



Direct and Inverse Proportion

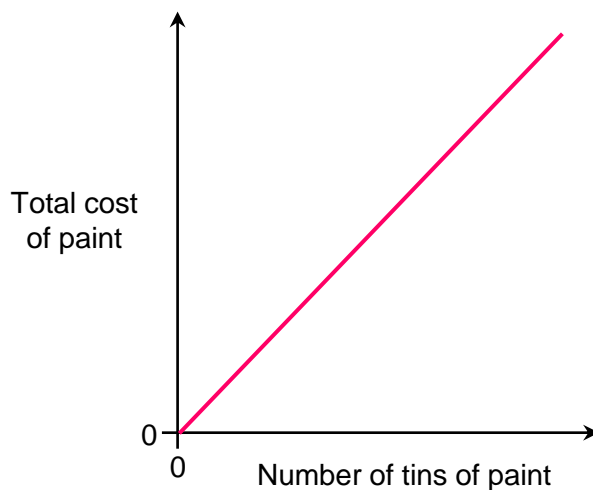


REVISE THIS
TOPIC

- 1 Sarah is painting the walls of her house.
The total cost of the paint needed is directly proportional to the number of tins of paint that Sarah buys.

- 1 (a) On the axes, sketch a graph showing this relationship.

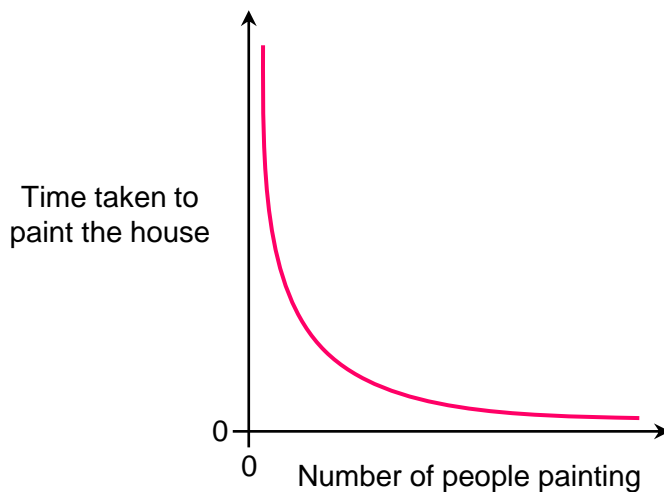
[1 mark]



Sarah asks her friends to help her paint the house.
The time taken to paint the house is inversely proportional to the number of people painting.

- 1 (b) On the axes, sketch a graph showing this relationship.

[1 mark]

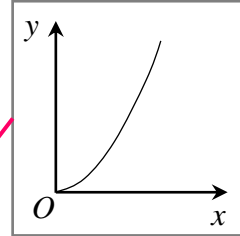


2 Match each expression on the left with the equivalent one on the right.

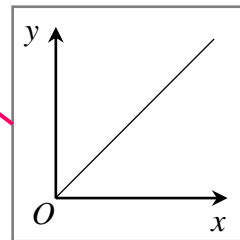
One has been done for you.

[3 marks]

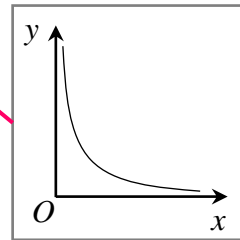
y is directly proportional to x



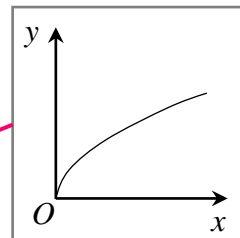
y is inversely proportional to x



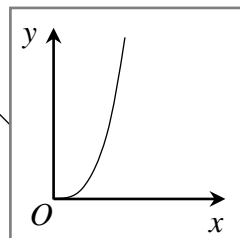
y is directly proportional to x^2



y is directly proportional to x^3



y is directly proportional to \sqrt{x}





- 3 y is directly proportional to x
 $y = 60$ when $x = 4$

- 3 (a) Work out an equation connecting y and x

[3 marks]

$$y = kx$$

$$60 = k \times 4$$

$$k = \frac{60}{4}$$

$$k = 15$$

Answer

$$y = 15x$$

- 3 (b) Work out the value of y when $x = 20$

[2 marks]

$$y = 15x$$

$$y = 15 \times 20$$

Answer

$$300$$





- 4 R is inversely proportional to G
 $R = 80$ when $G = 3$

- 4 (a) Work out an equation connecting R and G .

[3 marks]

$$R = \frac{k}{G}$$

$$80 = \frac{k}{3}$$

$$k = 80 \times 3$$

$$k = 240$$

Answer $R = \frac{240}{G}$

- 4 (b) Work out the value of R when $G = 16$

[2 marks]

$$R = \frac{240}{G}$$

$$R = \frac{240}{16}$$

Answer 15





- 5 H is directly proportional to p^2
 $H = 50$ when $p = 10$

- 5 (a) Work out an equation connecting H and p .

[3 marks]

$$H = kp^2$$

$$50 = k \times 10^2$$

$$50 = k \times 100$$

$$k = \frac{50}{100}$$

$$k = 0.5$$

Answer

$$H = 0.5p^2$$

- 5 (b) Work out the value of H when $p = 6$

[2 marks]

$$H = 0.5p^2$$

$$H = 0.5 \times 6^2$$

$$H = 0.5 \times 36$$

Answer

$$18$$





- 6 J is inversely proportional to \sqrt{F}
 $J = 12$ when $F = 9$

- 6 (a) Work out an equation connecting J and F .

[3 marks]

$$J = \frac{k}{\sqrt{F}}$$

$$12 = \frac{k}{\sqrt{9}}$$

$$k = 12 \times 3$$

$$k = 36$$

$$12 = \frac{k}{3}$$

$$J = \frac{36}{\sqrt{F}}$$

Answer

- 6 (b) Work out the value of J when $F = 4$

[2 marks]

$$J = \frac{36}{\sqrt{F}}$$

$$J = \frac{36}{\sqrt{4}}$$

$$J = \frac{36}{2}$$

Answer

18





- 7 q is directly proportional to r^3
 $q = 250$ when $r = 10$

- 7 (a) Work out an equation connecting q and r .

[3 marks]

$$q = kr^3$$

$$250 = k \times 10^3$$

$$250 = k \times 1000$$

$$k = \frac{250}{1000}$$

$$k = \frac{1}{4}$$

Answer

$$q = \frac{1}{4}r^3$$

- 7 (b) Work out the value of q when $r = 3$
Give your answer as a mixed number.

[2 marks]

$$q = \frac{1}{4} \times 3^3$$

$$q = \frac{1}{4} \times 27$$

$$q = \frac{27}{4}$$

Answer

$$6\frac{3}{4}$$





- 8 v is inversely proportional to the square of e
 $v = 0.2$ when $e = 8$

- 8 (a) Work out an equation connecting v and e .

[3 marks]

$$v = \frac{k}{e^2}$$

$$0.2 = \frac{k}{8^2}$$

$$k = 0.2 \times 64$$

$$0.2 = \frac{k}{64}$$

$$k = 12.8$$

Answer $v = \frac{12.8}{e^2}$

- 8 (b) Work out the value of v when $e = 10$

[2 marks]

$$v = \frac{12.8}{10^2}$$

$$v = \frac{12.8}{100}$$

Answer 0.128





- 9 m is directly proportional to the cube root of n
 $m = 8$ when $n = 8000$

- 9 (a) Work out an equation connecting m and n .

[3 marks]

$$m = k \times \sqrt[3]{n}$$

$$8 = k \times \sqrt[3]{8000}$$

$$8 = k \times 20$$

$$k = \frac{8}{20}$$

$$k = \frac{2}{5}$$

Answer

$$m = \frac{2}{5} \sqrt[3]{n}$$

- 9 (b) Work out the value of m when $n = 1.25 \times 10^{-4}$

[2 marks]

$$m = \frac{2}{5} \times \sqrt[3]{0.000125}$$

$$m = \frac{2}{5} \times 0.05$$

Answer

$$0.02$$



10 x is directly proportional to y

Complete the table.

[2 marks]

x	2	6	9
y	1	3	4.5

11 b is inversely proportional to c

Complete the table.

[2 marks]

b	5	10	200
c	4	2	0.1

12 p is directly proportional to q^2

Complete the table.

[2 marks]

p	4	64	100
q	1	4	5

13 m is inversely proportional to \sqrt{n}

Complete the table.

[2 marks]

m	24	6	30
n	25	400	16



14 The table shows a set of values for x , y and z

x	2	8	50
y	100	25	4
z	1	2	5

For each of the following questions tick the correct statement.
You should tick **one** box for each question.

14 (a)

[1 mark]

☐

x is directly proportional to y

☐

x is directly proportional to y^2

☒

x is inversely proportional to y

☐

x is inversely proportional to y^2

14 (b)

[1 mark]

☐

y is directly proportional to z

☐

y is directly proportional to z^2

☐

y is inversely proportional to z

☒

y is inversely proportional to z^2

14 (c)

[1 mark]

☐

x is directly proportional to z

☒

x is directly proportional to z^2

☐

x is inversely proportional to z

☐

x is inversely proportional to z^2



15

An athlete throws a javelin with velocity v metres per second.
The distance recorded by the judges for the throw is d metres.

v is directly proportional to d .

An athlete throws the javelin with a velocity of 20 metres per second.
The javelin travels a distance 64 metres.

15 (a) Work out an equation connecting v and d .

[3 marks]

$$v = kd$$

$$20 = k \times 64$$

$$k = \frac{20}{64}$$

$$k = \frac{5}{16}$$

Answer $v = \frac{5}{16}d$

15 (b) At a javelin competition

[3 marks]

Steve throws his javelin with velocity 25 metres per second.
Jan throws his javelin with velocity 27.5 metres per second.

How much further does Jan throw the Javelin than Steve?

$$\text{Steve: } 25 = \frac{5}{16}d \quad \text{Jan: } 27.5 = \frac{5}{16}d$$

$$d = \frac{25 \times 16}{5}$$

$$d = 80\text{m}$$

$$d = \frac{27.5 \times 16}{5}$$

$$d = 88\text{m}$$

Answer 8 metres





16

The amount of paint needed to paint a sculpture is p ml.
The height of the sculpture is h cm.

p is directly proportional to the square of h .

2 ml of paint is needed to paint a sculpture that is 4 cm tall.

16 (a) Work out an equation connecting p and h .

[3 marks]

$$\begin{aligned}p &= kh^2 \\2 &= k \times 4^2 \\2 &= k \times 16 \\k &= \frac{2}{16} \\k &= \frac{1}{8}\end{aligned}$$

Answer

$$p = \frac{1}{8}h^2$$

16 (b) Work the amount of paint needed to paint a sculpture that is 3 metres tall.
Give your answer in litres.

300 cm [3 marks]

$$p = \frac{1}{8} \times 300^2$$

$$p = 11250 \text{ ml}$$

$$p = (11250 \div 1000) \text{ litres}$$

$$p = 11.25 \text{ L}$$

Answer

$$11.25$$

litres





17

 A is directly proportional to B^3 When $A = 6$, $B = 2$ B is inversely proportional to C When $B = 150$, $C = 0.2$ Work out the value of A when $C = 45$

Give your answer as a fraction in its simplest form.

[5 marks]

$$A = k_1 B^3$$

$$6 = k_1 \times 2^3$$

$$6 = k_1 \times 8$$

$$k_1 = \frac{6}{8}$$

$$k_1 = \frac{3}{4}$$

$$A = \frac{3}{4} B^3$$

$$B = \frac{k_2}{C}$$

$$150 = \frac{k_2}{0.2}$$

$$k_2 = 150 \times 0.2$$

$$k_2 = 30$$

$$B = \frac{30}{C}$$

$$\text{when } C = 45$$

$$B = \frac{30}{45}$$

$$B = \frac{2}{3}$$

$$\text{when } B = \frac{2}{3}$$

$$A = \frac{3}{4} \times \left(\frac{2}{3}\right)^3$$

$$A = \frac{3}{4} \times \frac{8}{27}$$

$$A = \frac{24}{108}$$

$$\frac{2}{9}$$

Answer _____





18

 F , G and H have positive values. F is directly proportional to the square root of G When $F = 1$, $G = 0.16$ G is inversely proportional to the cube of H When $G = 1000$, $H = 0.2$ Work out the value of H when $F = 20$

[5 marks]

$$F = k_1 \sqrt{G}$$

$$G = \frac{k_2}{H^3}$$

$$1 = k_1 \times \sqrt{0.16}$$

$$1 = k_1 \times 0.4$$

$$k_1 = \frac{1}{0.4}$$

$$k_1 = 2.5$$

$$F = 2.5 \sqrt{G}$$

$$1000 = \frac{k_2}{0.2^3}$$

$$k_2 = 1000 \times 0.2^3$$

$$k_2 = 8$$

$$G = \frac{8}{H^3}$$

$$\text{when } F = 20$$

$$\div 2.5 \quad 20 = 2.5 \sqrt{G} \quad \downarrow \div 2.5$$

$$8 = \sqrt{G}$$

$$8^2 = G$$

$$G = 64$$

$$\text{when } G = 64$$

$$64 = \frac{8}{H^3}$$

$$H^3 = \frac{8}{64}$$

$$H = \sqrt[3]{\frac{1}{8}}$$

$$\frac{1}{2}$$

Answer _____





19

When a stone is thrown upwards with a speed of v metres per second it reaches a maximum height of h metres.

h is directly proportional to v^2

When $v = 10$, $h = 5$

Rosie and Jim both throw stones upwards.

Rosie throws her stone 10 metres per second faster than Jim.

The maximum height reached by Rosie's stone is 17 metres more than the maximum height reached by Jim's stone.

With what velocity did Jim throw his stone?

[5 marks]

$$h = kv^2$$

$$h_J = \frac{1}{20} v_J^2$$

$$5 = k \times 10^2$$

$$5 = k \times 100$$

$$h_J + 17 = \frac{1}{20} (v_J + 10)^2$$

$$k = \frac{5}{100}$$

substitute h_J for $\frac{1}{20} v_J^2$

$$k = \frac{1}{20}$$

$$\frac{1}{20} v_J^2 + 17 = \frac{1}{20} (v_J + 10)^2$$

$$\frac{1}{20} v_J^2 + 17 = \frac{1}{20} (v_J + 10)(v_J + 10)$$

$$\frac{1}{20} v_J^2 + 17 = \frac{1}{20} (v_J^2 + 20v_J + 100)$$

$$\cancel{\frac{1}{20} v_J^2} + 17 = \cancel{\frac{1}{20} v_J^2} + v_J + 5$$

$$17 = v_J + 5$$

$$12 = v_J$$

Answer 12 metres per second

