Variable Acceleration

SCAN ME

REVISE THIS TOPIC

1 A fixed point O lies on a straight line.

A particle, P, moves along the straight line.

At time t seconds, $0 \leq t \leq 10$, the distance, s metres, of P from O is given by

$$s = 10t - t^2$$

When $t = 1$, P is at point X.

When $t = T$, P is at point Y, where $OY = 21$ m

Find,

(a) the distance OX. **(1)**

(b) two possible values for T. **(2)**

(c) the maximum distance of point P from the point O in the interval $0 \leq t \leq 10$ **(2)**

(d) the total distance travelled by the point P in the interval $0 \leq t \leq 10$ **(2)**

(a) $s(1) = 10(1) - 1^2 = 9$

$OX =$ **9 m**

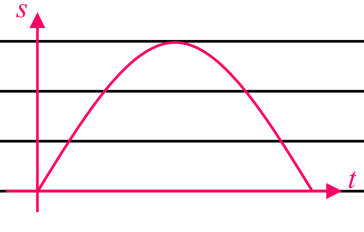
(b) $10T - T^2 = 21$

$T^2 - 10T + 21 = 0$

$(T - 3)(T - 7) = 0$

$T =$ **3 seconds** $\quad T =$ **7 seconds**

(c)



$s = -[t^2 - 10t]$

$s = -[(t - 5)^2 - 25]$

$s = -(t - 5)^2 + 25$

So turning point has coordinates (5, 25)

Maximum distance = **25 m**

(d) $s(0) = 10(0) - 0^2 = 0$

$s(10) = 10(10) - 10^2 = 0$

$25 + 25 =$ **50 m**

(Total for Question 1 is 7 marks)

2 A fixed point O lies on a straight line.

A particle, P, moves along the straight line.

At time t seconds, $t \geq 0$, the distance, s metres, of P from O is given by

$$s = 36t - t^3$$

When $t = 1$, P is at point X.
When $t = 2$, P is at point Y.
When $t = T$, $T > 0$, P is at point O.

Find,

(a) XY **(2)**

(b) T **(2)**

(c) the maximum distance of point P from the point O in the interval $0 \leq t \leq T$ **(3)**

(d) the total distance travelled by P in the interval $0 \leq t \leq T$ **(2)**

(a) $s(1) = 36(1) - 1^3 = 35$

$s(2) = 36(2) - 2^3 = 64$

$XY = 64 - 35$

$XY = \mathbf{29\ m}$

(b) When P is at O, $s = 0$

$36T - T^3 = 0$

$T(36 - T^2) = 0$

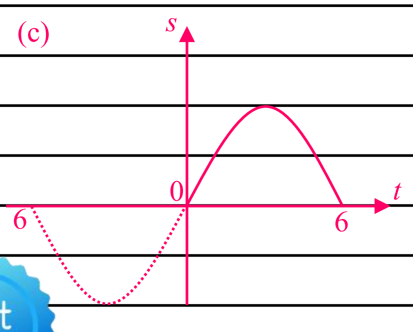
$T = 0 \qquad 36 - T^2 = 0$

$T^2 = 36$

$T = 6 \qquad T = -6$

$T > 0$, so $\boldsymbol{T = 6}$

(c)



(c) ctd $\dfrac{ds}{dt} = 36 - 3t^2$

Maximum distance when $\dfrac{ds}{dt} = 0$

$36 - 3t^2 = 0$

$3t^2 = 36$

$t^2 = 12$

$t = 2\sqrt{3}$ (ignore $-2\sqrt{3}$ as $t \geq 0$)

When $t = 2\sqrt{3}$ $\quad s = 36(2\sqrt{3}) - (2\sqrt{3})^3$

$s = \mathbf{48\sqrt{3}\ m}$

(d) $48\sqrt{3} + 48\sqrt{3} = \mathbf{96\sqrt{3}\ m}$

(Total for Question 2 is 9 marks)

3 A particle, P, moves along the x-axis. At time t seconds, $t \geq 0$, the displacement, x metres, of P from the origin O, is given by $x = 4t^3 - 28t^2 + 49t$

(a) Find the velocity of P at time t seconds. **(2)**

(b) Find the speed of P when $t = 2$ **(2)**

(c) Find the values of t when P is instantaneously at rest. **(3)**

When $t = 0$, P is at point O.

(d) Find the other value of t when P is at point O. **(2)**

(e) Show that P will never move along the negative x-axis. **(2)**

(a) $v = \frac{dx}{dt} = 12t^2 - 56t + 49$

$\mathbf{v = 12t^2 - 56t + 49}$

(b) $v(2) = 12(2)^2 - 56(2) + 49 = -15$

Speed = **15 ms^{-1}**

(c) Instantaneous rest when $v = 0$

$12t^2 - 56t + 49 = 0$

$(6t - 7)(2t - 7) = 0$

$\mathbf{t = \frac{7}{6}}$ **seconds** $\quad \mathbf{t = \frac{7}{2}}$ **seconds**

(d) When P is at O, $x = 0$

$4t^3 - 28t^2 + 49t = 0$

$t(4t^2 - 28t + 49) = 0$

$t(2t - 7)(2t - 7) = 0$

$t = 0 \qquad 2t - 7 = 0$

$\mathbf{t = \frac{7}{2}}$ **seconds**

(e) $x = t(2t - 7)^2$

Since $t \geq 0$ (given in question)

and $(2t - 7)^2 \geq 0$ (for all values of t)

$t(2t - 7)^2 \geq 0$

Therefore $x \geq 0$ (never negative)

(Total for Question 3 is 11 marks)

4 A fixed point O lies on a straight line.

A particle, P, moves along the straight line.

At time t seconds, $t \geq 0$, the distance, s metres, of P from O is given by

$$s = \frac{1}{3}t^3 - 4t^2 + 7t$$

Find,

(a) the maximum speed of P in the interval $0 \leq t \leq 8$ **(4)**

(b) the magnitude of the acceleration of P when $t = 3$ **(2)**

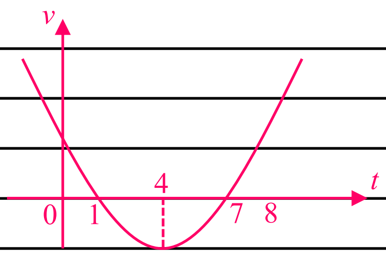
(c) The total distance travelled by P in the interval $0 \leq t \leq 8$ **(4)**

(a) $v = \frac{ds}{dt} = t^2 - 8t + 7$

$v = t^2 - 8t + 7$

$v = (t-1)(t-7)$

$v = 0$ when $t = 1$ and $t = 7$



Check v at turning point ($t = 4$)

and end points ($t = 0$, $t = 8$)

$v(4) = (4)^2 - 8(4) + 7 = -9 \text{ ms}^{-1}$

$v(0) = (0)^2 - 8(0) + 7 = 7 \text{ ms}^{-1}$

$v(8) = (8)^2 - 8(8) + 7 = 7 \text{ ms}^{-1}$

Therefore max speed = $\mathbf{9 \text{ ms}^{-1}}$

(b) $v = t^2 - 8t + 7$

$a = \frac{dv}{dt} = 2t - 8$

$a(3) = 2(3) - 8 = -2$

Magnitude of acceleration = $\mathbf{2 \text{ ms}^{-2}}$

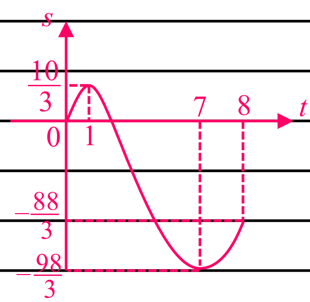
(c) P changes direction when $t = 1$ and $t = 7$

$s(0) = \frac{1}{3}(0)^3 - 4(0)^2 + 7(0) = 0$

$s(1) = \frac{1}{3}(1)^3 - 4(1)^2 + 7(1) = \frac{10}{3}$

$s(7) = \frac{1}{3}(7)^3 - 4(7)^2 + 7(7) = -\frac{98}{3}$

$s(8) = \frac{1}{3}(8)^3 - 4(8)^2 + 7(8) = -\frac{88}{3}$



Total distance = $\frac{10}{3} + \left(\frac{10}{3} + \frac{98}{3}\right) + \left(\frac{98}{3} - \frac{88}{3}\right) = \mathbf{\frac{128}{3}}$ **m**

(Total for Question 4 is 10 marks)

5 A particle, P, moves along the x-axis. At time t seconds, $t \geq 0$, the displacement, x metres, of P from the origin O, is given by $\quad x = \frac{1}{3}t^3 - 3t^2 + 5t$

At $t = 6$, P is at the point M.

(a) Find the distance OM. **(2)**

(b) Find the values of t for which P is at instantaneous rest. **(3)**

(c) Find the total distance travelled by P in the interval $0 \leq t \leq 6$ **(4)**

(d) Find the value of t at which the acceleration of P is zero. **(2)**

(a) $x(6) = \frac{1}{3}(6)^3 - 3(6)^2 + 5(6) = -6 \qquad OM =$ **6 m**

(b) $v = \frac{dx}{dt} = t^2 - 6t + 5$

Instantaneous rest when $v = 0$

$t^2 - 6t + 5 = 0$

$(t - 1)(t - 5) = 0$

$t =$ **1 second** $\qquad t =$ **5 seconds**

(d) $a = \frac{dv}{dt} = 2t - 6$

$0 = 2t - 6$

$2t = 6$

$t =$ **3 seconds**

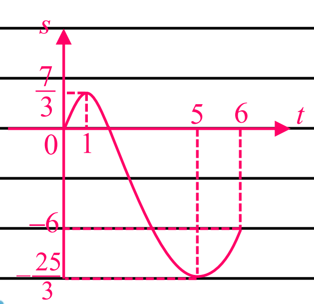
(c) P changes direction when $t = 1$ and $t = 5$

$x(0) = \frac{1}{3}(0)^3 - 3(0)^2 + 5(0) = 0$

$x(1) = \frac{1}{3}(1)^3 - 3(1)^2 + 5(1) = \frac{7}{3}$

$x(5) = \frac{1}{3}(5)^3 - 3(5)^2 + 5(5) = -\frac{25}{3}$

$x(6) = -6 \quad$ [from part (a)]



Total distance $= \frac{7}{3} + \left(\frac{7}{3} + \frac{25}{3}\right) + \left(\frac{25}{3} - 6\right) = \mathbf{\frac{46}{3}}$ **m**

(Total for Question 5 is 11 marks)

6 A particle, P, moves along a straight line such that at time t seconds, $t \geq 0$, the velocity of P, v ms^{-1}, is modelled as

$$v = t^3 + 3t^2$$

(a) Find the velocity of P when $t = 5$ **(1)**

(b) Find the acceleration of P at time t seconds. **(2)**

(c) Show that P never changes direction. **(2)**

(d) Find the total distance travelled by P in the interval $0 \leq t \leq 4$ **(3)**

(a) $v(5) = 5^3 + 3(5)^2 = \mathbf{200\ ms^{-1}}$

(b) $a = \frac{dv}{dt} = \mathbf{3t^2 + 6t}$

(c) $v = t^3 + 3t^2$

$v = t^2(t + 3)$

Since $t \geq 0$ (given in question)

$(t + 3) \geq 0$ and $t^2 \geq 0$

$t^2(t + 3) \geq 0$

Therefore $v \geq 0$ (so P always travels in the positive direction)

(d) $s = \int v \, dt$

$s = \int t^3 + 3t^2 \, dt = \frac{1}{4}t^4 + t^3 + c$

Since P never changes direction just check end points.

$s(0) = \frac{1}{4}(0)^4 + (0)^3 + c = c$

$s(4) = \frac{1}{4}(4)^4 + (4)^3 + c = 128 + c$

Distance travelled $= s(4) - s(0)$

$= (128 + c) - (c)$

$= \mathbf{128\ m}$

(Total for Question 6 is 8 marks)

7 A fixed point O lies on a straight line.

A particle, P, moves along the straight line such that at time t seconds, $t \geq 0$, after passing through point O, the velocity of P, v ms^{-1} is modelled as

$$v = t^2 - 6t + 8$$

(a) Find the range of values of t for which the velocity of P is negative. **(2)**

(b) Find the greatest speed of P in the interval $0 \leq t \leq 5$ **(3)**

(c) Find the magnitude of the acceleration of P the first time that P at instantaneous rest. **(4)**

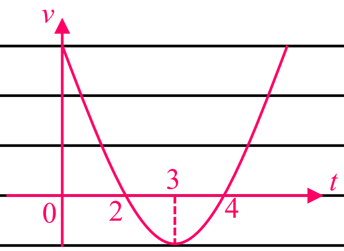
When $t = 3$, P is at point X.

(d) Find the distance OX. **(3)**

(a) $v < 0$

$t^2 - 6t + 8 < 0$

$(t - 2)(t - 4) < 0$



$2 < t < 4$

(b) Check v at turning point ($t = 3$)

and end points ($t = 0$, $t = 5$)

$v(3) = (3)^2 - 6(3) + 8 = -1$ ms^{-1}

$v(0) = (0)^2 - 6(0) + 8 = 8$ ms^{-1}

$v(5) = (5)^2 - 6(5) + 8 = 3$ ms^{-1}

Therefore max speed = **8 ms^{-1}**

(c) Instantaneous rest when $v = 0$

$t^2 - 6t + 8 = 0$

$(t - 2)(t - 4) = 0$

$t = 2$ seconds $\quad t = 4$ seconds

So $t = 2$ is the **first** time

$a = \dfrac{dv}{dt} = 2t - 6$

When $t = 2$, $\quad a = 2(2) - 6 = -2$

Magnitude of acceleration = **2 ms^{-2}**

(d) $s = \int v \, dt$

$s = \int t^2 - 6t + 8 \, dt = \frac{1}{3}t^3 - 3t^2 + 8t + c$

When $t = 0$, $s = 0$, so $\quad 0 = \frac{1}{3}(0)^3 - 3(0)^2 + 8(0) + c$

$c = 0$

$s = \frac{1}{3}t^3 - 3t^2 + 8t$

$s(3) = \frac{1}{3}(3)^3 - 3(3)^2 + 8(3) =$ **6 m**

(Total for Question 7 is 12 marks)

8 A particle, P, moves along a straight line such that at time t seconds, $t \geq 0$, the velocity of P, v ms^{-1}, is modelled as

$$v = 4t^3 - 24t^2 + 20t$$

(a) Find the values of t for which P is at instantaneous rest. **(3)**

(b) Find the magnitude of the acceleration of P when $t = 1$. **(2)**

M and N are fixed points on the straight line.

When $t = 0$, the displacement of P from point M is 25 metres.
When $t = 3$, P is at point N.

(c) Find the distance MN. **(4)**

(a) Instantaneous rest when $v = 0$

$4t^3 - 24t^2 + 20t = 0$

$4t(t^2 - 6t + 5) = 0$

$4t(t - 1)(t - 5) = 0$

$\boldsymbol{t = 0 \quad t = 1 \quad t = 5}$

(b) $a = \dfrac{dv}{dt} = 12t^2 - 48t + 20$

When $t = 1$, $a = 12(1)^2 - 48(1) + 20 = -16$

Magnitude of acceleration = **16 ms^{-2}**

(c) $s = \int v \, dt$

$s = \int 4t^3 - 24t^2 + 20t \; dt = t^4 - 8t^3 + 10t^2 + c$

Define s as displacement from M

When $t = 0$, $s = 25$, so $25 = (0)^4 - 8(0)^3 + 10(0)^2 + c$

$c = 25$

When $t = 3$, $s = (3)^4 - 8(3)^3 + 10(3)^2 + 25$

$s = -20$ (So N is 20 m from M in the negative direction) $MN =$ **20 m**

(Total for Question 8 is 9 marks)

9 A particle, P, moves along a straight line such that at time t seconds, $t \geq 0$, the velocity of P, v ms^{-1}, is modelled as

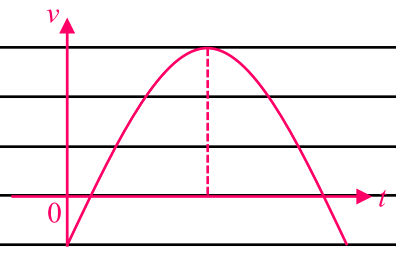
$$v = 36t - 3t^2 - 33$$

(a) Find the greatest speed of P in the interval $0 \leq t \leq 12$ **(4)**

(b) Find the range of values of t for which the acceleration of P is positive. **(3)**

(c) Find the total distance travelled by P in the interval $0 \leq t \leq 12$ **(5)**

(a)



$v = -3[t^2 - 12t + 11]$

$v = -3[(t-6)^2 - 25]$

$v = -3(t-6)^2 + 75$

So turning point has coordinates (6, 75)

Check v at turning point ($t = 6$)

and end points ($t = 0$, $t = 12$)

$v(6) = 36(6) - 3(6)^2 - 33 = 75$ ms^{-1}

$v(0) = 36(0) - 3(0)^2 - 33 = -33$ ms^{-1}

$v(12) = 36(12) - 3(12)^2 - 33 = -33$ ms^{-1}

Therefore max speed = **75 ms^{-1}**

(b) $a = \dfrac{dv}{dt} = 36 - 6t$

when $a > 0$, $36 - 6t > 0$

$36 > 6t$

$6 > t$

$t < 6$

Since $t \geq 0$ $\quad$ $\mathbf{0 \leq t < 6}$

(c) Check for potential direction changes

$36t - 3t^2 - 33 = 0$

$3t^2 - 36t + 33 = 0$

$t^2 - 12t + 11 = 0$

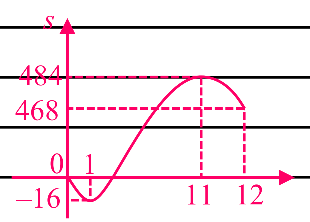
$(t-1)(t-11) = 0$

$\mathbf{t = 1} \qquad \mathbf{t = 11}$

$s = \int v \, dt$

$s = \int 36t - 3t^2 - 33 \, dt = 18t^2 - t^3 - 33t + c$

When $t = 0$, distance from starting point = 0, so $c = 0$



$s(0) = 18(0)^2 - (0)^3 - 33(0) = 0$

$s(1) = 18(1)^2 - (1)^3 - 33(1) = -16$

$s(11) = 18(11)^2 - (11)^3 - 33(11) = 484$

$s(12) = 18(12)^2 - (12)^3 - 33(12) = 468$

Total distance = $16 + [484 - (-16)] + [484 - 468]$

$= 16 + 500 + 16$

$=$ **532 m**

(Total for Question 9 is 12 marks)

10 A fixed point O lies on a straight line.

A particle, P, moves along the straight line such that at time t seconds, $t \geq 0$, after passing through point O, the velocity of P, v ms^{-1} is modelled as

$$v = 8 - t^3$$

(a) Show that the acceleration of P is never positive. **(2)**

(b) Find the distance travelled by P when it is at instantaneous rest. **(4)**

(c) Determine if P is moving towards or away from O when $t = 4$ **(3)**

(a) $a = \dfrac{dv}{dt} = -3t^2$

Since $t \geq 0$ (given in question)

$3t^2 \geq 0$ and so $-3t^2 \leq 0$

Therefore $a \leq 0$ (so acceleration is never positive)

(b) Instantaneous rest when $v = 0$

$8 - t^3 = 0$

$t^3 = 8$

$t = 2$

$s = \int v\,dt$

$s = \int 8 - t^3\,dt = 8t - \frac{1}{4}t^4 + c$

When $t = 0$, $s = 0$, so $0 = 8(0) - \frac{1}{4}(0)^4 + c$

$c = 0$

$s = 8t - \frac{1}{4}t^4$

P is at instantaneous rest when $t = 2$

$s(2) = 8(2) - \frac{1}{4}(2)^4$

$= \mathbf{12\ m}$

(c) when $t = 3$

$s(4) = 8(4) - \frac{1}{4}(4)^4 = -32$ m

$v(4) = 8 - (4)^3 = -56$ m

P is on the negative side of O

but moving in the negative direction

so P is moving **away from** O.

(Total for Question 10 is 9 marks)

11 A particle, P, moves along the x-axis such that at time t seconds, $t > 0$, the velocity of P, v ms^{-1} is modelled as

$$v = 4\sqrt{t} - 5t$$

(a) Find the magnitude of the acceleration when P is at instantaneous rest. **(4)**

When $t = 1$, the displacement of P from the origin is 6 metres.

(b) Find the displacement of P from the origin at the instant when P stops accelerating. **(6)**

(a) Instantaneous rest when $v = 0$

$4\sqrt{t} - 5t = 0$

$\sqrt{t}(4 - 5\sqrt{t}) = 0$

$\sqrt{t} = 0$

$t = 0$ (invalid as $t > 0$ in question)

$4 - 5\sqrt{t} = 0$

$5\sqrt{t} = 4$

$\sqrt{t} = \frac{4}{5}$

$t =$

$a = \frac{dv}{dt} = \frac{2}{\sqrt{t}} - 5$

$a\left(\frac{16}{25}\right) = \frac{2}{\sqrt{\frac{16}{25}}} - 5$

$= $ **-2.5 ms^{-2}**

(b) Stops accelerating $a = 0$

$\frac{2}{\sqrt{t}} - 5 = 0$

$\frac{2}{\sqrt{t}} = 5$

$\sqrt{t} = \frac{2}{5}$

$t = \frac{4}{25}$

$s = \int v\, dt$

$s = \int 4\sqrt{t} - 5t\, dt = \frac{8}{3}t^{\frac{3}{2}} - \frac{5}{2}t^2 + c$

When $t = 1$, $s = 6$, so $6 = \frac{8}{3}(1)^{\frac{3}{2}} - \frac{5}{2}(1)^2 + c$

$6 = \frac{8}{3} - \frac{5}{2} + c$

$c = \frac{35}{6}$

$s = \frac{8}{3}t^{\frac{3}{2}} - \frac{5}{2}t^2 + \frac{35}{6}$

When $t = \frac{4}{25}$ $s = \frac{8}{3}\left(\frac{4}{25}\right)^{\frac{3}{2}} - \frac{5}{2}\left(\frac{4}{25}\right)^2 + \frac{35}{6}$

$s =$ **5.94 m**

(Total for Question 11 is 10 marks)

12 A fixed point O lies on a straight line.

A particle, P, moves along the straight line such that at time t seconds, $t \geq 0$, after passing through point O, the velocity of P, v ms^{-1} is modelled as

$$v = t^2 - kt + 18$$

where k is a constant.

(a) Given that P at instantaneous rest when $t = 2$, find the value of k. **(2)**

(b) Find the magnitude of the acceleration of P when $t = 3$ **(2)**

(c) Find the maximum distance of P from O in the interval $0 \leq t \leq 10$ **(4)**

(a) Instantaneous rest when $v = 0$, so $v(2) = 0$

$0 = (2)^2 - k(2) + 18$

$0 = 4 - 2k + 18$

$0 = 22 - 2k$

$2k = 22$

$\boldsymbol{k = 11}$

(b) $a = \dfrac{dv}{dt} = 2t - 11$

When $t = 3$, $a = 2(3) - 11 = -5$

Magnitude of acceleration = **5 ms^{-2}**

(c) $s = \int v \, dt$

$\int t^2 - 11t + 18 \, dt = \frac{1}{3}t^3 - \frac{11}{2}t^2 + 18t + c$

When $t = 0$, $s = 0$, so $0 = \frac{1}{3}(0)^3 - \frac{11}{2}(0)^2 + 18(0) + c$

$c = 0$

$s = \frac{1}{3}t^3 - \frac{11}{2}t^2 + 18t$

(c) ctd check for potential direction changes ($v = 0$)

$t^2 - 11t + 18 = 0$

$(t - 2)(t - 9) = 0$

$t = 2 \quad t = 9$

Check s at $t = 2$, $t = 9$

and end points ($t = 0$, $t = 10$)

$s(2) = \frac{1}{3}(2)^3 - \frac{11}{2}(2)^2 + 18(2) = \frac{50}{3}$ m

$s(9) = \frac{1}{3}(9)^3 - \frac{11}{2}(9)^2 + 18(9) = -\frac{81}{2}$ m

$s(0) = \frac{1}{3}(0)^3 - \frac{11}{2}(0)^2 + 18(0) = 0$ m

$s(10) = \frac{1}{3}(10)^3 - \frac{11}{2}(10)^2 + 18(10) = -\frac{110}{3}$ m

Maximum distance from O = **$\frac{81}{2}$ m (40.5 m)**

(Total for Question 12 is 8 marks)

13 Particles P and Q, move along the x-axis. At time t seconds, $t \geq 0$, the displacement, x metres, of P from the origin O, is given by $\quad x = t^3 - 20t^2 + 77t$

At time t seconds, $t \geq 0$, the acceleration of Q, a ms^{-2}, is modelled as $\quad a = 2t - 9$

When $t = 0$, Q is moving with a velocity of 12 ms^{-1}

(a) Find a range of values of t for which the velocity of Q is greater than the velocity of P. **(6)**

When $t = 0$, the displacement of Q from the origin O is 6 metres.

(b) Find the distance between P and Q when $t = 4$ **(4)**

(a) $v_P = \dfrac{dx}{dt} = 3t^2 - 40t + 77$

$v_Q = \int a \, dt$

$v_Q = \int 2t - 9 \, dt = t^2 - 9t + c$

When $t = 0$, $v = 0$, so $\quad 12 = (0)^2 - 9(0) + c$

$c = 12$

$v_Q = t^2 - 9t + 12$

$v_Q > v_P$

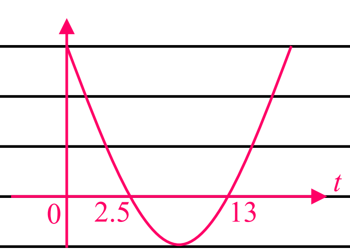
$t^2 - 9t + 12 > 3t^2 - 40t + 77$

$0 > 2t^2 - 31t + 65$

$2t^2 - 31t + 65 < 0$

$(2t - 5)(t - 13) < 0$

Critical values $t = 2.5$, $t = 13$



$2.5 < t < 13$

(b) $s_Q = \int v_Q \, dt$

$s_Q = \int t^2 - 9t + 12 \, dt = \frac{1}{3}t^3 - \frac{9}{2}t^2 + 12t + c$

When $t = 0$, $s = 6$, so $\quad 6 = \frac{1}{3}(0)^3 - \frac{9}{2}(0)^2 + 12(0) + c$

$c = 6$

$s_Q = \frac{1}{3}t^3 - \frac{9}{2}t^2 + 12t + 6$

$s_Q(4) = \frac{1}{3}(4)^3 - \frac{9}{2}(4)^2 + 12(4) + 6 = \frac{10}{3}$

$s_P(4) = (4)^3 - 20(4)^2 + 77(4) = 52$

$52 - \frac{10}{3} = \mathbf{\frac{146}{3}}$ **m**

(Total for Question 13 is 10 marks)

14 A fixed point O lies on a straight line.

A particle, P, moves along the straight line such that at time t seconds, $t \geq 0$, the acceleration of P, a ms^{-2} is modelled as

$$a = 13 - 6t$$

When $t = 0$, the velocity of P is 10 ms^{-1} and the displacement of P from point O is 2 metres.

(a) Find the value of t for which P is at instantaneous rest. **(4)**

(b) Find the displacement of P from O when $t = 3$ **(4)**

(c) Find the total distance travelled by P in the interval $0 \leq t \leq 6$ **(4)**

(a) $v = \int a \, dt$

$v = \int 13 - 6t \, dt = 13t - 3t^2 + c$

When $t = 0$, $v = 10$, so $10 = 13(0) - 3(0)^2 + c$

$c = 10$

$v = 13t - 3t^2 + 10$

Instantaneous rest when $v = 0$

$0 = 13t - 3t^2 + 10$

$3t^2 - 13t - 10 = 0$

$(3t + 2)(t - 5) = 0$

$t = -\frac{2}{3}$ $\quad t = 5$

$t \geq 0$, so $\boldsymbol{t = 5}$

(b) $s = \int v \, dt$

$s = \int 13t - 3t^2 + 10 \, dt = \frac{13}{2}t^2 - t^3 + 10t + c$

When $t = 0$, $s = 2$, so $2 = \frac{13}{2}(0)t^2 - (0)^3 + 10(0) + c$

$c = 2$

$s = \frac{13}{2}t^2 - t^3 + 10t + 2$

$s(3) = \frac{13}{2}(3)^2 - (3)^3 + 10(3) + 2 = \mathbf{63.5\ m}$

(c) Check change of direction ($t = 5$)

and end points ($t = 0$, $t = 6$)

$s(5) = \frac{13}{2}(5)^2 - (5)^3 + 10(5) + 2 = 89.5$ m

$s(0) = \frac{13}{2}(0)^2 - (0)^3 + 10(0) + 2 = 2$ m

$s(6) = \frac{13}{2}(6)^2 - (6)^3 + 10(6) + 2 = 80$ m

Total distance $= [89.5 - 2] + [89.5 - 80]$

$= 87.5 + 89.5$

$= \mathbf{97\ m}$

(Total for Question 14 is 12 marks)

15 An electric scooter travels along a straight horizontal road.

At time t seconds, $0 \leq t \leq k$, where k is an integer, the acceleration of the scooter, a ms^{-2} is modelled as

$$a = 2 - 0.4t$$

When $t = 0$, the velocity of the scooter is 2.2 ms^{-1}

The scooter comes to rest when $t = k$

(a) Find the value of k **(4)**

(b) Find the magnitude of the acceleration of the scooter when $t = k$ **(2)**

(c) Find the distance travelled by the scooter in the interval $0 \leq t \leq k$ **(4)**

(a) $v = \int a \, dt$

$v = \int 2 - 0.4t \, dt = 2t - 0.2t^2 + c$

When $t = 0$, $v = 2.2$, so $2.2 = 2(0) - 0.2(0)^2 + c$

$c = 2.2$

$v = 2t - 0.2t^2 + 2.2$

Instantaneous rest when $v = 0$

$0 = 2(k) - 0.2(k)^2 + 2.2$

$0 = 2k - 0.2k^2 + 2.2$

$0 = 20k - 2k^2 + 22$

$2k^2 - 20k - 22 = 0$

$k^2 - 10k - 11 = 0$

$(k + 1)(k - 11) = 0$

$k = -1 \qquad k = 11$

Given $0 \leq t \leq k$, $\mathbf{k = 11}$

(b) When $t = 11$, $a = 2 - 0.4(11) = -2.4$

Magnitude of acceleration = **2.4 ms^{-2}**

(c) In the interval $0 \leq t \leq 11$, $v = 0$ only when $t = 11$ so no change of direction.

$s = \int v \, dt$

$s = \int 2t - 0.2t^2 + 2.2 \, dt = t^2 - \frac{0.2}{3}t^3 + 2.2t + c$

$s(0) = (0)^2 - \frac{0.2}{3}(0)^3 + 2.2(0) + c = c$

$s(11) = (11)^2 - \frac{0.2}{3}(11)^3 + 2.2(11) + c = 56.46\ldots + c$

Total distance $= [56.46\ldots + c] - c$

$=$ **56.5 m**

(Total for Question 15 is 10 marks)

16 At time $t = 0$, a drone takes off from rest at ground level and moves vertically such that at time t seconds, $0 \le t \le 4$, the velocity of the drone, v ms^{-1} is modelled as

$$v = t^3 - 7t^2 + 12t$$

The drone is at instantaneous rest when $t = 0$

(a) Find the other values of t for which the drone is at instantaneous rest. **(2)**

(b) Find the greatest height above ground level reached by the drone in the interval $0 \le t \le 4$ **(4)**

The acceleration of the drone, a ms^{-2}, is such that for $0 \le t \le 4$,

$$a \ge k, \quad \text{where } k \text{ is a constant}$$

(c) Find the greatest possible value for k. **(4)**

(a) Instantaneous rest when $v = 0$

$t^3 - 7t^2 + 12t = 0$

$t(t^2 - 7t + 12) = 0$

$t(t-3)(t-4) = 0$

$t = 0$ (given) $\boldsymbol{t = 4}$ $\boldsymbol{t = 5}$

(b) $s = \int v\, dt$

$s = \int t^3 - 7t^2 + 12t\, dt = \frac{1}{4}t^4 - \frac{7}{3}t^3 + 6t^2 + c$

When $t = 0$, $s = 0$, so $0 = \frac{1}{4}(0)^4 - \frac{7}{3}(0)^3 + 6(0)^2 + c$

$c = 0$

$s = \frac{1}{4}t^4 - \frac{7}{3}t^3 + 6t^2$

Check end points/when drone at instantaneous rest

$s(0) = \frac{1}{4}(0)^4 - \frac{7}{3}(0)^3 + 6(0)^2 = 0$ m

$s(3) = \frac{1}{4}(3)^4 - \frac{7}{3}(3)^3 + 6(3)^2 = 11.25$ m

$s(4) = \frac{1}{4}(4)^4 - \frac{7}{3}(4)^3 + 6(4)^2 = 10.666\ldots$ m

Maximum height = **11.25 m**

(c) $a = \frac{dv}{dt} = 3t^2 - 14t + 12$

Maximum k occurs at minimum value of a in $0 \le t \le 4$

$a = 3t^2 - 14t + 12$

$= 3[t^2 - \frac{14}{3}t] + 12$

$= 3[(t - \frac{7}{3})^2 - \frac{49}{9}] + 12$

$= 3(t - \frac{7}{3})^2 - \frac{49}{3} + 12$

$= 3(t - \frac{7}{3})^2 - \frac{13}{3}$

Minimum a occurs when $t = \frac{7}{3}$

$a = 3(\frac{7}{3} - \frac{7}{3})^2 - \frac{13}{3} = -\frac{13}{3}$

so $\boldsymbol{k = -\frac{13}{3}}$

(Total for Question 16 is 10 marks)