Functions and Equations

1 $f(x) = 2x - 9$ $\qquad\qquad$ $g(x) = 7x + 1$

(a) Solve $fg(x) = 35$

$$fg(x) = 2(7x + 1) - 9$$
$$= 14x + 2 - 9$$
$$= 14x - 7$$

$$14x - 7 = 35$$
$$14x = 42$$

$$\underline{x = 3}$$

(3)

(b) Solve $f^{-1}(x) + g^{-1}(x) = 5$

$$y = 2x - 9 \qquad\qquad y = 7x + 1$$
$$x = 2y - 9 \qquad\qquad x = 7y + 1$$
$$c + 9 = 2y \qquad\qquad x - 1 = 7y$$
$$\frac{x+9}{2} = y \qquad\qquad \frac{x-1}{7} = y$$
$$f^{-1}(x) = \frac{x+9}{2} \qquad g^{-1}(x) = \frac{x-1}{7}$$

$$\frac{x+9}{2} + \frac{x-1}{7} = 5$$
$$\frac{7(x+9) + 2(x-1)}{14} = 5$$

$$7x + 63 + 2x - 2 = 70$$
$$9x + 61 = 70$$
$$9x = 9$$
$$x = 1$$

$$\underline{x = 1}$$

(4)

(Total for Question 1 is 7 marks)

1

2 $f(x) = \dfrac{8}{x}$ $\qquad$ $g(x) = x - 3$ $\qquad$ $h(x) = x^2$

(a) Solve $f(x) + g(x) = 3$

$$\frac{8}{x} + x - 3 = 3 \qquad\qquad x^2 - 6x + 8 = 0$$

$$\times x \left(\frac{8}{x} + x = 6 \right) \times x \qquad (x-2)(x-4) = 0$$

$$8 + x^2 = 6x \qquad\qquad x = 2 \quad x = 4$$

$$\underline{x = 2 \quad x = 4}$$
(3)

(b) Solve $h(x) = g(4x)$

$$x^2 = 4x - 3$$

$$x^2 - 4x + 3 = 0$$

$$(x - 3)(x - 1) = 0$$

$$x = 3 \quad x = 1$$

$$\underline{x = 3 \quad x = 1}$$
(3)

(c) $h^{-1}(100) + g^{-1}(3) = f(k)$ $\quad$ where k is a constant.
Work out the value of k.

$$h^{-1}(x) = \sqrt{x} \qquad\qquad h^{-1}(100) + g^{-1}(3) = f(k)$$

$$g^{-1}(x) = x + 3 \qquad\qquad 10 + 6 = \frac{8}{k}$$

$$16 = \frac{8}{k}$$

$$k = \frac{8}{16}$$

$k = \dfrac{1}{2}$
(4)

(Total for Question 2 is 10 marks)

1st

2

© 1stclassmaths

3 $f(x) = \dfrac{36}{x^2}$ $\qquad\qquad$ $g(x) = \sin(x)$ $\qquad\qquad$ $h(x) = 3x$

(a) Show that $f^{-1}(3) \times g(60)$ is an integer.

$$y = \frac{36}{x^2}$$

$$x = \frac{36}{y^2}$$

$$y^2 = \frac{36}{x}$$

$$y = \sqrt{\frac{36}{x}}$$

$$y = \frac{\sqrt{36}}{\sqrt{x}}$$

$$y = \frac{6}{\sqrt{x}}$$

$$f^{-1}(x) = \frac{6}{\sqrt{x}}$$

$$f^{-1}(3) \times g(60)$$

$$= \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$

$$= \frac{6\sqrt{3}}{2\sqrt{3}}$$

$$= \frac{6}{2}$$

$$= 3 \text{ (integer)}$$

(4)

(b) Solve $hf(x) - fh(x) = 26$

$$hf(x) = 3 \times \frac{36}{x^2}$$

$$= \frac{108}{x^2}$$

$$fh(x) = \frac{36}{(3x)^2}$$

$$= \frac{36}{9x^2}$$

$$= \frac{4}{x^2}$$

$$\frac{108}{x^2} - \frac{4}{x^2} = 26$$

$$\frac{104}{x^2} = 26$$

$$\frac{104}{26} = x^2$$

$$4 = x^2$$

$$x = \pm\sqrt{4}$$

$$x = 2 \qquad x = -2$$

(4)

(Total for Question 3 is 8 marks)

1st

3

4 $f(x) = x^2$ $g(x) = x + 4$ $h(x) = x + 2$

(a) Show that $fg(x) - fh(x) = 2g(x) + 2h(x)$

$fg(x) = (x+4)^2$ $fh(x) = (x+2)^2$
$\qquad\quad = x^2 + 8x + 16$ $\qquad\quad = x^2 + 4x + 4$

$fg(x) - fh(x) = x^2 + 8x + 16 - (x^2 + 4x + 4)$
$\qquad\qquad\quad = x^2 + 8x + 16 - x^2 - 4x - 4$
$\qquad\qquad\quad = 4x + 12$
$\qquad\qquad\quad = 2(2x + 6)$
$\qquad\qquad\quad = 2(x + 4 + x + 2)$
$\qquad\qquad\quad = 2(g(x) + h(x))$
$\qquad\qquad\quad = 2g(x) + 2h(x)$

(5)

(b) Solve $gf^{-1}(x) = 9$

$f^{-1}(x) = \sqrt{x}$ $\sqrt{x} + 4 = 9$
$gf^{-1}(x) = \sqrt{x} + 4$ $\sqrt{x} = 5$
$\qquad\qquad\qquad\qquad\qquad\qquad x = 25$

$x = 25$

(3)

(Total for Question 4 is 8 marks)

1st

4

5 $f(x) = x^2$ $\qquad$ $g(x) = \dfrac{x+8}{11}$ $\qquad$ $h(x) = ax + b$

(a) Solve $f(x + 2) = g^{-1}(x)$

$$f(x+2) = (x+2)^2$$
$$= x^2 + 4x + 4$$

$$g^{-1}(x) = 11x - 8$$

$$x^2 + 4x + 4 = 11x - 8$$
$$x^2 - 7x + 12 = 0$$
$$(x - 3)(x - 4) = 0$$
$$x = 3 \qquad x = 4$$

$$\underline{x = 3 \qquad x = 4}$$

(4)

$h(3) = 7$
$h^{-1}(55) = 15$

(b) Work out the values of a and b.

$$y = ax + b$$
$$x = ay + b$$
$$x - b = ay$$
$$\frac{x-b}{a} = y$$
$$h^{-1}(x) = \frac{x-b}{a}$$

$$h(3) = 7$$
$$3a + b = 7$$

$$h^{-1}(55) = 15$$
$$\frac{55 - b}{a} = 15$$
$$55 - b = 15a$$
$$55 = 15a + b$$

$$15a + b = 55$$
$$- \quad \underline{3a + b = 7}$$
$$12a \qquad = 48$$
$$a = 4$$

$$3a + b = 7$$
$$12 + b = 7$$
$$b = -5$$

$a =$ .... **4** ....

$b =$ .... **−5** ....

(5)

(Total for Question 5 is 9 marks)

1st