Perpendicular Lines

SCAN ME

REVISE THIS TOPIC

1 The equation of line $\mathbf{L_1}$ is $y = 5x + 1$
The equation of line $\mathbf{L_2}$ is $5y + x = 20$

Show that these two lines are perpendicular.

$y = 5x + 1$

gradient = 5

$5y + x = 20$

$5y = 20 - x$

$y = 4 - \frac{1}{5}x$

gradient = $-\frac{1}{5}$

$5 \times -\frac{1}{5} = -1$ therefore L_1 and L_2 are perpendicular

(Total for Question 1 is 2 marks)

2 The equation of line $\mathbf{L_1}$ is $y = 8 - 3x$
The equation of line $\mathbf{L_2}$ is $9y - 3x - 6 = 0$

Show that these two lines are perpendicular

$y = 8 - 3x$

gradient = -3

$9y - 3x = 6$

$9y = 6 + 3x$

$y = \frac{6}{9} + \frac{3}{9}x$

gradient = $\frac{1}{3}$

$-3 \times \frac{1}{3} = -1$ therefore L_1 and L_2 are perpendicular

(Total for Question 2 is 2 marks)

3 The equation of line $\mathbf{L_1}$ is $2y = x + 10$
The equation of line $\mathbf{L_2}$ is $4y + 8x = 16$

Show that these two lines are perpendicular.

$2y = x + 10$

$y = \frac{1}{2}x + 5$

gradient $= \frac{1}{2}$

$4y + 8x = 16$

$4y = 16 - 8x$

$y = 4 - 2x$

gradient $= -2$

$\frac{1}{2}x - 2 = -1$ therefore L_1 and L_2 are perpendicular

(Total for Question 3 is 2 marks)

4 The equation of line $\mathbf{L_1}$ is $y = \frac{3}{4}x + 1$

The equation of line $\mathbf{L_2}$ is $6y + 8x = 30$

Show that these two lines are perpendicular.

$y = \frac{3}{4}x + 1$

gradient $= \frac{3}{4}$

$6y + 8x = 30$

$6y = 30 - 8x$

$y = 5 - \frac{8}{6}x$

gradient $= -\frac{4}{3}$

$\frac{3}{4} \times -\frac{4}{3} = -1$ therefore L_1 and L_2 are perpendicular

(Total for Question 4 is 2 marks)

5 The equation of line $\mathbf{L_1}$ is $2y = 3x - 4$
The equation of line $\mathbf{L_2}$ is $8y - 12x - 40 = 0$

Show that these two lines are **not** perpendicular.

$2y = 3x - 6$

$y = \frac{3}{2}x - 3$

gradient $= \frac{3}{2}$

$8y - 12x = 40$

$8y = 40 + 12x$

$y = 5 + \frac{12}{8}x$

gradient $= \frac{3}{2}$

Both gradients are the same so L_1 and L_2 are parallel not perpendicular

(Total for Question 5 is 2 marks)

6 The equation of line $\mathbf{L_1}$ is $y = kx + 4$
The equation of line $\mathbf{L_2}$ is $2y + 4x = 10$

Lines $\mathbf{L_1}$ and $\mathbf{L_2}$ are perpendicular.
Work out the value of k.

$y = kx + 4$
gradient $= k$

$2y + 4x = 10$
$2y = 10 - 4x$
$y = 5 - 2x$
gradient $= -2$

$-2 \times k = -1$
$k = -\frac{1}{-2}$

$k =$ $\frac{1}{2}$

(Total for Question 6 is 2 marks)

7 The equation of line $\mathbf{L_1}$ is $2y = kx - 2$
The equation of line $\mathbf{L_2}$ is $3y + x = 18$

Lines $\mathbf{L_1}$ and $\mathbf{L_2}$ are perpendicular.
Work out the value of k.

$2y = kx - 2$
$y = \frac{k}{2}x - 1$
gradient $= \frac{k}{2}$

$3y + x = 18$
$3y = 18 - x$
$y = 18 - \frac{1}{3}x$
gradient $= -\frac{1}{3}$

$\frac{k}{2} \times -\frac{1}{3} = -1$
$-\frac{k}{6} = -1$
$-k = -6$
$k = 6$

$k =$ 6

(Total for Question 7 is 2 marks)

8 The equation of line $\mathbf{L_1}$ is $y = 3 - \frac{2}{5}x$
The equation of line $\mathbf{L_2}$ is $ky - 6x - 20 = 0$

Lines $\mathbf{L_1}$ and $\mathbf{L_2}$ are perpendicular.
Work out the value of k.

$y = 3 - \frac{2}{5}x$
gradient $= -\frac{2}{5}$

$ky = 6x + 20$
$y = \frac{6}{k}x + \frac{20}{k}$
gradient $= \frac{6}{k}$

$-\frac{2}{5} \times \frac{6}{k} = -1$
$-\frac{12}{5k} = -1$
$-12 = -5k$

$k =$ $\frac{12}{5}$

(Total for Question 8 is 2 marks)

9 The straight line **L** has the equation $y = 3x + 1$
The point A has coordinates (9, 4)

Find an equation of the straight line that is perpendicular to **L** and passes through A.

gradient of $L_1 = 3$
gradient of $L_2 = -\frac{1}{3}$
$y = -\frac{1}{3}x + c$
$4 = -\frac{1}{3}(9) + c$
$4 = -3 + c$
$c = 7$

$y = -\frac{1}{3}x + 7$

(Total for Question 9 is 3 marks)

10 The straight line **L** has the equation $y = 5 - 4x$
The point A has coordinates (4, 12)

Find an equation of the straight line that is perpendicular to **L** and passes through A.

gradient of $L_1 = -4$
gradient of $L_2 = \frac{1}{4}$
$y = \frac{1}{4}x + c$
$12 = \frac{1}{4}(4) + c$
$12 = 1 + c$
$c = 11$

$y = \frac{1}{4}x + 11$

(Total for Question 10 is 3 marks)

11 The straight line **L** has the equation $y = \frac{1}{2}x + 3$

The point *A* has coordinates (-3, 7)

Find an equation of the straight line that is perpendicular to **L** and passes through *A*.

gradient of $L_1 = \frac{1}{2}$

gradient of $L_2 = -2$

$y = -2x + c$

$7 = -2(-3) + c$

$7 = 6 + c$

$c = 1$

$y = -2x + 1$

(Total for Question 11 is 3 marks)

12 The straight line **L** has the equation $y = 2 - \frac{1}{6}x$

The point *A* has coordinates (2, 7)

Find an equation of the straight line that is perpendicular to **L** and passes through *A*.

gradient of $L_1 = -\frac{1}{6}$

gradient of $L_2 = 6$

$y = 6x + c$

$7 = 6(2) + c$

$7 = 12 + c$

$c = -5$

$y = 6x - 5$

(Total for Question 12 is 3 marks)

13 $A = (2, 6)$ $B = (1, 9)$ $C = (15, 2)$

Find an equation of the straight line that is perpendicular to AB and passes through C.

gradient of AB $= \frac{9-6}{1-2}$

$= \frac{3}{-1}$

$= -3$

$y = \frac{1}{3}x + c$

$2 = \frac{1}{3}(15) + c$

$2 = 5 + c$

$c = -3$

$y = \frac{1}{3}x - 3$

(Total for Question 13 is 4 marks)

14 $A = (0, 6)$ $B = (3, 8)$ $C = (6, 6)$

Find an equation of the straight line that is perpendicular to AB and passes through C.

gradient of AB $= \frac{8-6}{3-0}$

$= \frac{2}{3}$

$y = -\frac{3}{2}x + c$

$6 = -\frac{3}{2}(6) + c$

$6 = -9 + c$

$c = 15$

$y = -\frac{3}{2}x + 15$

(Total for Question 14 is 4 marks)

15 $A = (5, -3)$ $B = (3, 5)$ $C = (-5, 2)$

Find an equation of the straight line that is perpendicular to AB and passes through C.

gradient of AB $= \frac{5-(-3)}{3-5}$

$= \frac{8}{-2}$

$= -4$

$y = \frac{1}{4}x + c$

$2 = \frac{1}{4}(-5) + c$

$2 = -\frac{5}{4} + c$

$c = 2 + \frac{5}{4}$

$c = \frac{8}{4} + \frac{5}{4}$

$c = \frac{13}{4}$

$y = \frac{1}{4}x + \frac{13}{4}$

(Total for Question 15 is 4 marks)

16 $A = (-4, 5)$ $B = (6, 1)$ $C = (-8, -9)$

Find an equation of the straight line that is perpendicular to AB and passes through C.

gradient of AB $= \frac{1-5}{6-(-4)}$

$= \frac{-4}{10}$

$= -\frac{2}{5}$

$y = \frac{5}{2}x + c$

$-9 = \frac{5}{2}(-8) + c$

$-9 = -20 + c$

$c = 11$

$y = \frac{5}{2}x + 11$

(Total for Question 16 is 4 marks)

17 The straight line $\mathbf{L_1}$ has the equation $y = 3 - 2x$
The point A has coordinates (6, 2)

Line $\mathbf{L_2}$ is perpendicular to $\mathbf{L_1}$ and passes through A.

(a) Work out the coordinates of the point where line $\mathbf{L_2}$ intersects the x-axis.

gradient of $L_1 = -2$
gradient of $L_2 = \frac{1}{2}$

$y = \frac{1}{2}x + c$
$2 = \frac{1}{2}(6) + c$
$2 = 3 + c$
$c = -1$

At x-axis $y = 0$
$y = \frac{1}{2}x - 1$
$0 = \frac{1}{2}x - 1$
$1 = \frac{1}{2}x$ $\times 2$
$2 = x$ $\times 2$

(2 , 0)

(3)

(b) Work out the coordinates of the point where line $\mathbf{L_2}$ intersects the y-axis.

At y-axis $x = 0$
$y = \frac{1}{2}(0) - 1$
$y = -1$

(0 , −1)

(2)

(Total for Question 17 is 5 marks)

18 The straight line $\mathbf{L_1}$ has the equation $y = 2x + 2$
The point A has coordinates (-8, 11)

Line $\mathbf{L_2}$ is perpendicular to $\mathbf{L_1}$ and passes through A.

Lines $\mathbf{L_1}$ and $\mathbf{L_2}$ intersect at the point P.
Line $\mathbf{L_1}$ intersects the x-axis at the point Q.
Line $\mathbf{L_2}$ intersects the x-axis at the point R.

Work out the area of triangle PQR.

gradient of $L_1 = 2$
gradient of $L_2 = -\frac{1}{2}$

$y = -\frac{1}{2}x + c$
$11 = -\frac{1}{2}(-8) + c$
$11 = 4 + c$
$c = 7$
L_2 is $y = -\frac{1}{2}x + 7$

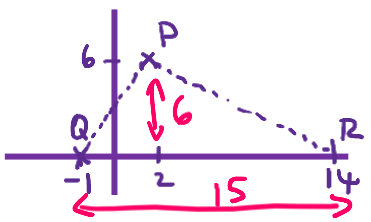
L_1 and L_2 intersect when
$2x + 2 = -\frac{1}{2}x + 7$
$\times 2$ ($4x + 4 = -x + 14$) $\times 2$
$5x = 10$
$x = 2$

$y = 2(2) + 2$
$y = 6$

$P = (2, 6)$

$2x + 2 = 0$
$2x = -2$
$x = -1$
$Q = (-1, 0)$

$-\frac{1}{2}x + 7 = 0$
$7 = \frac{1}{2}x$
$x = 14$
$R = (14, 0)$



Area $= \frac{1}{2} \times 15 \times 6$

45 ..... units^2

(Total for Question 18 is 6 marks)