



# Inverse Functions



REVISE THIS TOPIC

1  $f(x) = 2x + 9$

$g(x) = \sqrt{x-3}$

$h(x) = x^3 + 4$



(a) Find  $f^{-1}(x)$

$$\begin{aligned} y &= 2x + 9 \\ x &= 2y + 9 \\ x - 9 &= 2y \end{aligned}$$

$$\frac{x-9}{2} = y$$

$$f^{-1}(x) = \frac{x-9}{2} \quad (2)$$

(b) Find  $g^{-1}(x)$

$$\begin{aligned} y &= \sqrt{x-3} \\ x &= \sqrt{y-3} \\ x^2 &= y-3 \end{aligned}$$

$$x^2 + 3 = y$$

$$g^{-1}(x) = x^2 + 3 \quad (2)$$

(c) Find  $h^{-1}(31)$

$$\begin{aligned} y &= x^3 + 4 \\ x &= y^3 + 4 \\ x - 4 &= y^3 \end{aligned}$$

$$\begin{aligned} \sqrt[3]{x-4} &= y \\ h^{-1}(x) &= \sqrt[3]{x-4} \\ h^{-1}(31) &= \sqrt[3]{31-4} \end{aligned}$$

$$\underline{\hspace{10em}} \quad 3 \quad (2)$$

(Total for Question 1 is 6 marks)



2  $f(x) = \frac{2x+3}{4}$

$g(x) = x^2 - 6$



(a) Find  $f^{-1}(x)$

$$y = \frac{2x+3}{4}$$

$$4x = 2y+3$$

$$\frac{4x-3}{2} = y$$

$$x = \frac{2y+3}{4}$$

$$4x-3 = 2y$$

$$\frac{4x-3}{2}$$

$f^{-1}(x) = \dots\dots\dots$   
(2)

(b) Find  $g^{-1}(x)$

$$y = x^2 - 6$$

$$\sqrt{x+6} = y$$

$$x = y^2 - 6$$

$$x+6 = y^2$$

$g^{-1}(x) = \dots\dots\dots$   
(2)

(Total for Question 2 is 4 marks)

3  $f(x) = 50 - x^2$

$g(x) = 4x^2 - 1$



(a) Find  $f^{-1}(1)$

$$y = 50 - x^2$$

$$y^2 = 50 - x$$

$$x = 50 - y^2$$

$$y = \sqrt{50-x}$$

$$x + y^2 = 50$$

$$f^{-1}(1) = \sqrt{50-1}$$

$$7$$

$\dots\dots\dots$   
(2)

(b) Find  $g^{-1}(0)$

$$y = 4x^2 - 1$$

$$\frac{x+1}{4} = y^2$$

$$g^{-1}(x) = \sqrt{\frac{x+1}{4}}$$

$$x = 4y^2 - 1$$

$$\sqrt{\frac{x+1}{4}} = y$$

$$g^{-1}(0) = \sqrt{\frac{0+1}{4}}$$

$$\frac{1}{2}$$

$\dots\dots\dots$   
(2)

(Total for Question 3 is 4 marks)



4  $f(x) = \frac{2x^3}{5}$

$g(x) = \frac{x}{4} - 3$



(a) Find  $f^{-1}(x)$

$$y = \frac{2x^3}{5}$$

$$5x = 2y^2$$

$$y = \sqrt{\frac{5x}{2}}$$

$$x = \frac{2y^2}{5}$$

$$\frac{5x}{2} = y^2$$

$$f^{-1}(x) = \sqrt{\frac{5x}{2}}$$

(2)

(b) Find  $g^{-1}(x)$

$$y = \frac{x}{4} - 3$$

$$x + 3 = \frac{y}{4}$$

$$x = \frac{y}{4} - 3$$

$$4(x + 3) = y$$

$$g^{-1}(x) = 4(x + 3)$$

(2)

(Total for Question 4 is 4 marks)

5  $f(x) = \sqrt[3]{100 - x}$

$g(x) = 2(x + 14)$



(a) Find  $f^{-1}(4)$

$$y = \sqrt[3]{100 - x}$$

$$x^3 + y = 100$$

$$x = \sqrt[3]{100 - y}$$

$$y = 100 - x^3$$

$$x^3 = 100 - y$$

$$f^{-1}(4) = 100 - 4^3$$

$$36$$

(2)

(b) Find  $g^{-1}(26)$

$$y = 2(x + 14)$$

$$\frac{x}{2} - 14 = y$$

$$x = 2(y + 14)$$

$$g^{-1}(26) = \frac{26}{2} - 14$$

$$\frac{x}{2} = y + 14$$

$$-1$$

(2)

(Total for Question 5 is 4 marks)



6  $f(x) = \frac{5}{x+10}$

$g(x) = \sqrt{2x^3 - 3}$



(a) Find  $f^{-1}(x)$

$$y = \frac{5}{x+10}$$

$$x = \frac{5}{y+10}$$

$$x(y+10) = 5$$

$$y+10 = \frac{5}{x}$$

$$y = \frac{5}{x} - 10$$

$$f^{-1}(x) = \frac{5}{x} - 10$$

(2)

(b) Find  $g^{-1}(x)$

$$y = \sqrt{2x^3 - 3}$$

$$x = \sqrt{2y^3 - 3}$$

$$x^2 = 2y^3 - 3$$

$$x^2 + 3 = 2y^3$$

$$\frac{x^2 + 3}{2} = y^3$$

$$y = \sqrt[3]{\frac{x^2 + 3}{2}}$$

$$g^{-1}(x) = \sqrt[3]{\frac{x^2 + 3}{2}}$$

(2)

(Total for Question 6 is 4 marks)

7  $f(x) = 3 - \frac{2}{x}$

$g(x) = (x - 5)^3$



(a) Find  $f^{-1}(2.5)$

$$y = 3 - \frac{2}{x}$$

$$x + \frac{2}{y} = 3$$

$$\frac{2}{3-x} = y$$

$$x = 3 - \frac{2}{y}$$

$$\frac{2}{y} = 3 - x$$

$$f^{-1}(2.5) = \frac{2}{3-2.5}$$

4

(2)

(b) Find  $g^{-1}(-27)$

$$y = (x-5)^3$$

$$x = (y-5)^3$$

$$\sqrt[3]{x} = y - 5$$

$$\sqrt[3]{x} + 5 = y$$

$$g^{-1}(-27) = \sqrt[3]{-27} + 5$$

8

(2)

(Total for Question 7 is 4 marks)





8  $f(x) = \frac{x+4}{x-3}$        $g(x) = \sqrt{3x}$        $h(x) = 2x + 1$

(a) Find  $f^{-1}(x)$

$$\begin{aligned}
 y &= \frac{x+4}{x-3} & x(y-3) &= y+4 & y &= \frac{4+3x}{x-1} \\
 x &= \frac{y+4}{y-3} & xy - 3x &= y+4 & & \\
 & & xy - y &= 4+3x & & \\
 & & y(x-1) &= 4+3x & & 
 \end{aligned}$$

$$f^{-1}(x) = \frac{4+3x}{x-1} \quad (3)$$

(b) Find  $g^{-1}(9)$

$$\begin{aligned}
 y &= \sqrt{3x} & \frac{x^2}{3} &= y \\
 x &= \sqrt{3y} & g^{-1}(9) &= \frac{9^2}{3} \\
 x^2 &= 3y & & 
 \end{aligned}$$

$$\frac{27}{(2)}$$

(c) Find  $(gh)^{-1}(x)$

$$\begin{aligned}
 gh(x) &= \sqrt{3(2x+1)} & x &= \sqrt{6y+3} \\
 gh(x) &= \sqrt{6x+3} & x^2 &= 6y+3 \\
 (gh)^{-1}(x) &= \sqrt{6x+3} & x^2 - 3 &= 6y \\
 y &= \sqrt{6x+3} & \frac{x^2 - 3}{6} &= y
 \end{aligned}$$

$$(gh)^{-1}(x) = \frac{x^2 - 3}{6} \quad (4)$$

(Total for Question 8 is 9 marks)

