



SCAN ME

# Iteration



REVISE THIS  
TOPIC

1 (a) Use the iteration formula  $x_{n+1} = \frac{(x_n)^2 + 3}{5}$  to find the values of  $x_1$ ,  $x_2$  and  $x_3$

Start with  $x_0 = \sqrt{3}$

$$x_1 = \frac{(\sqrt{3})^2 + 3}{5} \\ = 1.2$$

$$x_3 = \frac{0.888^2 + 3}{5} \\ = 0.7577088$$

$$x_2 = \frac{1.2^2 + 3}{5} \\ = 0.888$$

$$x_1 = 1.2 \\ x_2 = 0.888 \\ x_3 = 0.7577088$$

(3)

(b) Explain the relationship between the values of  $x_1$ ,  $x_2$  and  $x_3$  and the equation  $x^2 - 5x + 3 = 0$

$x = \frac{x^2 + 3}{5}$  can be rearranged to give  
 $x^2 - 5x + 3 = 0$ . This means  $x_1$ ,  $x_2$  and  $x_3$   
 are estimates to a solution of the equation.  
 $x^2 - 5x + 3 = 0$

(2)

(Total for Question 1 is 5 marks)



1

2 (a) Use the iteration formula  $x_{n+1} = \sqrt{\frac{x_n}{10} + 2}$  to find the values of  $x_1$ ,  $x_2$  and  $x_3$

Start with  $x_0 = 42.5$

$$x_1 = \sqrt{\frac{42.5}{10} + 2}$$

$$= 2.5$$

$$x_2 = \sqrt{\frac{2.5}{10} + 2}$$

$$= 1.5$$

$$x_3 = \sqrt{\frac{1.5}{10} + 2}$$

$$= 1.46628783$$

$$x_1 = 2.5$$

$$x_2 = 1.5$$

$$x_3 = 1.46628783$$

(3)

The values of  $x_1$ ,  $x_2$  and  $x_3$  found in part (a) are estimates to the solution of an equation in the form  $ax^2 - x + b = 0$  where  $a$  and  $b$  are integers.

(b) Find the values of  $a$  and  $b$ .

$$x = \sqrt{\frac{x}{10} + 2}$$

$$10x^2 = x + 20$$

$$a = 10$$

$$x^2 = \frac{x}{10} + 2$$

$$10x^2 - x - 20 = 0$$

$$b = -20$$

(2)

(Total for Question 2 is 5 marks)

3 (a) Use the iteration formula  $x_{n+1} = \sqrt{80 - 5x_n}$  to find the values of  $x_1$ ,  $x_2$  and  $x_3$

Start with  $x_0 = 12.8$

$$x_1 = \sqrt{80 - 5(12.8)}$$

$$= 4$$

$$x_2 = \sqrt{80 - 5(4)}$$

$$= 7.745966692$$

$$x_3 = \sqrt{80 - 5(7.74...)}$$

$$= 6.42418606$$

$$x_1 = 4$$

$$x_2 = 7.745966692$$

$$x_3 = 6.42418606$$

(3)

The values of  $x_1$ ,  $x_2$  and  $x_3$  found in part (a) are estimates to the solution of an equation in the form  $x^2 + ax + b = 0$  where  $a$  and  $b$  are integers.

(b) Find the values of  $a$  and  $b$ .

$$x = \sqrt{80 - 5x}$$

$$a = 5$$

$$x^2 = 80 - 5x$$

$$x^2 + 5x - 80 = 0$$

$$b = -80$$

(2)

(Total for Question 3 is 5 marks)

1st

4 (a) Use the iteration formula  $x_{n+1} = \sqrt[3]{8 - (x_n)^2}$  to find the values of  $x_1$ ,  $x_2$  and  $x_3$

Start with  $x_0 = 1.8$

$$x_1 = \sqrt[3]{8 - 1.8^2}$$

$$= 1.682166517$$

$$x_2 = \sqrt[3]{8 - 1.68...^2}$$

$$= 1.729175302$$

$$x_3 = \sqrt[3]{8 - 1.72...^2}$$

$$= 1.711109795$$

$$x_1 = \underline{1.682166517}$$

$$x_2 = \underline{1.729175302}$$

$$x_3 = \underline{1.711109795}$$

(3)

(b) Explain the relationship between the values of  $x_1$ ,  $x_2$  and  $x_3$  and the equation  $x^3 + x^2 - 8 = 0$

$x = \sqrt[3]{8 - x^2}$  can be rearranged to give  
 $x^3 + x^2 - 8 = 0$ . This means  $x_1$ ,  $x_2$  and  $x_3$   
 are estimates to a solution of the equation.  
 $x^3 + x^2 - 8 = 0$

(2)

(Total for Question 4 is 5 marks)



5 (a) Show that the equation  $x^3 - x - 4 = 0$  has a solution between  $x = 1$  and  $x = 2$

$$1^3 - 1 - 4 = -4$$

$$2^3 - 2 - 4 = 2$$

change of sign so there must be a solution between  $x = 1$  and  $x = 2$

(b) Show that the equation  $x^3 - x - 4 = 0$  can be rearranged to give  $x = \sqrt[3]{x + 4}$  (2)

$$x^3 - 4 = x$$

$$x^3 = x + 4$$

$$x = \sqrt[3]{x + 4}$$

(2)

(c) Starting with  $x_0 = 2$ , use the iteration formula  $x_{n+1} = \sqrt[3]{x_n + 4}$  three times to find an estimate for the solution of  $x^3 - x - 4 = 0$

$$x_1 = \sqrt[3]{2 + 4} \\ = 1.817120593$$

$$x_2 = \sqrt[3]{1.81... + 4} \\ = 1.798467893$$

$$x_3 = \sqrt[3]{1.79... + 4} \\ = 1.796543562$$

1.796543562

(3)

(d) By substituting your answer to part (c) into  $x^3 - x - 4$  comment on the accuracy of your estimate for the solution to  $x^3 - x - 4 = 0$

$$(1.796...)^3 - (1.796...) - 4 = 0.00192431...$$

This is close to zero so an accurate solution

(2)

(Total for Question 5 is 9 marks)

1st

6 (a) Show that the equation  $x^2 + x - 13 = 0$  has a solution between  $x = 3$  and  $x = 4$

$3^2 + 3 - 13 = -1$  change of sign so there must  
 $4^2 + 4 - 13 = 7$  be a solution between  
 $x = 3$  and  $x = 4$

(2)

(b) Show that the equation  $x^2 + x - 13 = 0$  can be rearranged to give  $x = \sqrt{13 - x}$

$$\begin{aligned}x^2 + x - 13 &= 0 \\x^2 + x &= 13 \\x^2 &= 13 - x \\x &= \sqrt{13 - x}\end{aligned}$$

(2)

(c) Starting with  $x_0 = 3$ , use the iteration formula  $x_{n+1} = \sqrt{13 - x_n}$  three times to find an estimate for the solution of  $x^2 + x - 13 = 0$

$$\begin{aligned}x_1 &= \sqrt{13 - 3} \\&= 3.16227766 \\x_2 &= \sqrt{13 - 3.16\dots} \\&= 3.136514361 \\x_3 &= \sqrt{13 - 3.14\dots} \\&= 3.140618671\end{aligned}$$

3.140618671  
 (3)

(d) By substituting your answer to part (c) into  $x^2 + x - 13 = 0$   
 comment on the accuracy of your estimate for the solution to  $x^2 + x - 13 = 0$

$(3.14\dots)^2 + (3.14\dots) - 13 = 0.004104309809$

This is close to zero so an accurate solution

1st

(2)

(Total for Question 6 is 9 marks)

7 (a) Show that the equation  $x^2 - 10x + 6 = 0$  has a solution between  $x = 0$  and  $x = 1$

$$0^2 - 10(0) + 6 = 6$$

$$1^2 - 10(1) + 6 = -3$$

change of sign so there must be a solution between  $x = 0$  and  $x = 1$

(2)

(b) Show that the equation  $x^2 - 10x + 6 = 0$  can be rearranged to give  $x = \frac{x^2 + 6}{10}$

$$x^2 - 10x + 6 = 0$$

$$x^2 + 6 = 10x$$

$$\frac{x^2 + 6}{10} = x$$

$$x = \frac{x^2 + 6}{10}$$

(2)

(c) Starting with  $x_0 = 1$ , use the iteration formula  $x_{n+1} = \frac{(x_n)^2 + 6}{10}$  three times to find an estimate for the solution of  $x^2 - 10x + 6 = 0$

$$x_1 = \frac{1^2 + 6}{10} = 0.7$$

$$x_3 = \frac{0.649^2 + 6}{10} = 0.6421201$$

$$x_2 = \frac{0.7^2 + 6}{10} = 0.649$$

0.6421201

(3)

(d) By substituting your answer to part (c) into  $x^2 - 10x + 6 = 0$  comment on the accuracy of your estimate for the solution to  $x^2 - 10x + 6 = 0$

$$(0.642\ldots)^2 - 10(0.642\ldots) + 6 = -0.008882777\ldots$$

This is close to zero so an accurate solution

(2)

(Total for Question 7 is 9 marks)

1st

8 (a) Show that the equation  $x^3 - 20x^2 + 100x - 8 = 0$  has a solution between  $x = 10$  and  $x = 11$

$10^3 - 20(10)^2 + 100(10) - 8 = -8$  change of sign so there must  
 $11^3 - 20(11)^2 + 100(11) - 8 = 3$  be a solution between  
 $x = 10$  and  $x = 11$  (2)

(b) Show that the equation  $x^3 - 20x^2 + 100x - 8 = 0$  can be rearranged to give  $x = \sqrt{\frac{8}{x}} + 10$

$$\begin{aligned}x^3 - 20x^2 + 100x &= 8 \\x(x^2 - 20x + 100) &= 8 \\x^2 - 20x + 100 &= \frac{8}{x} \\(x-10)(x-10) &= \frac{8}{x} \\(x-10)^2 &= \frac{8}{x} \\x-10 &= \sqrt{\frac{8}{x}}\end{aligned}$$

$$x = \sqrt{\frac{8}{x}} + 10$$

(4)

(c) Starting with  $x_0 = 2$ , use the iteration formula  $x_{n+1} = \sqrt{\frac{8}{x_n}} + 10$  three times to find an

estimate for the solution of  $x^3 - 20x^2 + 100x - 8 = 0$

$$\begin{aligned}x_1 &= \sqrt{\frac{8}{2}} + 10 & x_2 &= \sqrt{\frac{8}{12}} + 10 & x_3 &= \sqrt{\frac{8}{10.8...}} + 10 \\&= 12 & &= 10.81649658 & &= 10.8600064\end{aligned}$$

$$\underline{\underline{10.8600064}} \quad (3)$$

(d) By substituting your answer to part (c) into  $x^3 - 20x^2 + 100x - 8$

comment on the accuracy of your estimate for the solution to  $x^3 - 20x^2 + 100x - 8 = 0$

$$\underline{\underline{(10.86...)}^3 - 20(10.86...)^2 + 100(10.86...)} - 8 = 0.03218034378$$

This is close to zero so an accurate solution

(2)  
 (Total for Question 8 is 9 marks)

1st