# Discrete Random Variables

REVISE THIS TOPIC

**1** $X$ and $Y$ are independent discrete random variables with the following probability distributions.

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(X = x)$ | $p$ | 0.25 | 0.25 | 0.35 |

| $y$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $P(Y = y)$ | 0.2 | 0.3 | 0.4 | $q$ |

(a) Find the values of $p$ and $q$ **(2)**

(b) Find

(i) $P(X > 2)$ **(1)**

(ii) $P(Y < 6)$ **(1)**

(iii) $P(Y \leq 6)$ **(1)**

(iv) $P(X > Y)$ **(2)**

$Z$ is a discrete random variable with values $z = 0, 1, 2, 3, 4$

The probability distribution for $Z$ is a discrete uniform distribution.

(c) Find

(i) $P(Z = 3)$ **(1)**

(ii) $P(Z > 1)$ **(1)**

(a) $p + 0.25 + 0.25 + 0.35 = 1$ $\qquad$ $q + 0.2 + 0.3 + 0.4 = 1$

$p + 0.85 = 1$ $\qquad$ $q + 0.9 = 1$

$p = 0.15$ $\qquad$ $q = 0.1$

(b) (i) $P(X > 2) = 0.25 + 0.35 = 0.6$

(ii) $P(Y < 6) = 0.2 + 0.3 = 0.5$

(iii) $P(Y \leq 6) = 0.2 + 0.3 + 0.4 = 0.9$

(iv) $P(X > Y) = P(X = 4 \text{ and } Y = 2)$ or $P(X = 3 \text{ and } Y = 2)$

$= (0.35 \times 0.2) + (0.25 \times 0.2)$

$= 0.12$

(c)

| $z$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(Z = z)$ | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |

(i) $P(Z = 3) = 0.2$

(i) $P(Z > 1) = 0.2 + 0.2 + 0.2 = 0.6$

**(Total for Question 1 is 9 marks)**

**2** $X$ is a random variable with the following probability distribution.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P(X = x)$ | $p$ | $q$ | $r$ | 0.14 | 0.16 | 0.19 | 0.21 |

(a) Find $P(4 \le X < 6)$ **(1)**

$P(X > 1) = 0.88$
$P(X = 0) = 2 \times P(X = 1)$

(b) Find the values of $p$, $q$ and $r$. **(3)**

The random variable $Y = X^2$

(c) Find
(i) $P(Y = 3)$ **(1)**
(ii) $P(Y > 4)$ **(1)**
(iii) $P(X < Y)$ **(1)**

(a) $P(4 \le X < 6) = 0.16 + 0.19 = 0.35$

(b) $P(X > 1) = 0.88$

$r + 0.14 + 0.16 + 0.19 + 0.21 = 0.88$

$r + 0.7 = 0.88$

$\mathbf{r = 0.18}$

$P(X = 0) = 2 \times P(X = 1)$

$p = 2q$

$p + q + 0.88 = 1$

$p + q = 0.12$

$2q + q = 0.12$

$3q = 0.12$

$\mathbf{q = 0.04}$

$p = 2q$

$p = 2(0.04)$

$\mathbf{p = 0.08}$

(c) (i) $P(Y = 3) = P(X = \sqrt{3}) = 0$

(ii) $P(Y > 4) = P(X = 2)$

$= 0.14 + 0.16 + 0.19 + 0.21$

$= 0.7$

(iii) $P(X < Y) = P(X > 1)$

$= 0.88$

(Since when $X = 0$, $Y = 0$, and when $X = 1$, $Y = 1$, otherwise $X < Y$)

**(Total for Question 2 is 7 marks)**

**3** $X$ is a random variable with the following probability distribution.

| $x$ | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| $P(X = x)$ | $k$ | $k$ | $3k$ | $5k$ | $5k$ |

(a) Find the value of $k$ **(2)**

(b) Find
(i) $P(X \leq 6)$ **(1)**
(ii) $P(5 < X < 9)$ **(1)**

The random variable $Y = 4X$

$n$ is an integer with $P(X + Y < n) > 0.5$

(c) Find the minimum possible value for $n$. **(3)**

(a) $k + k + 3k + 5k + 5k = 1$

$15k = 1$

$k = \frac{1}{15}$

(b) (i) $P(X \leq 6) = k + k = \frac{2}{15}$

(ii) $P(5 < X < 9) = k + 3k + 5k = \frac{9}{15}$

(c) $P(X + Y < n) = P(X + 4X < n) = P(5X < n)$

| $x$ | 25 | 30 | 35 | 40 | 45 |
|---|---|---|---|---|---|
| $P(5X = x)$ | $\frac{1}{15}$ | $\frac{1}{15}$ | $\frac{3}{15}$ | $\frac{5}{15}$ | $\frac{5}{15}$ |

$P(5X < 40) = \frac{1}{15} + \frac{1}{15} + \frac{3}{15} = \frac{4}{15} = 0.266\ldots < 0.5$

$P(5X < 41) = \frac{1}{15} + \frac{1}{15} + \frac{3}{15} + \frac{5}{15} = \frac{10}{15} = 0.666\ldots > 0.5$

Minimum $n = 41$

**(Total for Question 3 is 7 marks)**

**4** $X$ is a random variable with the following probability distribution.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | $p$ | $p + q$ | $p - q$ | $4q$ | $p + 0.08$ |

$P(X < 3) = 0.305$

(a) Find $P(X = 2)$ **(4)**

$Y$ is a random variable with the following probability distribution.

| $y$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(Y = y)$ | $a$ | $12b^2$ | $4c$ |

The probability distribution for $Y$ is a discrete uniform distribution.

(b) Given that $a$, $b$ and $c$ are all positive, find the value of $a + b + c$. **(4)**

(a) $p + p + q + p - q + 4q + p + 0.08 = 1$

$4p + 4q + 0.08 = 1$

$4p + 4q = 0.92$

$p + q = 0.23$

$P(X < 3) = 0.305$

$p + p + q = 0.305$

$p + 0.23 = 0.305$

$\boldsymbol{p = 0.075}$

$p + q = 0.23$

$0.075 + q = 0.23$

$\boldsymbol{q = 0.155}$

(b) $a = \frac{1}{3}$

$12b^2 = \frac{1}{3}$

$b^2 = \frac{1}{36}$

$b = \pm \frac{1}{6}$

$b = \frac{1}{6}$ (as $b > 0$)

$4c = \frac{1}{3}$

$c = \frac{1}{12}$

$a + b + c = \frac{1}{3} + \frac{1}{6} + \frac{1}{12}$

$= \frac{7}{12}$

**(Total for Question 4 is 8 marks)**

**5** $X$, $Y$ and $Z$ are independent discrete random variables with the following probability distributions.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.2 | 0.1 | 0.05 | 0.25 | 0.4 |

| $y$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $P(Y = y)$ | 0.24 | 0.18 | 0.11 | 0.15 | 0.32 |

| $z$ | 5 | 6 | 7 | 8 |
|---|---|---|---|---|
| $P(Z = z)$ | 0.4 | 0.07 | 0.13 | 0.4 |

(a) Find
(i) $P(2 < X < 5)$ **(1)**
(ii) $P(Y > 2)$ **(1)**

(b) Find
(i) $P(X + Y > 11)$ **(2)**
(ii) $P(X + Z = 11)$ **(2)**
(iii) $P(Y > Z)$ **(3)**

(a) (i) $P(2 < X < 5) = 0.05 + 0.25 = 0.3$

(ii) $P(Y > 2) = 0.11 + 0.15 + 0.32 = 0.58$

(b) (i) $P(X + Y > 11) = P(X = 5 \text{ and } Y = 8) \text{ or } P(X = 4 \text{ and } Y = 8)$

$= (0.4 \times 0.32) + (0.25 \times 0.32)$

$= 0.208$

(ii) $P(X + Z = 11) = P(X = 3 \text{ and } Z = 8) \text{ or } P(X = 4 \text{ and } Z = 7) \text{ or } P(X = 5 \text{ and } Z = 6)$

$= (0.05 \times 0.4) + (0.25 \times 0.13) + (0.4 \times 0.07)$

$= 0.0805$

(iii) $P(Y > Z) = P(Y = 8 \text{ and } Z = 7) \text{ or } P(Y = 8 \text{ and } Z = 6) \text{ or } P(Y = 8 \text{ and } Z = 5) \text{ or } P(Y = 6 \text{ and } Z = 5)$

$= (0.32 \times 0.13) + (0.32 \times 0.07) + (0.32 \times 0.4) + (0.15 \times 0.4)$

$= 0.252$

**(Total for Question 5 is 9 marks)**

**6** A bag contains 5 blue counters, 1 red counters and 4 green counters.

Three counters are taken at random from the bag without replacement.

The discrete random variable $X$ represents the number of red counters selected.
The discrete random variable $Y$ represents the number of green counters selected.

(a) Find the complete probability distribution of $X$. **(3)**
(b) Find the complete probability distribution of $Y$. **(4)**

A different bag contains 4 blue counters and 1 red counter.

Tom randomly takes counters from the bag until he has taken out the red counter.
The discrete random variable $M$ represents the number of counters Tom takes from the bag.

(c) Find the complete probability distribution for $M$. **(3)**

(a) $x = 0, 1$

$P(X = 0) = P(NR, NR, NR) = \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8} = \frac{7}{10}$

$P(X = 1) = 1 - P(X = 0)$

$= 1 - \frac{7}{10}$

$= \frac{3}{10}$

| $x$ | 0 | 1 |
|---|---|---|
| $P(X = x)$ | $\frac{7}{10}$ | $\frac{3}{10}$ |

(b) $y = 0, 1, 2, 3$

$P(Y = 0) = P(NG, NG, NG) = \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{1}{6}$

$P(Y = 1) = P(G, NG, NG) + P(NG, G, NG) + P(NG, NG, G)$

$= \left(\frac{4}{10} \times \frac{6}{9} \times \frac{5}{8}\right) + \left(\frac{6}{10} \times \frac{4}{9} \times \frac{5}{8}\right) + \left(\frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}\right)$

$= \frac{1}{2}$

$P(Y = 2) = P(G, G, NG) + P(G, NG, G) + P(NG, G, G)$

$= \left(\frac{4}{10} \times \frac{3}{9} \times \frac{6}{8}\right) + \left(\frac{4}{10} \times \frac{6}{9} \times \frac{3}{8}\right) + \left(\frac{6}{10} \times \frac{4}{9} \times \frac{3}{8}\right)$

$= \frac{3}{10}$

$P(Y = 3) = P(G, G, G) = \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8} = \frac{1}{30}$

| $y$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(Y = y)$ | $\frac{1}{6}$ | $\frac{1}{2}$ | $\frac{3}{10}$ | $\frac{1}{30}$ |

(c) $m = 1, 2, 3, 4, 5$

$P(M = 1) = P(R) = \frac{1}{5}$

$P(M = 2) = P(NR, R) = \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$

$P(M = 3) = P(NR, NR, R) = \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} = \frac{1}{5}$

$P(M = 4) = P(NR, NR, NR, R) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{5}$

$P(M = 5) = P(NR, NR, NR, NR, R) = \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{5}$

| $m$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(M = m)$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ |

**(Total for Question 6 is 11 marks)**



 

7 At the end of the year Hannah completes exams in three subjects.

The probability that Hannah passes each of the subjects is shown below.

| Subject | Maths | Chemistry | History |
|---|---|---|---|
| Probability of passing | 0.9 | $p$ | 0.6 |

The outcome of each exam is independent of the others.
The probability that Hannah passes all three exams is 0.459

(a) Find the value of $p$. **(1)**

The discrete random variable $X$ represents the number of exams that Hannah passes.

(b) Find the complete probability distribution of $X$. **(4)**

(a) $0.9 \times p \times 0.6 = 0.459$

$p = 0.85$

(b) $x = 0, 1, 2, 3$

| Subject | Maths | Chemistry | History |
|---|---|---|---|
| Probability of passing | 0.9 | 0.85 | 0.6 |
| Probability of NOT passing | 0.1 | 0.15 | 0.4 |

$P(X = 0) = P(NP, NP, NP) = 0.1 \times 0.15 \times 0.4 = 0.006$

$P(X = 1) = P(P, NP, NP) + P(NP, P, NP) + P(NP, NP, P)$

$= (0.9 \times 0.15 \times 0.4) + (0.1 \times 0.85 \times 0.4) + (0.1 \times 0.15 \times 0.6)$

$= 0.097$

$P(X = 2) = P(P, P, NP) + P(P, NP, P) + P(NP, P, P)$

$= (0.9 \times 0.85 \times 0.4) + (0.9 \times 0.15 \times 0.6) + (0.1 \times 0.85 \times 0.6)$

$= 0.438$

$P(X = 3) = P(P, P, P) = 0.9 \times 0.85 \times 0.6 = 0.459$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | 0.006 | 0.097 | 0.438 | 0.459 |

**(Total for Question 7 is 5 marks)**

**8** Kat plays a game at a fair that costs £5 per play.

The probability that Kat wins the game is 0.3

Kat has £20 and plays the game either until she has won the game twice or until she has no money left.

The discrete random variable $X$ represents the number of times that Kat plays the game.

(a) Find the complete probability distribution of $X$. **(4)**

The discrete random variable $Y$ represents the number of times that Kat wins the game.

(b) Find the complete probability distribution of $Y$. **(4)**

(a) $x = 2, 3, 4$

$P(X = 2) = P(W, W) = 0.3 \times 0.3 = 0.09$

$P(X = 3) = P(L, W, W) + P(W, L, W) = (0.7 \times 0.3 \times 0.3) + (0.3 \times 0.7 \times 0.3) = 0.126$

[Note: P(W, W, L) cannot happen as Kat stops at 2 wins]

$P(X = 4) = 1 - P(X = 2) - P(X = 3)$

$= 1 - 0.09 - 0.126$

$= 0.784$

| $x$ | 2 | 3 | 4 |
|---|---|---|---|
| $P(X = x)$ | 0.09 | 0.126 | 0.784 |

(b) $y = 0, 1, 2$ Let W be the event Kat wins the game and L be the event Kat does not win the game.

$P(Y = 0) = P(L, L, L, L) = 0.7 \times 0.7 \times 0.7 \times 0.7 = 0.2401$

$P(Y = 1) = P(W, L, L, L) + P(L, W, L, L) + P(L, L, W, L) + P(L, L, L, W)$

$= (0.3 \times 0.7 \times 0.7 \times 0.7) + (0.7 \times 0.3 \times 0.7 \times 0.7) + (0.7 \times 0.7 \times 0.3 \times 0.7) + (0.7 \times 0.7 \times 0.7 \times 0.3)$

$= 0.4116$

$P(Y = 2) = 1 - P(Y = 0) - P(Y = 1)$

$= 1 - 0.2401 - 0.4116$

$= 0.3483$

| $y$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(Y = y)$ | 0.2401 | 0.4116 | 0.3483 |

**(Total for Question 8 is 8 marks)**

**9** A biased dice can land on the numbers 1, 2, 3, 4, 5, or 6.

The random variable $X$ represents the number that the dice lands on.

$P(X = r) = P(X = 7 - r)$ for $r = 1, 2, 3$

Given that $P(X = 3) = 0.05$ and $P(X = 2) = 2 \times P(X = 1)$

(a) Find the complete probability distribution of $X$. **(4)**

The dice is rolled 3 times.

The random variable $Y$ represents the number of times that the dice lands on the number 2.

(b) Find the complete probability distribution of $Y$. **(4)**

(a) $P(X = 1) = P(X = 6) = p$

$P(X = 2) = P(X = 5) = q$

$P(X = 3) = P(X = 4) = 0.05$

$p + q + 0.05 + 0.05 + q + p = 1$

$2p + 2q + 0.1 = 1$

$2p + 2q = 0.9$

$p + q = 0.45$

$P(X = 2) = 2 \times P(X = 1)$

$q = 2p$

$p + 2p = 0.45$

$3p = 0.45$

$p = 0.15$

$p + q = 0.45$

$0.15 + q = 0.45$

$q = 0.3$

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(X = x)$ | 0.15 | 0.3 | 0.05 | 0.05 | 0.3 | 0.15 |

(b) $y = 0, 1, 2, 3$ Let Y be the event a 2 is rolled and N the event a 2 is not rolled.

$P(Y = 0) = P(N, N, N) = 0.7 \times 0.7 \times 0.7 = 0.343$

$P(Y = 1) = P(Y, N, N) + P(N, Y, N) + P(N, N, Y)$

$= (0.3 \times 0.7 \times 0.7) + (0.7 \times 0.3 \times 0.7) + (0.7 \times 0.7 \times 0.3)$

$= 0.441$

$P(Y = 2) = P(Y, Y, N) + P(Y, N, Y) + P(N, Y, Y)$

$= (0.3 \times 0.3 \times 0.7) + (0.3 \times 0.7 \times 0.3) + (0.7 \times 0.3 \times 0.3)$

$= 0.189$

$P(Y = 3) = P(Y, Y, Y) = 0.3 \times 0.3 \times 0.3 = 0.027$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X = x)$ | 0.343 | 0.441 | 0.189 | 0.027 |

**(Total for Question 9 is 8 marks)**

**10** $X$ is a random variable with the following probability distribution.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.12 | 0.16 | 0.2 | 0.24 | 0.28 |

$Y = X^2$
$Z = 2X$

(a) Find
(i) $P(X < 3)$ **(1)**
(ii) $P(Y < 3)$ **(1)**
(iii) $P(2 < Z \leq 10)$ **(1)**

(b) Find
(i) $P(Y = Z)$ **(1)**
(ii) $P(Y - 4X = -3)$ **(3)**
(iii) $P(Y + 8 \leq 3Z)$ **(3)**

(a) (i) $P(X < 3) = 0.12 + 0.16 = 0.28$

(ii)

| $y$ | 1 | 4 | 9 | 16 | 25 |
|---|---|---|---|---|---|
| $P(Y = y)$ | 0.12 | 0.16 | 0.2 | 0.24 | 0.28 |

$P(Y < 3) = 0.12$

(iii)

| $z$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $P(Z = z)$ | 0.12 | 0.16 | 0.2 | 0.24 | 0.28 |

$P(2 < Z < 10) = 0.16 + 0.2 + 0.24 + 0.28 = 0.88$

(b) (i) $P(Y = Z) = P(Y = 4 \text{ and } Z = 4) = 0.16 \times 0.16 = 0.0256$

(ii) $P(Y - 4X = -3)$
$= P(X^2 - 4X = -3)$
$= P(X^2 - 4X + 3 = 0)$
$= P([X - 3][X - 1] = 0)$
$= P(X = 3) + P(X = 1)$
$= 0.12 + 0.2$
$= 0.32$

(iii) $P(Y + 8 \leq 3Z)$
$= P(X^2 + 8 \leq 6X)$
$= P(X^2 - 6X + 8 \leq 0)$
$= P([X - 2][X - 4] \leq 0)$
$= P(2 \leq X \leq 4)$
$= 0.16 + 0.2 + 0.24$
$= 0.6$

**(Total for Question 10 is 10 marks)**

**11** The random variable $X$ has a probability function

$$P(X = x) = kx^2 \qquad x = 1, 2, 3$$

where $k$ is a constant.

(a) Find the value of $k$ **(2)**

(b) Find $P(X = 3)$ **(2)**

The random variable $Y$ has a probability function

$$P(Y = y) = (c - 3)y \qquad y = 1, 2, 3, 4$$

where $c$ is a constant.

(c) Find the value of $c$ **(2)**

(d) Find $P(Y = 2)$ **(2)**

(a) $P(X = 1) + P(X = 2) + P(X = 3) = 1$

$k + 4k + 9k = 1$

$14k = 1$

$k = \frac{1}{14}$

(b) $P(X = 3) = 9k$

$= 9 \times \frac{1}{14}$

$= \frac{9}{14}$

(c) $P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) = 1$

$(c - 3) + 2(c - 3) + 3(c - 3) + 4(c - 3) = 1$

$10(c - 3) = 1$

$c - 3 = 0.1$

$c = 3.1$

(d) $P(Y = 2) = 2(c - 3)$

$= 2(3.1 - 3)$

$= 0.2$

**(Total for Question 11 is 8 marks)**

**12** The random variable $X$ has a probability function

$$P(X = x) = (k + 1)x \qquad x = 1, 2, 3$$

where $k$ is a constant.

(a) By working out the value of $k$, find the complete probability distribution of $X$. **(4)**

The random variable $Y$ has a probability function

$$P(Y = y) = \frac{c - y^2}{100} \qquad y = 1, 2, 3, 4$$

where $c$ is a constant.

(b) By working out the value of $c$, find the complete probability distribution of $Y$. **(4)**

(a) $P(X = 1) + P(X = 2) + P(X = 3) = 1$

$(k + 1) + 2(k + 1) + 3(k + 1) = 1$

$6(k + 1) = 1$

$(k + 1) = \frac{1}{6}$

$k = -\frac{5}{6}$

$P(X = 1) = (-\frac{5}{6} + 1) \times 1 = \frac{1}{6}$

$P(X = 2) = (-\frac{5}{6} + 1) \times 2 = \frac{2}{6} = \frac{1}{3}$

$P(X = 3) = (-\frac{5}{6} + 1) \times 3 = \frac{3}{6} = \frac{1}{2}$

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X = x)$ | $\frac{1}{6}$ | $\frac{1}{3}$ | $\frac{1}{2}$ |

(b) $P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) = 1$

$\frac{c - 1^2}{100} + \frac{c - 2^2}{100} + \frac{c - 3^2}{100} + \frac{c - 4^2}{100} = 1$

$c - 1^2 + c - 2^2 + c - 3^2 + c - 4^2 = 100$

$4c - 30 = 10$

$4c = 130$

$c = 32.5$

$P(Y = 1) = \frac{32.5 - 1^2}{100} = 0.315$

$P(Y = 2) = \frac{32.5 - 2^2}{100} = 0.285$

$P(Y = 3) = \frac{32.5 - 3^2}{100} = 0.235$

$P(Y = 4) = \frac{32.5 - 4^2}{100} = 0.165$

| $y$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(Y = y)$ | 0.315 | 0.285 | 0.235 | 0.165 |

**(Total for Question 12 is 8 marks)**

**13** The random variable $X$ has a probability function

$$P(X = x) = \frac{1}{k - x} \qquad x = 1, 3$$

where $k$ is a constant.

Find the value of $k$ giving your answer in the form $a + \sqrt{b}$, where $a$ and $b$ are integers. **(5)**

$P(X = 1) + P(X = 3) = 1$

$$\frac{1}{k-1} + \frac{1}{k-3} = 1$$

$$k - 3 + k - 1 = (k - 1)(k - 3)$$

$$2k - 4 = k^2 - 4k + 3$$

$$0 = k^2 - 6k + 7$$

$$k = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 7}}{2}$$

$$k = \frac{6 \pm \sqrt{8}}{2}$$

$$k = \frac{6 \pm 2\sqrt{2}}{2}$$

$$k = 3 \pm \sqrt{2}$$

If $k = 3 - \sqrt{2}$ then

$P(X = 1) = \frac{1}{3 - \sqrt{2} - 1} = 1.70\ldots > 1$ [probability cannot be greater than 1]

$P(X = 3) = \frac{1}{3 - \sqrt{2} - 3} = -0.70\ldots < 0$ [probability cannot be less than 0]

Therefore $\mathbf{k = 3 + \sqrt{2}}$ with $P(X = 1) = 0.292\ldots$ and $P(X = 3) = 0.707\ldots$

**(Total for Question 13 is 5 marks)**

**14** The random variable $X$ has a probability function

$$P(X = x) = kx - c \qquad x = 5, 6, 7, 8$$

where $k$ and $c$ are constants.

Given that $P(X = 5) = 0.01$ find the values of $k$ and $c$. **(4)**

$P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8) = 1$

$5k - c + 6k - c + 7k - c + 8k - c = 1$

$26k - 4c = 1$

$P(X = 5) = 0.01$

$5k - c = 0.01$

$26k - 4c = 1$

$5k - c = 0.01 \quad (\times 4)$

$26k - 4c = 1$

$-\ 20k - 4c = 0.04$

$6k = 0.96$

$\boldsymbol{k = 0.16}$

$5 \times 0.16 - c = 0.01$

$0.8 - c = 0.01$

$c = 0.8 - 0.01$

$\boldsymbol{c = 0.79}$

**(Total for Question 14 is 4 marks)**

**15** The random variable $X$ has a probability function

$$P(X = x) = \begin{cases} kx & x = 1, 2 \\ (k - 0.2)x & x = 3, 4 \end{cases}$$

where $k$ is a constant.

(a) By working out the value of $k$, find the complete probability distribution of $X$. **(5)**

The random variable $Y$ has a probability function

$$P(Y = y) = \begin{cases} c(y + 0.5) & y = 0, 1 \\ cy^2 & y = 2, 3, 4 \end{cases}$$

where $c$ is a constant.

(b) By working out the value of $c$, find the complete probability distribution of $Y$. **(5)**

(a) $P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$

$k + 2k + 3(k - 0.2) + 4(k - 0.2) = 1$

$k + 2k + 3k - 0.6 + 4k - 0.8 = 1$

$10k - 1.4 = 1$

$10k = 2.4$

$k = 0.24$

$P(X = 1) = 0.24$

$P(X = 2) = 0.24 \times 2 = 0.48$

$P(X = 3) = (0.24 - 0.2) \times 3 = 0.12$

$P(X = 4) = (0.24 - 0.2) \times 4 = 0.16$

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $P(X = x)$ | 0.24 | 0.48 | 0.12 | 0.16 |

(b) $P(Y = 0) + P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) = 1$

$c(0 + 0.5) + c(1 + 0.5) + c \times 2^2 + c \times 3^2 + c \times 4^2 = 1$

$0.5c + 1.5c + 4c + 9c + 16c = 1$

$31c = 1$

$c = \frac{1}{31}$

$P(Y = 0) = \frac{1}{31}(0 + 0.5) = \frac{1}{62}$

$P(Y = 1) = \frac{1}{31}(1 + 0.5) = \frac{3}{62}$

$P(Y = 2) = \frac{1}{31} \times 2^2 = \frac{4}{31}$

$P(Y = 3) = \frac{1}{31} \times 3^2 = \frac{9}{31}$

$P(Y = 4) = \frac{1}{31} \times 4^2 = \frac{16}{31}$

| $y$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(Y = y)$ | $\frac{1}{62}$ | $\frac{3}{62}$ | $\frac{4}{31}$ | $\frac{9}{31}$ | $\frac{16}{31}$ |

**(Total for Question 15 is 10 marks)**