 = 52$

Answer 7.2 cm

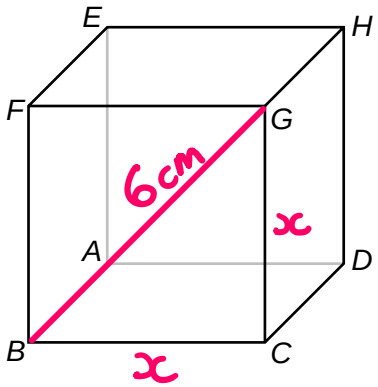
6 (c) Work out the size of angle BPF giving your answer to 1 decimal place. **[2 marks]**

$\tan(x) = \frac{7}{7.21\ldots}$ $\quad x = \tan^{-1}\left(\frac{7}{7.21\ldots}\right)$

$x = 44.148\ldots$

Answer 44.1 °

7 Here is a cube.
BG = 6 cm



Work out the volume of the cube giving your answer to 1 decimal place. **[4 marks]**

$BG^2 = BC^2 + CG^2$

$6^2 = x^2 + x^2$

$36 = 2x^2$ ÷2

$18 = x^2$ ÷2

$x = \sqrt{18}$

Volume $= \sqrt{18} \times \sqrt{18} \times \sqrt{18}$

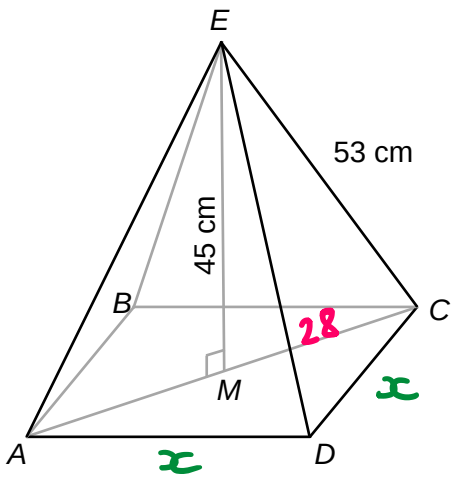
$= 76.367...$

Answer 76.4 cm^3

11

8 *ABCDE* is a square-based pyramid.
M is the midpoint of the line *AC* and *AC* is perpendicular to *ME*.

EC = 53 cm
EM = 45 cm



Volume of pyramid = $\frac{1}{3}$ × area of base × perpendicular height

Work out the volume of the pyramid. **[6 marks]**

$MC^2 = EC^2 - ME^2$
$MC^2 = 53^2 - 45^2$
$MC^2 = 784$
$MC = \sqrt{784}$
$MC = 28\text{cm}$
$AC = 2 \times MC$
$= 56\text{cm}$

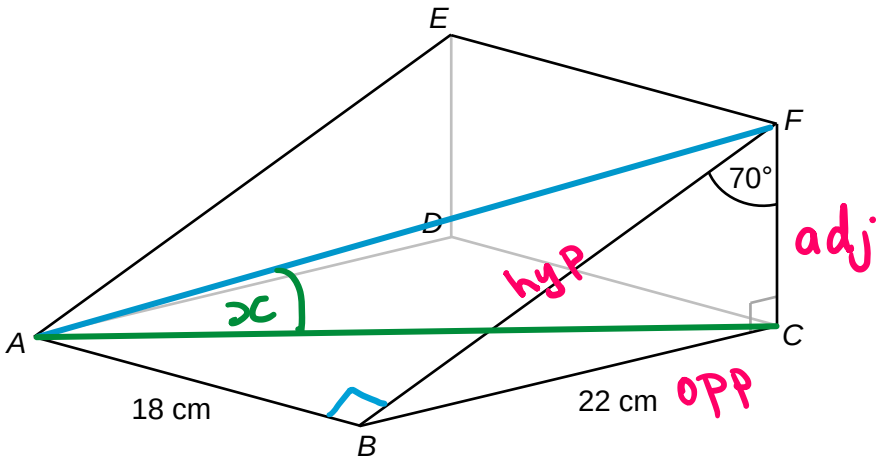
$AC^2 = x^2 + x^2$
$56^2 = x^2 + x^2$
$3136 = 2x^2$ ÷2
$1568 = x^2$ ÷2
$x = \sqrt{1568}$

$\text{Volume} = \frac{1}{3} \times \sqrt{1568} \times \sqrt{1568} \times 45$

Answer 23520 cm^3

9 Here is a triangular prism.

AB = 18 cm BC = 22 cm Angle BFC = 70°



9 (a) Work out the length of AF giving your answer to 1 decimal place. **[4 marks]**

$\sin(70) = \frac{22}{BF}$

$BF = \frac{22}{\sin(70)}$

$BF = 23.411\ldots$

$AF^2 = AB^2 + BF^2$

$AF^2 = 18^2 + 23.4\ldots^2$

$AF^2 = 872.11757\ldots$

$AF = \sqrt{872.11757\ldots}$

$AF = 29.5316\ldots$

Answer 29.5 cm

9 (b) Work out the size of angle FAC giving your answer to 1 decimal place. **[4 marks]**

$AC^2 = AB^2 + BC^2$

$AC^2 = 18^2 + 22^2$

$AC^2 = 808$

$AC = \sqrt{808}$

$\cos(x) = \frac{\sqrt{808}}{29.53\ldots}$

$\cos(x) = 0.9625\ldots$

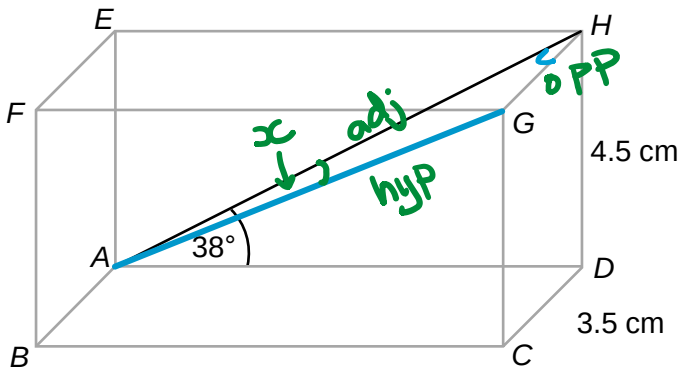
$x = \cos^{-1}(0.9625\ldots)$

Answer 15.7 °

12

10 Here is a cuboid.

CD = 3.5 cm DH = 4.5 cm Angle HAD = 38°



10 (a) Work out the length of AG giving your answer to 1 decimal place. **[4 marks]**

$\sin(38) = \frac{4.5}{AH}$

$AH = \frac{4.5}{\sin(38)}$

$= 7.309\ldots$

$AG^2 = AH^2 + HG^2$

$AG^2 = 7.309\ldots^2 + 3.5^2$

$AG^2 = 65.674\ldots$

$AG = \sqrt{65.674\ldots}$

$AG = 8.1039\ldots$

Answer 8.1 cm

10 (b) Work out the size of angle HAG giving your answer to 1 decimal place. **[2 marks]**

$\sin(x) = \frac{3.5}{8.10\ldots}$

$\sin(x) = 0.43188\ldots$

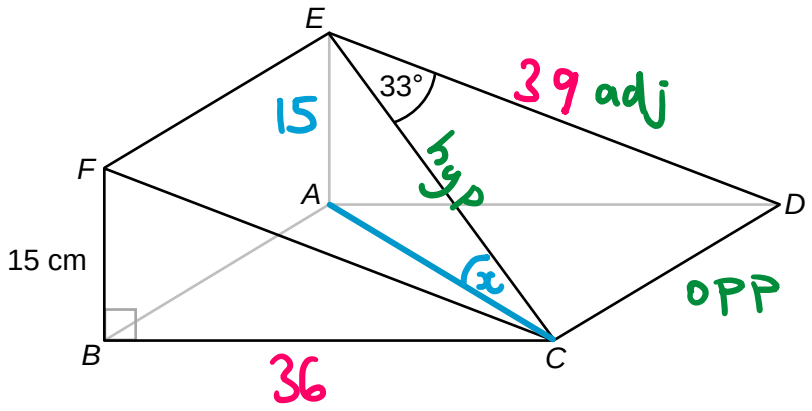
$x = \sin^{-1}(0.43188\ldots)$

$x = 25.5873\ldots$

Answer 25.6 °

11 Here is a triangular prism.

BF = 15 cm Angle *CED* = 33°
BF : *BC* = 5 : 12



Work out the size of angle *ACE* giving your answer to 1 decimal place. **[6 marks]**

$BC = \frac{15}{5} \times 12$

$= 36\text{cm}$

$FC^2 = 15^2 + 36^2$

$FC^2 = 1521$

$FC = \sqrt{1521}$

$FC = 39$

$FC = ED = 39$

$\cos(33) = \frac{39}{EC}$

$EC = \frac{39}{\cos(33)}$

$EC = 46.502\ldots$

$\sin(x) = \frac{15}{46.502\ldots}$

$\sin(x) = 0.3225\ldots$

$x = \sin^{-1}(0.322\ldots)$

$x = 18.81815\ldots$

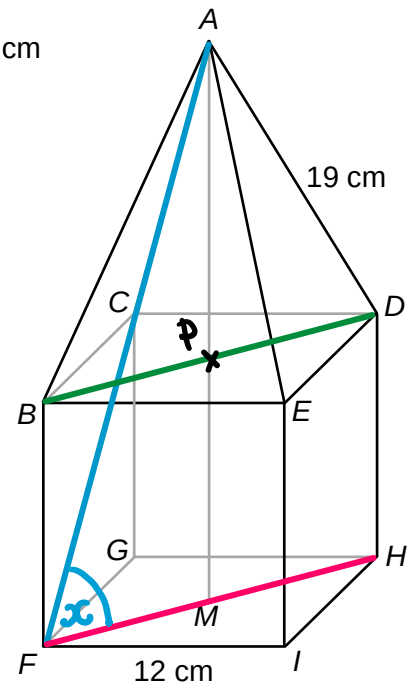
Answer 18.8 °

12

Turn over ►

12 *ABCDE* is a square-based pyramid placed on top of cube *BCDEFGHI*
M is the midpoint of the line *FH* with *FH* perpendicular to *MA*.

FI = 12 cm *AD* = 19 cm



Work out the size of angle *AFM* giving your answer to 1 decimal place. **[6 marks]**

$FH^2 = 12^2 + 12^2$

$FH^2 = 288$

$FH = \sqrt{288}$

$MH = \sqrt{288} \div 2$

$MH = 8.485\ldots$

$\tan(x) = \dfrac{29}{\sqrt{288} \div 2}$

$\tan(x) = 3.4176\ldots$

$PA^2 = 19^2 - PD^2$

$PA^2 = 19^2 - 8.485\ldots^2$

$PA^2 = 289$

$PA = \sqrt{289}$

$PA = 17$

$MA = 17 + 12$

$MA = 29$

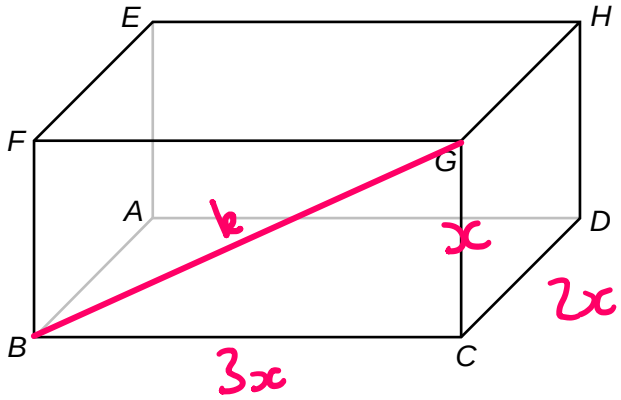
$x = \tan^{-1}(3.417\ldots)$

$x = 73.6907\ldots$

Answer 73.7 °

13 Here is a cuboid.

$CG : CD : CB = 1 : 2 : 3$
$BG = k$ cm



Show that the volume of the cuboid can be written in the form $\frac{3\sqrt{a}}{b}k^3$ where a and b are integers.

[6 marks]

$x^2 + (3x)^2 = k^2$

$x^2 + 9x^2 = k^2$

$10x^2 = k^2$

$x^2 = \frac{k^2}{10}$

$x = \frac{k}{\sqrt{10}}$

$x = \frac{k\sqrt{10}}{10}$

Volume $= x \times 2x \times 3x$

$= 6x^3$

$= 6 \times \left(\frac{k\sqrt{10}}{10}\right)^3$

$= 6 \times \frac{k^3 \times 10\sqrt{10}}{1000}$

$V = \frac{60\sqrt{10}\,k^3}{1000}$

$V = \frac{3\sqrt{10}}{50}k^3$

12