

Non-Linear Simultaneous Equations



REVISE THIS
TOPIC



SCAN ME

1 Solve algebraically the simultaneous equations

$$y = x^2 - 3x - 4$$
$$y = 2x - 10$$

$$x^2 - 3x - 4 = 2x - 10$$

$$x^2 - 5x - 4 = -10$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2 \quad x = 3$$

when $x = 2$

$$y = 2(2) - 10$$

$$y = -6$$

when $x = 3$

$$y = 2(3) - 10$$

$$y = -4$$

$$x = 2 \quad y = -6, \quad x = 3 \quad y = -4$$

(Total for Question 1 is 5 marks)

2 Solve algebraically the simultaneous equations

$$y = x^2 + 5x - 8$$
$$y - 4x = 4 \quad \leftarrow y = 4 + 4x$$

$$x^2 + 5x - 8 = 4 + 4x$$

$$x^2 + x - 8 = 4$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$x = -4 \quad x = 3$$

when $x = -4$

$$y = 4 + 4(-4)$$

$$y = -12$$

when $x = 3$

$$y = 4 + 4(3)$$

$$y = 16$$

$$x = -4 \quad y = -12, \quad x = 3 \quad y = 16$$

(Total for Question 2 is 5 marks)



3 Solve algebraically the simultaneous equations



$$y = 3x^2 + 2x - 8$$

$$y = 9x - 10$$

$$3x^2 + 2x - 8 = 9x - 10$$

$$3x^2 - 7x - 8 = -10$$

$$3x^2 - 7x + 2 = 0$$

$$(3x - 1)(x - 2) = 0$$

$$3x - 1 = 0 \quad x - 2 = 0$$

$$x = \frac{1}{3} \quad x = 2$$

$$\text{when } x = \frac{1}{3}$$

$$y = 9\left(\frac{1}{3}\right) - 8$$

$$y = -5$$

$$\text{when } x = 2$$

$$y = 9(2) - 8$$

$$y = 10$$

$$x = \frac{1}{3} \quad y = -5, \quad x = 2 \quad y = 10$$

(Total for Question 3 is 5 marks)

4 C is a graph with equation $y = x^2 - 5x + 10$



L is a straight line with equation $y = 3x - 6$

Using algebra, find the coordinates of the point of intersection of C and L.
You must show all your working.

$$x^2 - 5x + 10 = 3x - 6$$

$$x^2 - 8x + 10 = -6$$

$$x^2 - 8x + 16 = 0$$

$$(x - 4)(x - 4) = 0$$

$$x = 4$$

$$\text{when } x = 4$$

$$y = 3(4) - 6$$

$$y = 12 - 6$$

$$y = 6$$

$$(\dots 4 \dots, \dots 6 \dots)$$

(Total for Question 4 is 4 marks)



5 C is a graph with equation $y = 5x^2 - x - 15$



L is a straight line with equation $y = 10x - 3$

Using algebra, find the coordinates of the points of intersection of C and L.
You must show all your working.

$$\begin{aligned}
 5x^2 - x - 15 &= 10x - 3 && \text{when } x = -\frac{4}{5} \\
 5x^2 - 11x - 15 &= -3 && y = 10\left(-\frac{4}{5}\right) - 3 \\
 5x^2 - 11x - 12 &= 0 && y = -8 - 3 \\
 (5x + 4)(x - 3) &= 0 && y = -11 \\
 5x + 4 = 0 & \quad x - 3 = 0 && \text{when } x = 3 \\
 x = -\frac{4}{5} & \quad x = 3 && y = 10(3) - 3 \\
 &&& y = 27
 \end{aligned}$$

$\left(-\frac{4}{5}, -11\right)$
 $\left(3, 27\right)$

(Total for Question 5 is 5 marks)

6 Solve algebraically the simultaneous equations



$$\begin{aligned}
 y &= x^2 + x + 1 \\
 y &= x + 3
 \end{aligned}$$

Give your answers as exact values.

$$\begin{aligned}
 x^2 + x + 1 &= x + 3 && \text{when } x = \sqrt{2} \\
 x^2 + 1 &= 3 && y = \sqrt{2} + 3 \\
 x^2 - 2 &= 0 && \text{when } x = -\sqrt{2} \\
 x^2 &= 2 && y = -\sqrt{2} + 3 \\
 x &= \pm\sqrt{2}
 \end{aligned}$$

$$x = \sqrt{2} \quad y = 3 + \sqrt{2}, \quad x = -\sqrt{2} \quad y = 3 - \sqrt{2}$$

(Total for Question 6 is 5 marks)



7 Solve algebraically the simultaneous equations



$$x^2 + y^2 = 100$$

$$y = x - 2$$

$$x^2 + (x-2)^2 = 100$$

$$x^2 + x^2 - 2x - 2x + 4 = 100$$

$$2x^2 - 4x + 4 = 100$$

$$2x^2 - 4x - 96 = 0$$

$$x^2 - 2x - 48 = 0$$

$$(x-8)(x+6) = 0$$

$$x = 8 \quad x = -6$$

when $x = 8$

$$y = 8 - 2$$

$$y = 6$$

when $x = -6$

$$y = -6 - 2$$

$$y = -8$$

$$x = 8 \quad y = 6, \quad x = -6 \quad y = -8$$

(Total for Question 7 is 5 marks)

8 Solve algebraically the simultaneous equations



$$x^2 + y^2 = 200$$

$$y = 2x - 10$$

$$x^2 + (2x-10)^2 = 200$$

$$x^2 + 4x^2 - 20x - 20x + 100 = 200$$

$$5x^2 - 40x + 100 = 200$$

$$5x^2 - 40x - 100 = 0$$

$$x^2 - 8x - 20 = 0$$

$$(x-10)(x+2) = 0$$

$$x = 10 \quad x = -2$$

when $x = 10$

$$y = 2(10) - 10$$

$$y = 10$$

when $x = -2$

$$y = 2(-2) - 10$$

$$y = -14$$

$$x = 10 \quad y = 10, \quad x = -2 \quad y = -14$$

(Total for Question 8 is 5 marks)



9 C is a graph with equation $x^2 + y^2 = 65$



L is a straight line with equation $2y = 10 - x$ ← $x = 10 - 2y$

Using algebra, find the coordinates of the point of intersection of C and L.
You must show all your working.

$$\begin{aligned}
 (10 - 2y)^2 + y^2 &= 65 \\
 100 - 20y - 20y + 4y^2 + y^2 &= 65 \\
 5y^2 - 40y + 100 &= 65 \\
 5y^2 - 40y + 35 &= 0 \\
 y^2 - 8y + 7 &= 0 \\
 (y - 7)(y - 1) &= 0 \\
 y = 7 \quad y = 1
 \end{aligned}$$

when $y = 7$
 $x = 10 - 2(7)$

$x = -4$

when $y = 1$

$x = 10 - 2(1)$

$x = 8$

(.....,.....)
 (-4, 7)
 (.....,.....)
 (8, 1)

(Total for Question 9 is 5 marks)

10 C is a graph with equation $x^2 + y^2 = 85$



L is a straight line with equation $y + 3x = 25$ ← $y = 25 - 3x$

Using algebra, find the coordinates of the points of intersection of C and L.
You must show all your working.

$$\begin{aligned}
 x^2 + (25 - 3x)^2 &= 85 \\
 x^2 + 625 - 75x - 75x + 9x^2 &= 85 \\
 10x^2 - 150x + 625 &= 85 \\
 10x^2 - 150x + 540 &= 0 \\
 x^2 - 15x + 54 &= 0 \\
 (x - 6)(x - 9) &= 0 \\
 x = 6 \quad x = 9
 \end{aligned}$$

when $x = 6$
 $y = 25 - 3(6)$

$y = 7$

when $x = 9$

$y = 25 - 3(9)$

$y = -2$

(.....,.....)
 (6, 7)
 (.....,.....)
 (9, -2)

(Total for Question 10 is 5 marks)



11 Solve algebraically the simultaneous equations



$$\begin{aligned}
 x^2 - 2y^2 &= 7 \\
 2y &= x + 1 \quad \leftarrow x = 2y - 1
 \end{aligned}$$

$$\begin{aligned}
 (2y-1)^2 - 2y^2 &= 7 \\
 4y^2 - 2y - 2y + 1 - 2y^2 &= 7 \\
 2y^2 - 4y + 1 &= 7 \\
 2y^2 - 4y - 6 &= 0 \\
 y^2 - 2y - 3 &= 0 \\
 (y-3)(y+1) &= 0 \\
 y = 3 \quad y = -1
 \end{aligned}$$

$$\begin{aligned}
 \text{When } y = 3 \\
 x &= 2(3) - 1 \\
 x &= 5 \\
 \text{When } y = -1 \\
 x &= 2(-1) - 1 \\
 x &= -3
 \end{aligned}$$

$x = 5 \quad y = 3, \quad x = -3 \quad y = -1$

(Total for Question 11 is 5 marks)

12 C is a graph with equation $2x^2 - 3y^2 = 7$



L is a straight line with equation $y = x - 2$

Using algebra, find the coordinates of the points of intersection of C and L.
You must show all your working.

$$\begin{aligned}
 2x^2 - 3(x-2)^2 &= 7 \\
 2x^2 - 3(x^2 - 4x + 4) &= 7 \\
 2x^2 - 3x^2 + 4x + 4 &= 7 \\
 -x^2 + 4x + 4 &= 7 \\
 -x^2 + 4x - 3 &= 0 \\
 x^2 - 4x + 3 &= 0 \\
 (x-3)(x-1) &= 0 \\
 x = 3 \quad x = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{When } x = 3 \\
 y &= 3 - 2 \\
 y &= 1 \\
 \text{When } x = 1 \\
 y &= 1 - 2 \\
 y &= -1
 \end{aligned}$$

(..... 3 1)

(..... 1 0)

(Total for Question 12 is 5 marks)





13 C is a graph with equation $x^2 - 8y^2 = k$ where k is a positive integer.

L is a straight line with equation $4y = x - 1$

L and C intersect at points A and B.

The x -coordinate of point A is -7 .

(a) Work out the value of k .

$$\begin{aligned}
 \text{At A } x &= -7 \\
 4y &= x - 1 \\
 4y &= -7 - 1 \\
 4y &= -8 \\
 y &= -2 \\
 A &= (-7, -2)
 \end{aligned}$$

$$\begin{aligned}
 x^2 - 8y^2 &= k \\
 (-7)^2 - 8(-2)^2 &= k \\
 49 - 8(4) &= k \\
 49 - 32 &= k \\
 17 &= k
 \end{aligned}$$

$$k = \dots\dots\dots 17 \dots\dots\dots$$

(3)

(b) Work out the coordinates of point B.

$$\begin{aligned}
 x &= 4y + 1 \\
 (4y + 1)^2 - 8y^2 &= 17 \\
 16y^2 + 8y + 1 - 8y^2 &= 17 \\
 8y^2 + 8y + 1 &= 17 \\
 8y^2 + 8y - 16 &= 0 \\
 y^2 + y - 2 &= 0 \\
 (y + 2)(y - 1) &= 0 \\
 y &= -2 \quad y = 1 \\
 \swarrow \quad \quad \quad \nearrow \\
 A \quad \quad \quad B
 \end{aligned}$$

$$\begin{aligned}
 \text{when } y &= 1 \\
 x &= 4(1) + 1 \\
 x &= 5
 \end{aligned}$$

$$(\dots\dots\dots 1 \dots\dots\dots 5 \dots\dots\dots)$$

(4)

(Total for Question 13 is 7 marks)

