



Non-Linear Simultaneous Equations



REVISE THIS
TOPIC



1 Solve algebraically the simultaneous equations



$$y = x^2 - 3x - 4$$

$$y = 2x - 10$$

[5 marks]

$$x^2 - 3x - 4 = 2x - 10$$

$$\text{when } x = 2$$

$$x^2 - 5x - 4 = -10$$

$$y = 2(2) - 10$$

$$x^2 - 5x + 6 = 0$$

$$y = -6$$

$$(x - 2)(x - 3) = 0$$

$$\text{when } x = 3$$

$$x = 2 \quad x = 3$$

$$y = 2(3) - 10$$

$$y = -4$$

Answer $x = 2 \quad y = -6, x = 3 \quad y = -4$

2 Solve algebraically the simultaneous equations



$$y = x^2 + 5x - 8$$

$$y - 4x = 4 \quad \leftarrow y = 4 + 4x$$

[5 marks]

$$x^2 + 5x - 8 = 4 + 4x$$

$$\text{when } x = -4$$

$$x^2 + x - 8 = 4$$

$$y = 4 + 4(-4)$$

$$x^2 + x - 12 = 0$$

$$y = -12$$

$$(x + 4)(x - 3) = 0$$

$$\text{when } x = 3$$

$$x = -4 \quad x = 3$$

$$y = 4 + 4(3)$$

$$y = 16$$

Answer $x = -4 \quad y = -12, x = 3 \quad y = 16$



3

Solve algebraically the simultaneous equations



[5 marks]

$$y = 3x^2 + 2x - 8$$

$$y = 9x - 10$$

$$3x^2 + 2x - 8 = 9x - 10$$

$$\text{when } x = \frac{1}{3}$$

$$3x^2 - 7x - 8 = -10$$

$$y = 9\left(\frac{1}{3}\right) - 10$$

$$3x^2 - 7x + 2 = 0$$

$$y = -7$$

$$(3x - 1)(x - 2) = 0$$

$$\text{when } x = 2$$

$$3x - 1 = 0$$

$$x - 2 = 0$$

$$y = 9(2) - 10$$

$$x = \frac{1}{3}$$

$$x = 2$$

$$y = 8$$

$$\text{Answer } x = \frac{1}{3} \ y = -7, \ x = 2 \ y = 8$$

4

A curve has equation $y = x^2 - 5x + 10$

A line has equation $y = 3x - 6$


Find the coordinates of the point of intersection of the curve and the line. [4 marks]

$$x^2 - 5x + 10 = 3x - 6$$

$$\text{when } x = 4$$

$$x^2 - 8x + 10 = -6$$

$$y = 3(4) - 6$$

$$x^2 - 8x + 16 = 0$$

$$y = 12 - 6$$

$$(x - 4)(x - 4) = 0$$

$$y = 6$$

$$x = 4$$

$$\text{Answer } (\underline{4} , \underline{6})$$





5

A curve has equation $y = 5x^2 - x - 15$ A line has equation $y = 10x - 3$ 

Find the coordinates of the points of intersection of the curve and the line.

[5 marks]

$$\begin{aligned}
 5x^2 - x - 15 &= 10x - 3 & \text{when } x &= -\frac{4}{5} \\
 5x^2 - 11x - 15 &= -3 & y &= 10\left(-\frac{4}{5}\right) - 3 \\
 5x^2 - 11x - 12 &= 0 & y &= -8 - 3 \\
 (5x + 4)(x - 3) &= 0 & y &= -11 \\
 5x + 4 = 0 & \quad x - 3 = 0 & \text{when } x &= 3 \\
 x = -\frac{4}{5} & \quad x = 3 & y &= 10(3) - 3 \\
 & & y &= 27
 \end{aligned}$$

Answers $\left(-\frac{4}{5}, -11\right)$ and $(3, 27)$

6

Solve algebraically the simultaneous equations

$$y = x^2 + x + 1$$

$$y = x + 3$$



Give your answers as exact values.

[5 marks]

$$\begin{aligned}
 x^2 + x + 1 &= x + 3 & \text{when } x &= \sqrt{2} \\
 x^2 + 1 &= 3 & y &= \sqrt{2} + 3 \\
 x^2 - 2 &= 0 & \text{when } x &= -\sqrt{2} \\
 x^2 &= 2 & y &= -\sqrt{2} + 3 \\
 x &= \pm\sqrt{2}
 \end{aligned}$$

Answer $x = \sqrt{2} \quad y = 3 + \sqrt{2}, x = -\sqrt{2} \quad y = 3 - \sqrt{2}$



7

Solve algebraically the simultaneous equations



$$x^2 + y^2 = 100$$

$$y = x - 2$$

[5 marks]

$$x^2 + (x-2)^2 = 100$$

$$\text{when } x = 8$$

$$x^2 + x^2 - 2x - 2x + 4 = 100$$

$$y = 8 - 2$$

$$2x^2 - 4x + 4 = 100$$

$$y = 6$$

$$2x^2 - 4x - 96 = 0$$

$$\text{when } x = -6$$

$$x^2 - 2x - 48 = 0$$

$$y = -6 - 2$$

$$(x-8)(x+6) = 0$$

$$y = -8$$

$$x = 8 \quad x = -6$$

$$\text{Answer } x=8 \ y=6, \ x=-6 \ y=-8$$

8

Solve algebraically the simultaneous equations



$$x^2 + y^2 = 200$$

$$y = 2x - 10$$

[5 marks]

$$x^2 + (2x-10)^2 = 200$$

$$\text{when } x = 10$$

$$x^2 + 4x^2 - 20x - 20x + 100 = 200$$

$$y = 2(10) - 10$$

$$5x^2 - 40x + 100 = 200$$

$$y = 10$$

$$5x^2 - 40x - 100 = 0$$

$$x^2 - 8x - 20 = 0$$

$$\text{when } x = -2$$

$$(x-10)(x+2) = 0$$

$$y = 2(-2) - 10$$

$$x = 10 \quad x = -2$$

$$y = -14$$

$$\text{Answer } x=10 \ y=10, \ x=-2 \ y=-14$$





9

A circle has equation $x^2 + y^2 = 65$ A line has equation $2y = 10 - x \leftarrow x = 10 - 2y$ 

Find the coordinates of the points of intersection of the circle and the line.

[5 marks]

$$(10 - 2y)^2 + y^2 = 65 \quad \text{when } y = 7$$

$$100 - 20y - 20y + 4y^2 + y^2 = 65 \quad x = 10 - 2(7)$$

$$5y^2 - 40y + 100 = 65 \quad x = -4$$

$$5y^2 - 40y + 35 = 0$$

$$y^2 - 8y + 7 = 0 \quad \text{when } y = 1$$

$$(y - 7)(y - 1) = 0 \quad x = 10 - 2(1)$$

$$y = 7 \quad y = 1 \quad x = 8$$

Answers (-4 , 7) and (8 , 1)

10

A circle has equation $x^2 + y^2 = 85$ A line has equation $y + 3x = 25 \leftarrow y = 25 - 3x$ 

Find the coordinates of the points of intersection of the circle and the line.

[5 marks]

$$x^2 + (25 - 3x)^2 = 85$$

$$x^2 + 625 - 75x - 75x + 9x^2 = 85 \quad \text{when } x = 6$$

$$10x^2 - 150x + 625 = 85 \quad y = 25 - 3(6)$$

$$10x^2 - 150x + 540 = 0 \quad y = 7$$

$$x^2 - 15x + 54 = 0 \quad \text{when } x = 9$$

$$(x - 6)(x - 9) = 0 \quad y = 25 - 3(9)$$

$$x = 6 \quad x = 9 \quad y = -2$$

Answers (6 , 7) and (9 , -2)

11

Solve algebraically the simultaneous equations



$$x^2 - 2y^2 = 7$$

$$2y = x + 1 \quad \leftarrow x = 2y - 1$$

[5 marks]

$$(2y-1)^2 - 2y^2 = 7$$

$$\text{When } y = 3$$

$$4y^2 - 2y - 2y + 1 - 2y^2 = 7$$

$$x = 2(3) - 1$$

$$2y^2 - 4y + 1 = 7$$

$$x = 5$$

$$2y^2 - 4y - 6 = 0$$

$$\text{When } y = -1$$

$$y^2 - 2y - 3 = 0$$

$$x = 2(-1) - 1$$

$$(y-3)(y+1) = 0$$

$$x = -3$$

$$y = 3 \quad y = -1$$

$$\text{Answer } x = 5, y = 3, x = -3, y = -1$$

12

A curve has equation $2x^2 - 3y^2 = 15$

A line has equation $y = x - 2$


Find the coordinates of the points of intersection of the curve and the line.

[5 marks]

$$2x^2 - 3(x-2)^2 = 15$$

$$2x^2 - 3(x^2 - 4x + 4) = 15$$

$$\text{When } x = 3$$

$$2x^2 - 3x^2 + 12x - 12 = 15$$

$$y = 3 - 2$$

$$-x^2 + 12x - 12 = 15$$

$$y = 1$$

$$-x^2 + 12x - 27 = 0$$

$$x^2 - 12x + 27 = 0$$

$$\text{When } x = 9$$

$$(x-3)(x-9) = 0$$

$$y = 9 - 2$$

$$x = 3 \quad x = 9$$

$$y = 7$$

$$\text{Answers } (3, 1) \text{ and } (9, 7)$$



13

A curve has equation $x^2 - 8y^2 = k$ where k is a positive integer.
A line has equation $4y = x - 1$



The curve and the line intersect at the points A and B.
The x -coordinate of point A is -7.

13 (a)

Work out the value of k .

[3 marks]

$$\begin{aligned} \text{At A } x &= -7 & x^2 - 8y^2 &= k \\ 4y &= x - 1 & (-7)^2 - 8(-2)^2 &= k \\ 4y &= -7 - 1 & 49 - 8(4) &= k \\ 4y &= -8 & 49 - 32 &= k \\ y &= -2 & 17 &= k \\ A &= (-7, -2) \end{aligned}$$

$$k = 17$$

13 (b)

Work out the coordinates of point B.

[4 marks]

$$\begin{aligned} x &= 4y + 1 \\ (4y + 1)^2 - 8y^2 &= 17 & \text{when } y &= 1 \\ 16y^2 + 8y + 1 - 8y^2 &= 17 & x &= 4(1) + 1 \\ 8y^2 + 8y + 1 &= 17 & x &= 5 \\ 8y^2 + 8y - 16 &= 0 \\ y^2 + y - 2 &= 0 \\ (y + 2)(y - 1) &= 0 \\ y &= -2 & y &= 1 \\ \text{A} & \nearrow & \uparrow & \text{B} \end{aligned}$$

$$B = (5, 1)$$

