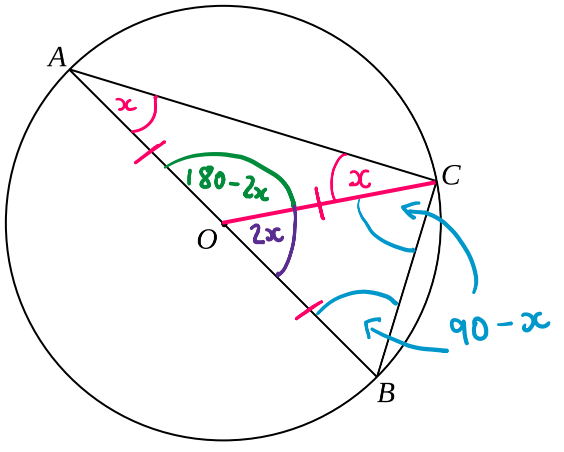
Circle Theorem Proofs

SCAN ME

REVISE THIS TOPIC

1



A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOB is a diameter of the circle.

Prove that angle *ACB* = 90°

OC = OA = OB (all radii)

Let angle OAC = x

Angle ACO = angle OAC = x
(Base angles in an isosceles triangle are equal)

Angle AOC = $180 - x - x$
$= 180 - 2x$ (angles in a triangle add to 180°)

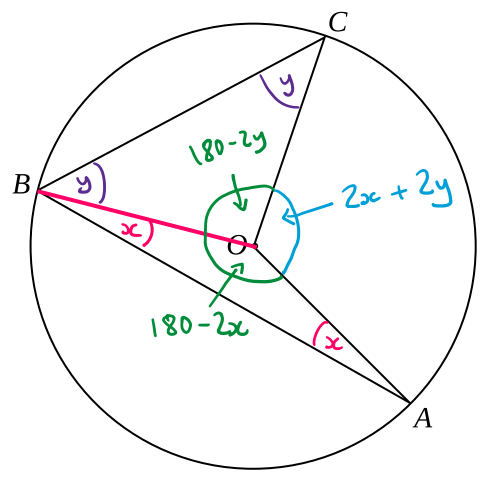
Angle BOC = $180 - (180 - 2x)$
$= 2x$ (angles on a straight line add to 180°)

Angle OBC = Angle OCB = $\frac{1}{2}(180 - 2x)$
$= 90 - x$
(Base angles in an isosceles triangle are equal)

Angle ACB = Angle ACO + Angle OCB
$= x + 90 - x$
$= 90°$

(Total for Question 1 is 4 marks)

2



A, *B* and *C* are points on the circumference of a circle, centre *O*.

Prove that angle *AOC* = 2 × angle *ABC*

OC = OA = OB (all radii)

Let angle OAB = x Let angle OCB = y

Angle OBA = x Angle OBC = y

Base angles in an isosceles triangle are equal

Angle COB = $180 - 2y$ Angle AOB = $180 - 2x$

Angles in a triangle add to 180°

Angle ABC = $x + y$

Angle AOC = $360 - (180 - 2y) - (180 - 2x)$

$= 360 - 180 + 2y - 180 + 2x$

$= 2x + 2y$

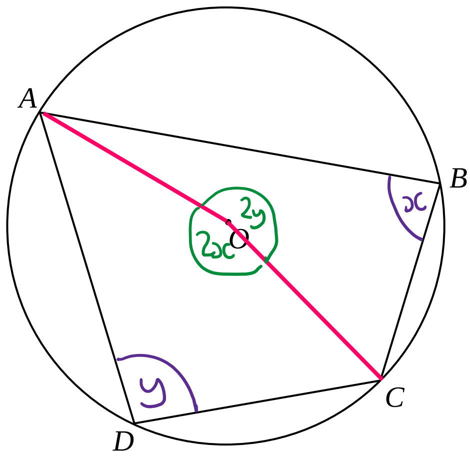
angles around a point add to 360°

$= 2(x + y)$

$= 2 \times$ Angle ABC

(Total for Question 2 is 4 marks)

3



A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.

Prove that angle *ABC* + angle *CDA* = 180°

Let angle ABC = x and Angle CDA = y

Minor angle AOC = $2x$ and major angle AOC = $2y$

as angle at the centre is twice the angle at the circumference

$2x + 2y = 360°$ (angles around a point add to 360°)

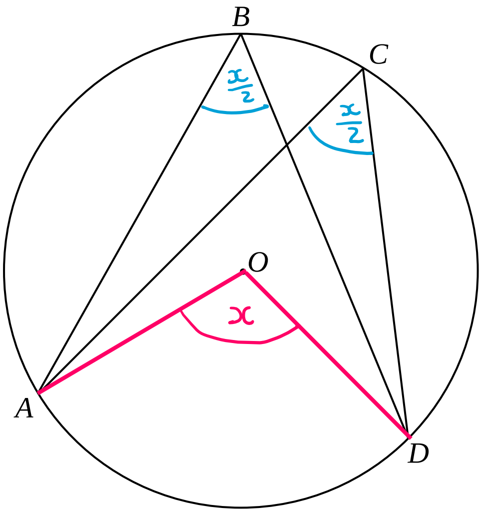
$\div 2$ $\div 2$

$x + y = 180°$

Angle ABC + Angle CDA = 180°

(Total for Question 3 is 4 marks)

4



A, B, C and D are points on the circumference of a circle, centre O.

Prove that angle ABC = angle ACD

Let angle $AOD = x$

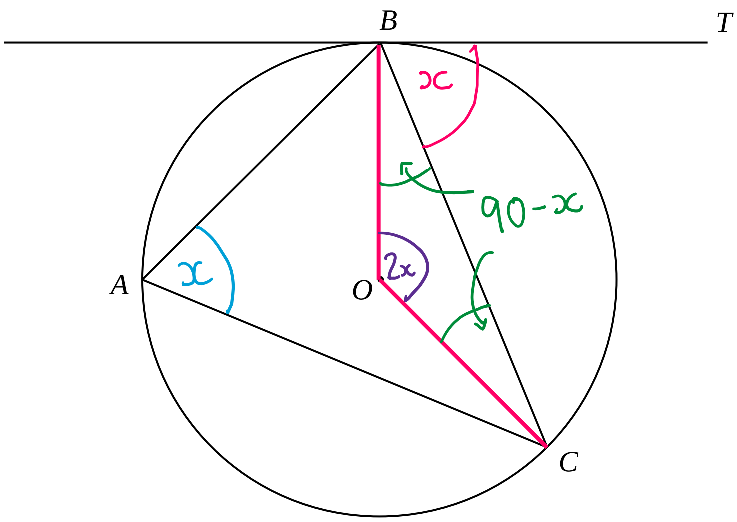
Angle $ABD = \frac{x}{2}$ } angle at the centre is

Angle $ACD = \frac{x}{2}$ } twice the angle at the circumference

Angle ABD = Angle $ACD = \frac{x}{2}$

(Total for Question 4 is 2 marks)

5



A, *B* and *C* are points on the circumference of a circle, centre *O*.
BT is the tangent to the circle at *B*.

Prove that angle *CAB* = angle *CBT*

Let angle $CBT = x$

Angle $OBC = 90 - x$ (a tangent meets a

Angle $BCO = 90 - x$ radius at $90°$)

Angle $COB = 180 - (90 - x) - (90 - x)$

$= 180 - 90 + x - 90 + x$

$= 2x$

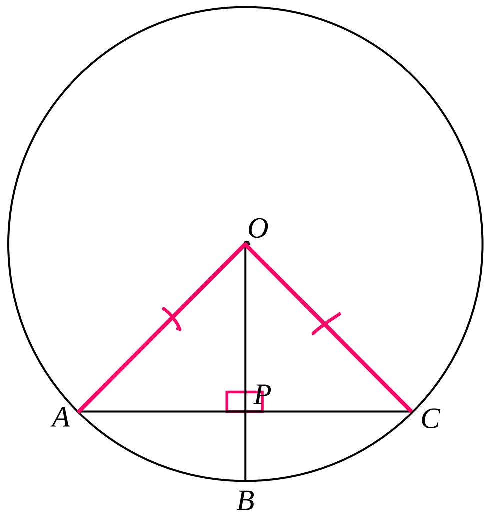
(angles in a triangle add to $180°$)

Angle $CAB = x$ angle at the centre is twice the angle at the circumference

Angle CAB = Angle $CBT = x$

(Total for Question 5 is 4 marks)

6



A, *B* and *C* are points on the circumference of a circle, centre *O*.
The lines *OB* and *AC* intersect at the point *P*.

Angle *APO* = angle *OPC* = 90°

Prove that *AP* = *PC*

OA = OC (both radii)

Triangles OAP and OCP both have side OP

Angle APO = angle OPC = 90°

Therefore triangles OAP and OCP are congruent due to RHS

If they are congruent then we have

OA = OB

OP = OP (common side)

So the remaining sides are equal and AP = PC

(Total for Question 6 is 4 marks)