



Iteration



REVISE THIS
TOPIC

- 1 (a) Use the iteration formula $x_{n+1} = \frac{(x_n)^2 + 3}{5}$ to find the values of x_1 , x_2 and x_3

Start with $x_0 = \sqrt{3}$

$$x_1 = \frac{(\sqrt{3})^2 + 3}{5} \\ = 1.2$$

$$x_3 = \frac{0.888^2 + 3}{5} \\ = 0.7577088$$

$$x_2 = \frac{1.2^2 + 3}{5} \\ = 0.888$$

$$x_1 = 1.2$$

$$x_2 = 0.888$$

$$x_3 = 0.7577088$$

(3)

- (b) Explain the relationship between the values of x_1 , x_2 and x_3 and the equation $x^2 - 5x + 3 = 0$

$x = \frac{x^2 + 3}{5}$ can be rearranged to give
 $x^2 - 5x + 3 = 0$. This means x_1 , x_2 and x_3
 are estimates to a solution of the equation.
 $x^2 - 5x + 3 = 0$

(2)

(Total for Question 1 is 5 marks)



- 2 (a) Use the iteration formula $x_{n+1} = \sqrt{\frac{x_n}{10} + 2}$ to find the values of x_1 , x_2 and x_3

Start with $x_0 = 42.5$

$$x_1 = \sqrt{\frac{42.5}{10} + 2} = 2.5$$

$$x_3 = \sqrt{\frac{1.5}{10} + 2} = 1.46628783$$

$$x_2 = \sqrt{\frac{2.5}{10} + 2} = 1.5$$

$$x_1 = 2.5$$

$$x_2 = 1.5$$

$$x_3 = 1.46628783$$

(3)

The values of x_1 , x_2 and x_3 found in part (a) are estimates to the solution of an equation in the form $ax^2 - x + b = 0$ where a and b are integers.

- (b) Find the values of a and b .

$$x = \sqrt{\frac{x}{10} + 2}$$

$$10x^2 = x + 20$$

$$10x^2 - x - 20 = 0$$

$$x^2 = \frac{x}{10} + 2$$

$$a = 10$$

$$b = -20$$

(2)

(Total for Question 2 is 5 marks)

- 3 (a) Use the iteration formula $x_{n+1} = \sqrt{80 - 5x_n}$ to find the values of x_1 , x_2 and x_3

Start with $x_0 = 12.8$

$$x_1 = \sqrt{80 - 5(12.8)} = 4$$

$$x_3 = \sqrt{80 - 5(7.74...)} = 6.42418606$$

$$x_2 = \sqrt{80 - 5(4)} = 7.745966692$$

$$x_1 = 4$$

$$x_2 = 7.745966692$$

$$x_3 = 6.42418606$$

(3)

The values of x_1 , x_2 and x_3 found in part (a) are estimates to the solution of an equation in the form $x^2 + ax + b = 0$ where a and b are integers.

- (b) Find the values of a and b .

$$x = \sqrt{80 - 5x}$$

$$x^2 = 80 - 5x$$

$$x^2 + 5x - 80 = 0$$

$$a = 5$$

$$b = -80$$

(2)

(Total for Question 3 is 5 marks)



- 4 (a) Use the iteration formula $x_{n+1} = \sqrt[3]{8 - (x_n)^2}$ to find the values of x_1 , x_2 and x_3

Start with $x_0 = 1.8$

$$x_1 = \sqrt[3]{8 - 1.8^2} = 1.682166517$$

$$x_2 = \sqrt[3]{8 - 1.68...^2} = 1.729175302$$

$$x_3 = \sqrt[3]{8 - 1.72...^2} = 1.711109795$$

$$\begin{array}{l}
 x_1 = 1.682166517 \\
 x_2 = 1.729175302 \\
 x_3 = 1.711109795
 \end{array}$$

(3)

- (b) Explain the relationship between the values of x_1 , x_2 and x_3 and the equation $x^3 + x^2 - 8 = 0$

$x = \sqrt[3]{8 - x^2}$ can be rearranged to give
 $x^3 + x^2 - 8 = 0$. This means x_1 , x_2 and x_3
 are estimates to a solution of the equation.
 $x^3 + x^2 - 8 = 0$

(2)

(Total for Question 4 is 5 marks)



- 5 (a) Show that the equation $x^3 - x - 4 = 0$ has a solution between $x = 1$ and $x = 2$

$$1^3 - 1 - 4 = -4$$

$$2^3 - 2 - 4 = 2$$

change of sign so there must be a solution between $x = 1$ and $x = 2$

- (b) Show that the equation $x^3 - x - 4 = 0$ can be rearranged to give $x = \sqrt[3]{x+4}$ (2)

$$x^3 - 4 = x$$

$$x^3 = x + 4$$

$$x = \sqrt[3]{x+4}$$

- (c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{x_n + 4}$ three times to find an estimate for the solution of $x^3 - x - 4 = 0$ (2)

$$x_1 = \sqrt[3]{2+4} = 1.817120593$$

$$x_2 = \sqrt[3]{1.81... + 4} = 1.798467893$$

$$x_3 = \sqrt[3]{1.79... + 4} = 1.796543562$$

$$\underline{1.796543562}$$

- (d) By substituting your answer to part (c) into $x^3 - x - 4$ comment on the accuracy of your estimate for the solution to $x^3 - x - 4 = 0$ (3)

$$(1.796...)^3 - (1.796...) - 4 = 0.001924331...$$

This is close to zero so an accurate solution



- 6 (a) Show that the equation $x^2 + x - 13 = 0$ has a solution between $x = 3$ and $x = 4$

$$\begin{aligned}
 3^2 + 3 - 13 &= -1 && \text{change of sign so there must} \\
 4^2 + 4 - 13 &= 7 && \text{be a solution between} \\
 &&& x=3 \text{ and } x=4
 \end{aligned}$$

(2)

- (b) Show that the equation $x^2 + x - 13 = 0$ can be rearranged to give $x = \sqrt{13 - x}$

$$\begin{aligned}
 x^2 + x - 13 &= 0 \\
 x^2 + x &= 13 \\
 x^2 &= 13 - x \\
 x &= \sqrt{13 - x}
 \end{aligned}$$

(2)

- (c) Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \sqrt{13 - x_n}$ three times to find an estimate for the solution of $x^2 + x - 13 = 0$

$$\begin{aligned}
 x_1 &= \sqrt{13 - 3} \\
 &= 3.16227766 \\
 x_2 &= \sqrt{13 - 3.16...} \\
 &= 3.136514361 \\
 x_3 &= \sqrt{13 - 3.14...} \\
 &= 3.140618671
 \end{aligned}$$

$$3.140618671$$

(3)

- (d) By substituting your answer to part (c) into $x^2 + x - 13$ comment on the accuracy of your estimate for the solution to $x^2 + x - 13 = 0$

$$(3.14...)^2 + (3.14...) - 13 = 0.004104309809$$

This is close to zero so an accurate solution

(2)

(Total for Question 6 is 9 marks)



7 (a) Show that the equation $x^2 - 10x + 6 = 0$ has a solution between $x = 0$ and $x = 1$

$$\begin{aligned}
 0^2 - 10(0) + 6 &= 6 \\
 1^2 - 10(1) + 6 &= -3
 \end{aligned}$$

change of sign so there must be a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^2 - 10x + 6 = 0$ can be rearranged to give $x = \frac{x^2 + 6}{10}$

$$\begin{aligned}
 x^2 - 10x + 6 &= 0 \\
 x^2 + 6 &= 10x \\
 \frac{x^2 + 6}{10} &= x
 \end{aligned}$$

$$x = \frac{x^2 + 6}{10}$$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \frac{(x_n)^2 + 6}{10}$ three times to find an estimate for the solution of $x^2 - 10x + 6 = 0$

$$\begin{aligned}
 x_1 &= \frac{1^2 + 6}{10} \\
 &= 0.7 \\
 x_2 &= \frac{0.7^2 + 6}{10} \\
 &= 0.649
 \end{aligned}$$

$$\begin{aligned}
 x_3 &= \frac{0.649^2 + 6}{10} \\
 &= 0.6421201
 \end{aligned}$$

$$\underline{0.6421201}$$

(3)

(d) By substituting your answer to part (c) into $x^2 - 10x + 6$ comment on the accuracy of your estimate for the solution to $x^2 - 10x + 6 = 0$

$$(0.642\dots)^2 - 10(0.642\dots) + 6 = -0.008882777\dots$$

This is close to zero so an accurate solution

(2)

(Total for Question 7 is 9 marks)



8 (a) Show that the equation $x^3 - 20x^2 + 100x - 8 = 0$ has a solution between $x = 10$ and $x = 11$

$$\begin{aligned}
 10^3 - 20(10)^2 + 100(10) - 8 &= -8 \text{ change of sign so there must} \\
 11^3 - 20(11)^2 + 100(11) - 8 &= 3 \text{ be a solution between} \\
 x &= 10 \text{ and } x = 11
 \end{aligned}$$

(2)

(b) Show that the equation $x^3 - 20x^2 + 100x - 8 = 0$ can be rearranged to give $x = \sqrt{\frac{8}{x}} + 10$

$$\begin{aligned}
 x^3 - 20x^2 + 100x &= 8 \\
 x(x^2 - 20x + 100) &= 8 \\
 x^2 - 20x + 100 &= \frac{8}{x} \\
 (x - 10)(x - 10) &= \frac{8}{x} \\
 (x - 10)^2 &= \frac{8}{x} \\
 x - 10 &= \sqrt{\frac{8}{x}}
 \end{aligned}$$

$$x = \sqrt{\frac{8}{x}} + 10$$

(4)

(c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt{\frac{8}{x_n}} + 10$ three times to find an

estimate for the solution of $x^3 - 20x^2 + 100x - 8 = 0$

$$\begin{aligned}
 x_1 &= \sqrt{\frac{8}{2}} + 10 = 12 \\
 x_2 &= \sqrt{\frac{8}{12}} + 10 = 10.81649658 \\
 x_3 &= \sqrt{\frac{8}{10.8...}} + 10 = 10.8600064
 \end{aligned}$$

$$10.8600064$$

(3)

(d) By substituting your answer to part (c) into $x^3 - 20x^2 + 100x - 8$ comment on the accuracy of your estimate for the solution to $x^3 - 20x^2 + 100x - 8 = 0$

$$(10.86...) ^3 - 20(10.86...) ^2 + 100(10.86...) - 8 = 0.03218034378$$

This is close to zero so an accurate solution

(2)

(Total for Question 8 is 9 marks)

