



# Iteration



REVISE THIS TOPIC

CHECK YOUR ANSWERS



1 (a) Use the iteration formula  $x_{n+1} = \frac{(x_n)^2 + 3}{5}$  to find the values of  $x_1, x_2$  and  $x_3$

Start with  $x_0 = \sqrt{3}$

$x_1 =$  .....

$x_2 =$  .....

$x_3 =$  .....

(3)

(b) Explain the relationship between the values of  $x_1, x_2$  and  $x_3$  and the equation  $x^2 - 5x + 3 = 0$

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(2)

(Total for Question 1 is 5 marks)



2 (a) Use the iteration formula  $x_{n+1} = \sqrt{\frac{x_n}{10} + 2}$  to find the values of  $x_1$ ,  $x_2$  and  $x_3$

Start with  $x_0 = 42.5$

$x_1 =$  .....

$x_2 =$  .....

$x_3 =$  .....

(3)

The values of  $x_1$ ,  $x_2$  and  $x_3$  found in part (a) are estimates to the solution of an equation in the form  $ax^2 - x + b = 0$  where  $a$  and  $b$  are integers.

(b) Find the values of  $a$  and  $b$ .

$a =$  .....

$b =$  .....

(2)

**(Total for Question 2 is 5 marks)**

3 (a) Use the iteration formula  $x_{n+1} = \sqrt{80 - 5x_n}$  to find the values of  $x_1$ ,  $x_2$  and  $x_3$

Start with  $x_0 = 12.8$

$x_1 =$  .....

$x_2 =$  .....

$x_3 =$  .....

(3)

The values of  $x_1$ ,  $x_2$  and  $x_3$  found in part (a) are estimates to the solution of an equation in the form  $x^2 + ax + b = 0$  where  $a$  and  $b$  are integers.

(b) Find the values of  $a$  and  $b$ .

$a =$  .....

$b =$  .....

(2)

**(Total for Question 3 is 5 marks)**



4 (a) Use the iteration formula  $x_{n+1} = \sqrt[3]{8 - (x_n)^2}$  to find the values of  $x_1$ ,  $x_2$  and  $x_3$

Start with  $x_0 = 1.8$

$x_1 =$  .....

$x_2 =$  .....

$x_3 =$  .....

(3)

(b) Explain the relationship between the values of  $x_1$ ,  $x_2$  and  $x_3$  and the equation  $x^3 + x^2 - 8 = 0$

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(2)

(Total for Question 4 is 5 marks)



5 (a) Show that the equation  $x^3 - x - 4 = 0$  has a solution between  $x = 1$  and  $x = 2$

(2)

(b) Show that the equation  $x^3 - x - 4 = 0$  can be rearranged to give  $x = \sqrt[3]{x + 4}$

(2)

(c) Starting with  $x_0 = 2$ , use the iteration formula  $x_{n+1} = \sqrt{x_n + 4}$  three times to find an estimate for the solution of  $x^3 - x - 4 = 0$

(3)

(d) By substituting your answer to part (c) into  $x^3 - x - 4$  comment on the accuracy of your estimate for the solution to  $x^3 - x - 4 = 0$

(2)

(Total for Question 5 is 9 marks)



6 (a) Show that the equation  $x^2 + x - 13 = 0$  has a solution between  $x = 3$  and  $x = 4$

(2)

(b) Show that the equation  $x^2 + x - 13 = 0$  can be rearranged to give  $x = \sqrt{13 - x}$

(2)

(c) Starting with  $x_0 = 3$ , use the iteration formula  $x_{n+1} = \sqrt{13 - x_n}$  three times to find an estimate for the solution of  $x^2 + x - 13 = 0$

(3)

(d) By substituting your answer to part (c) into  $x^2 + x - 13$  comment on the accuracy of your estimate for the solution to  $x^2 + x - 13 = 0$

(2)

(Total for Question 6 is 9 marks)



7 (a) Show that the equation  $x^2 - 10x + 6 = 0$  has a solution between  $x = 0$  and  $x = 1$

(b) Show that the equation  $x^2 - 10x + 6 = 0$  can be rearranged to give  $x = \frac{x^2 + 6}{10}$  (2)

(c) Starting with  $x_0 = 1$ , use the iteration formula  $x_{n+1} = \frac{(x_n)^2 + 6}{10}$  three times to find an estimate for the solution of  $x^2 - 10x + 6 = 0$  (2)

(d) By substituting your answer to part (c) into  $x^2 - 10x + 6$  comment on the accuracy of your estimate for the solution to  $x^2 - 10x + 6 = 0$  (3)

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(2)  
(Total for Question 7 is 9 marks)



8 (a) Show that the equation  $x^3 - 20x^2 + 100x - 8 = 0$  has a solution between  $x = 10$  and  $x = 11$

(2)

(b) Show that the equation  $x^3 - 20x^2 + 100x - 8 = 0$  can be rearranged to give  $x = \sqrt{\frac{8}{x}} + 10$

(4)

(c) Starting with  $x_0 = 2$ , use the iteration formula  $x_{n+1} = \sqrt{\frac{8}{x_n}} + 10$  three times to find an estimate for the solution of  $x^3 - 20x^2 + 100x - 8 = 0$

(3)

(d) By substituting your answer to part (c) into  $x^3 - 20x^2 + 100x - 8$  comment on the accuracy of your estimate for the solution to  $x^3 - 20x^2 + 100x - 8 = 0$

(2)

(Total for Question 8 is 9 marks)

