1 (a) Use the iteration formula $x_{n+1} = \dfrac{(x_n)^2 + 3}{5}$ to find the values of x_1, x_2 and x_3

Start with $x_0 = \sqrt{3}$

$x_1 =$ ....

$x_2 =$ ....

$x_3 =$ ....

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^2 - 5x + 3 = 0$

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(2)

(Total for Question 1 is 5 marks)

1

2 (a) Use the iteration formula $x_{n+1} = \sqrt{\dfrac{x_n}{10} + 2}$ to find the values of x_1, x_2 and x_3

Start with $x_0 = 42.5$

$x_1 =$ ....

$x_2 =$ ....

$x_3 =$ ....

(3)

The values of x_1, x_2 and x_3 found in part (a) are estimates to the solution of an equation in the form $ax^2 - x + b = 0$ where a and b are integers.

(b) Find the values of a and b.

$a =$ ....

$b =$ ....

(2)

(Total for Question 2 is 5 marks)

3 (a) Use the iteration formula $x_{n+1} = \sqrt{80 - 5x_n}$ to find the values of x_1, x_2 and x_3

Start with $x_0 = 12.8$

$x_1 =$ ....

$x_2 =$ ....

$x_3 =$ ....

(3)

The values of x_1, x_2 and x_3 found in part (a) are estimates to the solution of an equation in the form $x^2 + ax + b = 0$ where a and b are integers.

(b) Find the values of a and b.

$a =$ ....

$b =$ ....

(2)

(Total for Question 3 is 5 marks)

4 (a) Use the iteration formula $x_{n+1} = \sqrt[3]{8 - (x_n)^2}$ to find the values of x_1, x_2 and x_3

Start with $x_0 = 1.8$

$x_1 =$ ....

$x_2 =$ ....

$x_3 =$ ....

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 + x^2 - 8 = 0$

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(2)

(Total for Question 4 is 5 marks)

5 (a) Show that the equation $x^3 - x - 4 = 0$ has a solution between $x = 1$ and $x = 2$

(2)

(b) Show that the equation $x^3 - x - 4 = 0$ can be rearranged to give $x = \sqrt[3]{x + 4}$

(2)

(c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{x_n + 4}$ three times to find an estimate for the solution of $x^3 - x - 4 = 0$

....
(3)

(d) By substituting your answer to part (c) into $x^3 - x - 4$ comment on the accuracy of your estimate for the solution to $x^3 - x - 4 = 0$

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(2)

(Total for Question 5 is 9 marks)

6 (a) Show that the equation $x^2 + x - 13 = 0$ has a solution between $x = 3$ and $x = 4$

(2)

(b) Show that the equation $x^2 + x - 13 = 0$ can be rearranged to give $x = \sqrt{13 - x}$

(2)

(c) Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \sqrt{13 - x_n}$ three times to find an estimate for the solution of $x^2 + x - 13 = 0$

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(3)

(d) By substituting your answer to part (c) into $x^2 + x - 13$ comment on the accuracy of your estimate for the solution to $x^2 + x - 13 = 0$

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(2)

(Total for Question 6 is 9 marks)

7 (a) Show that the equation $x^2 - 10x + 6 = 0$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^2 - 10x + 6 = 0$ can be rearranged to give $x = \dfrac{x^2 + 6}{10}$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \dfrac{(x_n)^2 + 6}{10}$ three times to find an estimate for the solution of $x^2 - 10x + 6 = 0$

....

(3)

(d) By substituting your answer to part (c) into $x^2 - 10x + 6$ comment on the accuracy of your estimate for the solution to $x^2 - 10x + 6 = 0$

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(2)

(Total for Question 7 is 9 marks)

8 (a) Show that the equation $x^3 - 20x^2 + 100x - 8 = 0$ has a solution between $x = 10$ and $x = 11$

(2)

(b) Show that the equation $x^3 - 20x^2 + 100x - 8 = 0$ can be rearranged to give $x = \sqrt{\dfrac{8}{x}} + 10$

(4)

(c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt{\dfrac{8}{x_n}} + 10$ three times to find an

estimate for the solution of $x^3 - 20x^2 + 100x - 8 = 0$

....

(3)

(d) By substituting your answer to part (c) into $x^3 - 20x^2 + 100x - 8$
comment on the accuracy of your estimate for the solution to $x^3 - 20x^2 + 100x - 8 = 0$

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(2)

(Total for Question 8 is 9 marks)