Velocity-Time Graphs

SCAN ME

REVISE THIS TOPIC

1 A cyclist travels along a straight horizontal path between points A and B.

In a model of the motion of the cyclist, at time $t = 0$ the cyclist starts from rest at point A and accelerates uniformly for 5 seconds until they reach a speed of 10 ms^{-1}

For the next 30 seconds the cyclist travels at a constant speed of 10 ms^{-1} before decelerating uniformly for 2.5 seconds, coming to rest at point B.

Using the model

(a) Sketch a speed-time graph for the motion of the cyclist between points A and B. **(1)**

(b) Find the acceleration of the cyclist during the first 5 seconds. **(1)**

(c) Find the deceleration of the cyclist during the final 2.5 seconds. **(1)**

(d) Find the total distance travelled by the cyclist. **(2)**

(b) $\frac{10}{5} = \mathbf{2\ ms^{-2}}$

(c) $\frac{10}{2.5} = \mathbf{4\ ms^{-2}}$

(d) $0.5 \times (37.5 + (35 - 5)) \times 10$

$= \mathbf{337.5\ m}$

(Total for Question 1 is 5 marks)

2

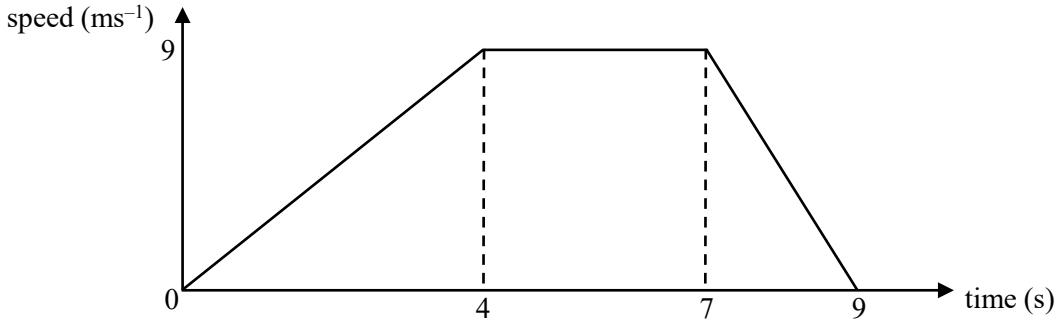


Figure 1

Figure 1 shows the speed-time graph for the journey of a dog chasing after a ball in straight line on horizontal ground.

At time $t = 0$, the dog is at rest at point P.

The dog then accelerates uniformly for 4 seconds until it reaches a speed of 9 ms^{-1}

For the next 3 seconds the dog travels at a constant speed of 9 ms^{-1}

The dog then decelerates uniformly until it comes to rest at point Q at time $t = 9$

(a) Find the acceleration of the dog during the first 4 seconds. **(1)**

(b) Find the deceleration of the dog during the final 2 seconds. **(1)**

(c) Find the distance PQ. **(2)**

(d) Find the average speed of the dog for the entire journey. **(1)**

(e) Suggest one limitation of the model which could affect your answers to parts (a) to (d) **(1)**

(a) $\frac{9}{4} = \mathbf{2.25\ ms^{-2}}$

(b) $\frac{9}{2} = \mathbf{4.5\ ms^{-2}}$

(c) $0.5 \times (9 + (7 - 4)) \times 9$

$= \mathbf{54\ m}$

(d) $\frac{54}{9} = \mathbf{6\ ms^{-1}}$

(e) Any suitable limitation e.g.

Dog unlikely to maintain constant speed or acceleration

Changes in acceleration would be smooth and not instantaneous

(Total for Question 2 is 6 marks)

3 A car travels along a straight horizontal road between points C and D.

In a model of the motion of the car, at time $t = 0$ the car starts from rest at point C and moves with constant acceleration 2.5 ms^{-2} until it reaches a speed of 18 ms^{-1}

The car then moves at a constant speed of 18 ms^{-1} before it moves with a constant deceleration for 4 seconds as it comes to rest at point D.

The total time for the journey between C and D is 40 seconds.

(a) For this model of the motion of the car between C and D,

(i) state the time for which the car is accelerating **(1)**

(ii) state the value of the constant deceleration **(1)**

(iii) sketch a speed-time graph **(1)**

(a) (i) $\frac{18}{2.5} = \mathbf{7.2\ seconds}$ (ii) $\frac{-18}{4} = \mathbf{-4.5\ ms^{-2}}$

(iii)

(b) Find the distance between points C and D. **(2)**

(b) $0.5 \times (40 + (36 - 7.2)) \times 18$

$= \mathbf{619.2\ m}$

(Total for Question 3 is 5 marks)

4

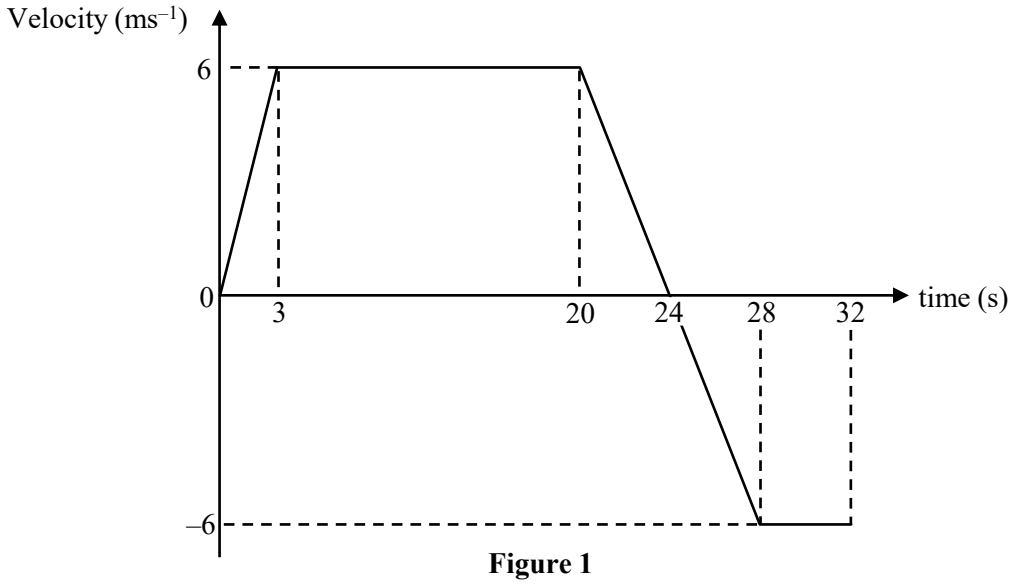


Figure 1

Figure 1 shows the velocity-time graph for a model of the motion of a person riding an electric scooter along a straight horizontal road.

(a) Find the acceleration during the first 3 seconds **(1)**

(b) Find the acceleration between 20 and 28 seconds. **(1)**

(c) Find the total distance travelled during the 32 second journey. **(3)**

(d) Find the displacement of the person, from their starting point, at time $t = 30$ **(2)**

(a) $\frac{6}{3} = \mathbf{2\ ms^{-2}}$

(b) $\frac{6-(-6)}{28-20} = \frac{-12}{8} = \mathbf{-1.5\ ms^{-2}}$

(c) $0.5 \times (24 + (20 - 3)) \times 6$

$= 123$ m

$0.5 \times 4 \times 6$

$= 12$ m

4×6

$= 24$ m

$123 + 12 + 24 = \mathbf{159m}$

(d) $123 - 12 - (6 \times 2) = \mathbf{99\ m}$

(Total for Question 4 is 7 marks)

5

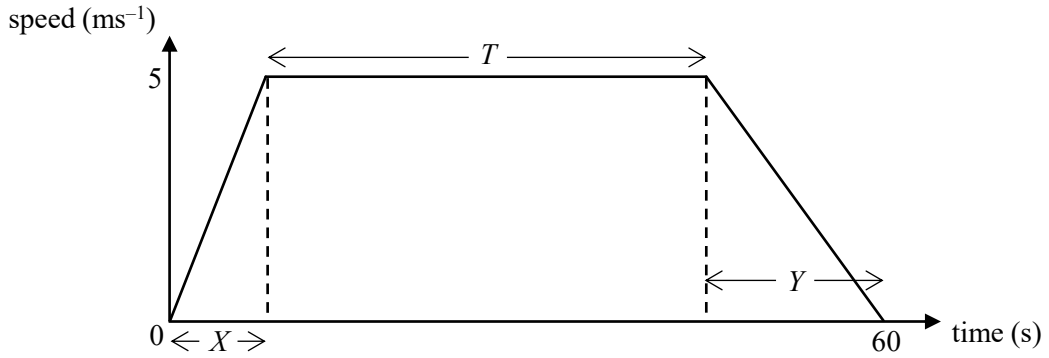


Figure 1

Figure 1 shows a sketch of the speed-time graph for a model of the motion of a runner travelling along a straight horizontal road from point A to point B in a time of 60 seconds.

The distance from point A to point B is 267 metres.

In the model of the motion, the runner

- starts from rest at A at time $t = 0$
- then moves with constant acceleration for X seconds until they reach a speed of 5 ms^{-1}
- then travels at a constant speed of 5 ms^{-1} for T seconds.
- then moves with constant deceleration for Y seconds, until coming to rest at B.

(a) Find the value of T **(3)**

(b) Given that $Y = 2X$, find

(i) the value of X **(2)**

(ii) the acceleration during the first X seconds. **(1)**

(a) $0.5 \times (60 + T) \times 5 = 267$

$2.5 \times (60 + T) = 267$

$60 + T = 106.8$

$T =$ **46.8 seconds**

(b) (i) $X + T + Y = 60$

$X + 46.8 + Y = 60$

$X + Y = 13.2$

$X + 2X = 13.2$

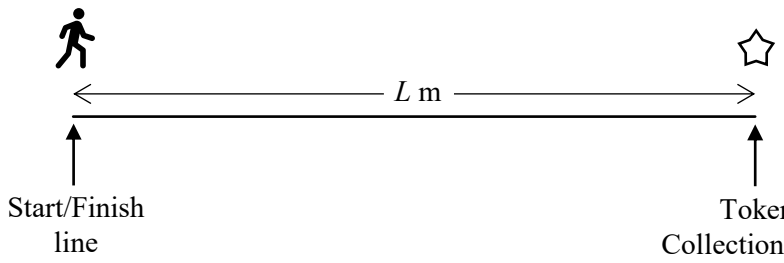
$3X = 13.2$

$X =$ **4.4 seconds**

(b) (ii) $\frac{5}{4.4} =$ **1.14 ms^{-2}**

(Total for Question 5 is 6 marks)

6 In a gameshow a contestant must run across a straight horizontal bridge of length L metres.
Once they reach the other side, they must collect a token, then run back to the start/finish line.



In the model of the motion, the contestant starts from rest at the start/finish line at time $t = 0$ then accelerates at a constant rate for 3 seconds, until they reach a velocity of 6 ms^{-1}

They then maintain a constant velocity of 6 ms^{-1} for 12 seconds before decelerating at a constant rate for 3 seconds until they come to rest at the token collection point.

It takes them 5 seconds to find the token, and then they accelerates back towards the start/finish line at a constant rate for 4 seconds until they reach a velocity of –6 ms^{-1}

They then maintain at a constant velocity of until they cross the start/finish line at time T seconds.

Figure 1 shows the velocity time graph for a model of the motion of a contestant on a gameshow.

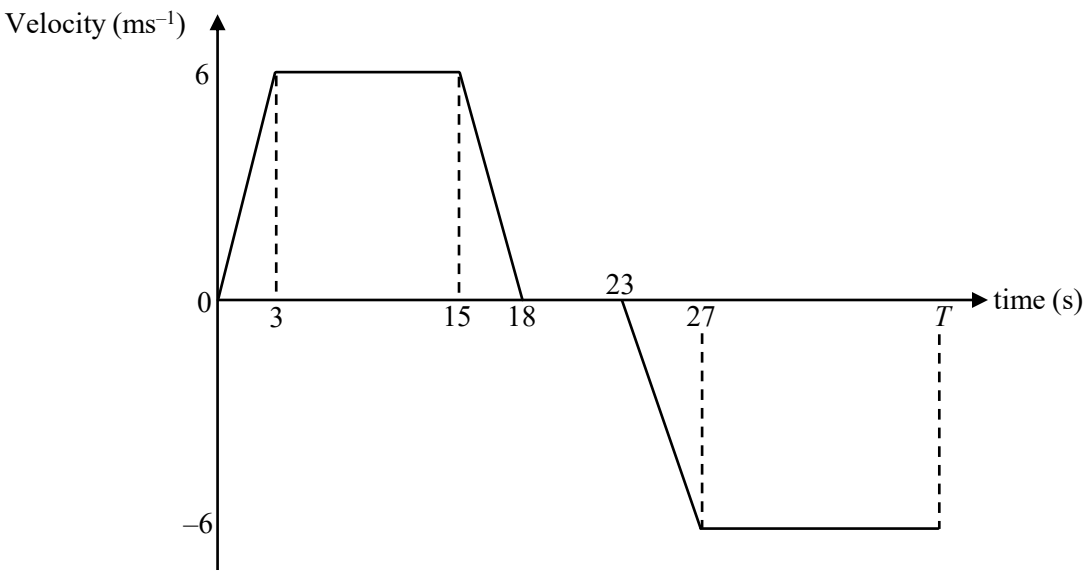


Figure 1

(a) Find the acceleration of the contestant during the first 3 seconds. Give the units of your answer. **(1)**

(b) Find the value of L **(2)**

(c) Find the value of T **(3)**

At time $t = P$ the displacement of the contestant from the start finish line is 60 m

(d) Find two possible values for P **(4)**

(a) $\frac{6}{3} = \mathbf{2\ ms^{-2}}$

(b) $L = 0.5 \times (18 + (15 - 3)) \times 6$

$= \mathbf{90\ m}$

(c) $0.5 \times ((T - 27) + (T - 23) \times 6 = 90$

$3 \times (2T - 50) = 90$

$2T - 50 = 30$

$2T = 80$

$T = \mathbf{40\ seconds}$

(d) Outward journey: $0.5 \times 6 \times 3 + 6(P - 3) = 60$

$9 + 6(P - 3) = 60$

$6(P - 3) = 51$

$P - 3 = 8.5$

$P = \mathbf{11.5\ seconds}$

Return journey: $0.5 \times 4 \times 6 + 6(P - 27) = 30$ [Note: displacement = 60 m and L = 90 m so must travel 30 m back towards start/finish line]

$12 + 6(P - 27) = 30$

$6(P - 27) = 18$

$P - 27 = 3$

$P = \mathbf{30\ seconds}$

(Total for Question 6 is 10 marks)

7

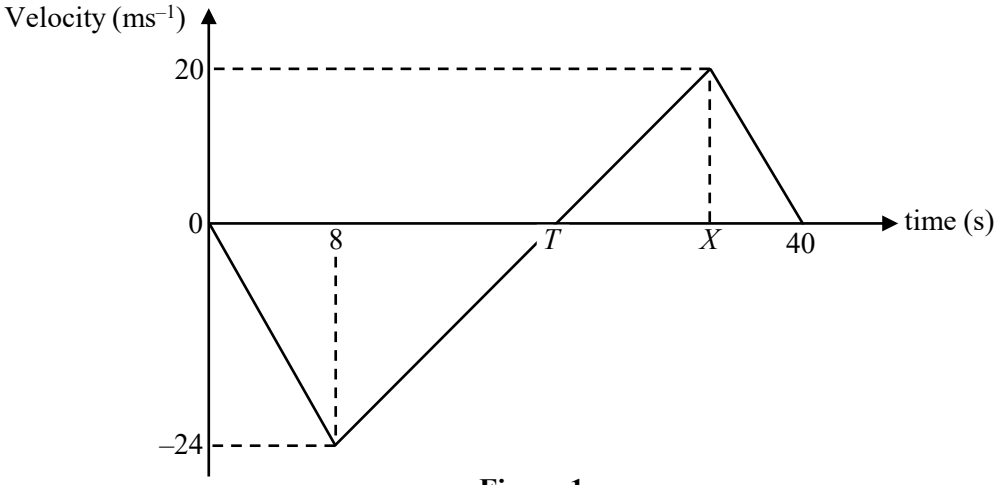


Figure 1

Figure 1 shows a sketch of the velocity-time graph for a model of the motion of a particle travelling in a straight line for 40 seconds.

In a model of the motion, the particle

- moves with uniform acceleration A ms^{-2} between $t = 0$ and $t = 8$
- moves with uniform acceleration B ms^{-2} between $t = 8$ and $t = X$
- moves with uniform acceleration C ms^{-2} between $t = X$ and $t = 40$

(a) Find the value of A **(1)**

(b) State the maximum **speed** of the particle during the 40 second journey. **(1)**

The velocity of the particle is 0 ms^{-1} when $t = 0$, $t = T$ and $t = 40$.
The total distance travelled by the particle during the 40 second journey is 446 metres.

(c) Find the value of T **(3)**

(d) Find the average velocity for the 40 second journey. **(3)**

(e) Find the value of B **(1)**

(f) Find the value of X **(2)**

(g) Find the value of C **(1)**

(a) $\frac{-24}{8} = \mathbf{-3\ ms^{-2}}$

(b) **24 ms^{-1}** [Remember, speed the magnitude of velocity]

(c) $(0.5 \times 24 \times T) + (0.5 \times 20 \times (40 - T)) = 446$

$12T + 10(40 - T) = 446$

$12T + 400 - 10T = 446$

$2T + 400 = 446$

$2T = 46$

$T =$ **23 seconds**

(d) Total displacement $= (0.5 \times (-24) \times 23) + (0.5 \times (40 - 23) \times 20)$

$= -106$ m

Average velocity $= \frac{-106}{40} = \mathbf{-2.65\ ms^{-1}}$

(e) $B = \frac{24}{23 - 8} = \mathbf{1.6\ ms^{-2}}$

(f) $1.6 = \frac{20}{X - 23}$

$X - 23 = \frac{20}{1.6}$

$X - 23 = 12.5$

$X =$ **35.5 seconds**

(g) $C = \frac{-20}{40 - 35.5} = \mathbf{-4.44\ ms^{-2}}$

(Total for Question 7 is 12 marks)

8

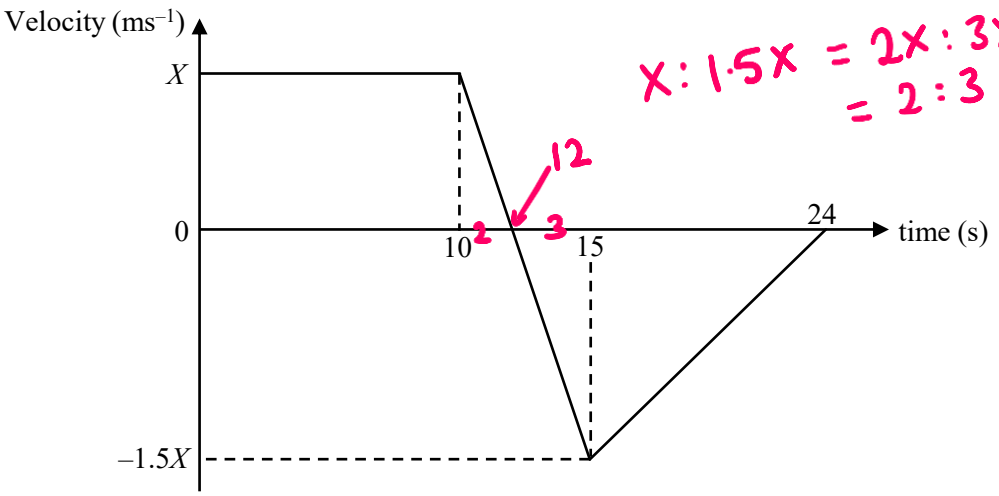


Figure 1

Figure 1 shows a sketch of the velocity-time graph for a model of the motion of a particle travelling in a straight line.

In the model of the motion, the particle

- starts travelling with constant velocity of X ms^{-1} at time $t = 0$ until time $t = 10$
- then moves with uniform acceleration a ms^{-2}, for 5 seconds until it reaches a velocity of $–1.5X$ ms^{-1}
- then moves with uniform acceleration b ms^{-2}, for 9 seconds, until coming to rest at time $t = 24$

At time $t = 24$, the displacement of the particle from its starting position is 7.2 metres.

Find the values of X, a and b. **(6)**

$7.2 = (0.5 \times (10 + 12) \times X) – (0.5 \times (24 – 12) \times 1.5X)$

$7.2 = (11X) – (9X)$

$7.2 = 2X$

$X =$ **3.6 ms^{-1}**

$a = \dfrac{3.6 – (–5.4)}{15 – 10}$ $a =$ **–1.8 ms^{-2}**

$b = \dfrac{5.4}{24 – 15}$ $b =$ **0.6 ms^{-2}**

(Total for Question 8 is 6 marks)

9

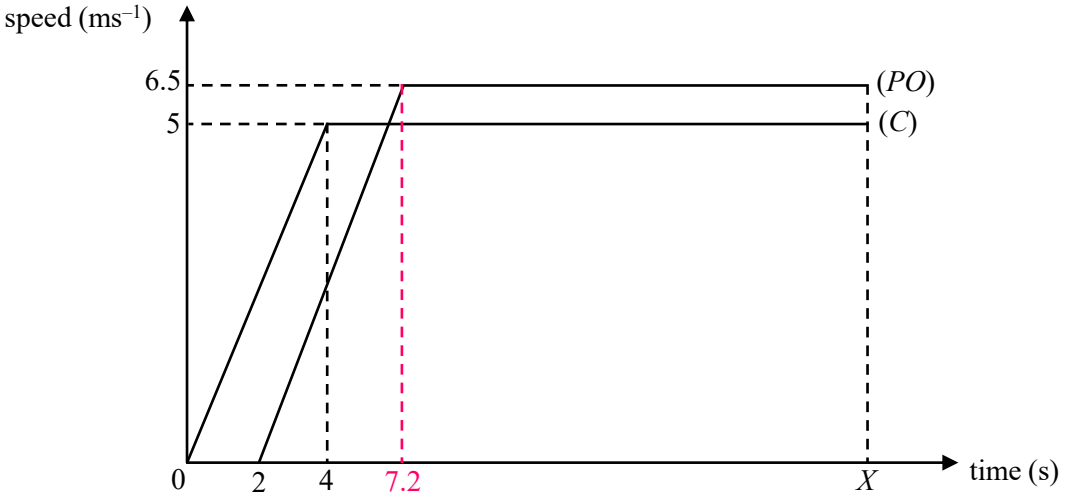


Figure 1

A police officer (PO) catches a criminal (C). The criminal manages to break free and runs away from the police officer in a straight line along a horizontal road. The police officer chases and eventually catches the criminal.

In a model of their motion, the criminal starts from rest at time $t = 0$ before travelling with constant acceleration for 4 seconds until they reach a speed of 5 ms^{-1}. The criminal maintains a constant speed of 5 ms^{-1} until they are caught by the police officer at time X seconds.

The police officer starts at rest at time $t = 0$ but remains at rest until time $t = 2$ when they accelerate with the same acceleration as the criminal until they reach a speed of 6.5 ms^{-1}. The police officer maintains a constant speed of 6.5 ms^{-1} until they catch the criminal at time X seconds.

The speed-time graphs shown in Figure 1 describe the model of the motion for the police officer and the criminal.

(a) Find the value of X. **(5)**

(b) Find the distance travelled by the criminal before they are caught. **(2)**

(c) Describe one limitation of the model that could affect your answers to parts (a) and (b) **(1)**

(a) Acceleration of criminal (and police officer) $= \frac{5}{4} = 1.25\ \text{ms}^{-2}$

Time spent accelerating (police officer) $= \frac{6.5}{1.25} = 5.2$ seconds

Distance travelled by criminal $= (0.5 \times 4 \times 5) + 5(X - 4)$

Distance travelled by police officer $= (0.5 \times 6.5 \times 5.2) + 6.5(X - 7.2)$

$(0.5 \times 4 \times 5) + 5(X - 4) = (0.5 \times 6.5 \times 5.2) + 6.5(X - 7.2)$

$10 + 5X - 20 = 16.9 + 6.5X - 46.8$

$5X - 10 = 6.5X - 29.9$

$19.9 = 1.5X$

$X =$ **13.3 seconds**

(b) Distance travelled by criminal $= (0.5 \times 4 \times 5) + 5(13.26\ldots - 4)$

= 56.3 m

(c) Any suitable limitation e.g.

Police officer/criminal unlikely to maintain constant speed or acceleration

Changes in acceleration would be smooth and not instantaneous

(Total for Question 9 is 8 marks)

10

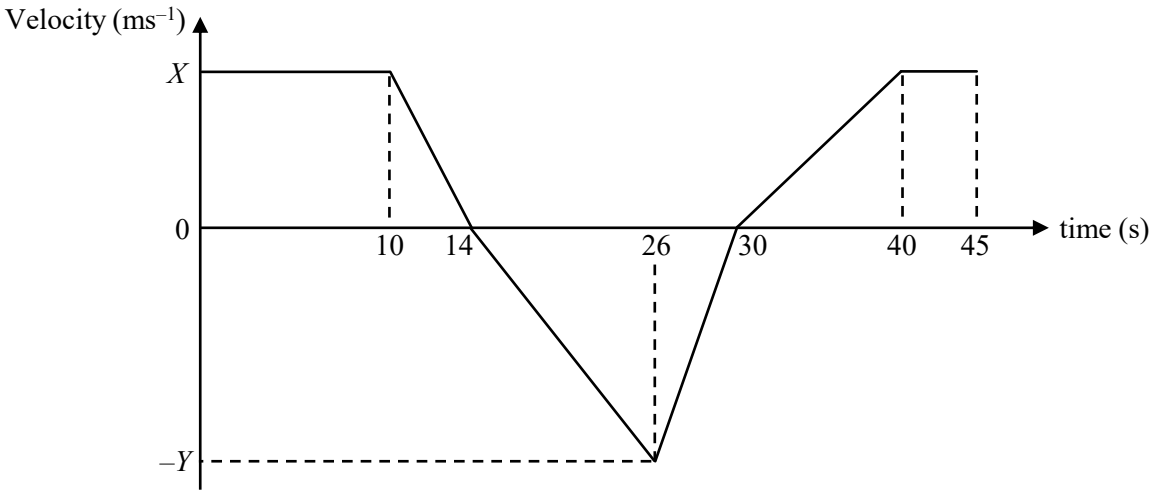


Figure 1

Figure 1 shows a sketch of the velocity-time graph for a model of the motion of a particle travelling in a straight line for 45 seconds.

In a model of the motion, the particle

- moves with constant velocity X ms^{-1}, between the following time intervals.

  $t = 0$ and $t = 10$
  $t = 40$ and $t = 45$

- moves with uniform acceleration between the following time intervals

  $t = 10$ and $t = 14$
  $t = 14$ and $t = 26$
  $t = 26$ and $t = 30$
  $t = 30$ and $t = 45$

The total displacement of the particle when $t = 30$ is 0 m
The total distance travelled by the particle for the 45 second journey is 289 m

Work out the average velocity for the 45 second journey. **(6)**

$10 \times X = 10X$

$0.5 \times (14 - 10) \times X = 2X$

$0.5 \times (30 - 14) \times Y = 8Y$

$0.5 \times (40 - 30) \times X = 5X$

$(45 - 40) \times X = 5X$

Displacement (when $t = 30$) $= 10X + 2X - 8Y$

$= 12X - 8Y$

Total distance $= 10X + 2X + 8Y + 5X + 5X$

$= 22X + 8Y$

$12X - 8Y = 0$

$22X + 8Y = 289$

$22X + 8Y = 289$

$12X - 8Y = 0$

$X = 8.5$ (From calculator)

$Y = 12.75$ (From calculator)

Total displacement $= 22X - 8Y$

$= 22(8.5) - 8(12.75)$

$= 85$ m

Average velocity $= \frac{85}{45} =$ **1.89 ms^{-1}**

(Total for Question 10 is 6 marks)