Algebraic Fractions (Equations)

1 Solve $\dfrac{x+9}{5} + \dfrac{x+2}{4} = 5$ **[3 marks]**

$$\frac{4(x+9) + 5(x+2)}{20} = 5$$

$$4x + 36 + 5x + 10 = 100$$
$$9x + 46 = 100$$
$$9x = 54$$

$$x = 6$$

Answer ___ $x = 6$ ___

2 Solve $\dfrac{x-1}{2} + \dfrac{x+4}{5} = 8$ **[3 marks]**

$$\frac{5(x-1) + 2(x+4)}{10} = 8$$

$$5x - 5 + 2x + 8 = 80$$
$$7x + 3 = 80$$
$$7x = 77$$

$$x = 11$$

Answer ___ $x = 11$ ___

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3 Solve $\dfrac{x+5}{3} - \dfrac{x-2}{4} = 3$ **[3 marks]**

$$\frac{4(x+5) - 3(x-2)}{12} = 3$$

$$4x + 20 - 3x + 6 = 36$$

$$x + 26 = 36$$

Answer $\quad x = 10$

4 Solve $\dfrac{x+2}{8} + \dfrac{5-x}{3} = 2$ **[3 marks]**

$$\frac{3(x+2) + 8(5-x)}{24} = 2$$

$$3x + 6 + 40 - 8x = 48$$

$$46 - 5x = 48$$

$$-5x = 2$$

$$x = -\frac{2}{5}$$

Answer $\quad x = -\dfrac{2}{5} \quad [\text{or } x = -0.4]$

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5 Solve $\dfrac{3}{x+5} + \dfrac{1}{x+3} = 2$ [4 marks]

$$\frac{3(x+3) + x + 5}{(x+5)(x+3)} = 2$$

$$3x + 9 + x + 5 = 2(x^2 + 8x + 15)$$
$$4x + 14 = 2x^2 + 16x + 30$$
$$0 = 2x^2 + 12x + 16$$
$$0 = x^2 + 6x + 8$$
$$0 = (x+4)(x+2)$$

Answer $\underline{\quad x = -4 \qquad x = -2 \quad}$

6 Solve $\dfrac{2}{2x+3} + \dfrac{3}{x-2} = 1$ [4 marks]

$$\frac{2(x-2) + 3(2x+3)}{(2x+3)(x-2)} = 1$$

$$2x - 4 + 6x + 9 = 2x^2 - x - 6$$
$$8x + 5 = 2x^2 - x - 6$$
$$0 = 2x^2 - 9x - 11$$
$$0 = (2x-11)(x+1)$$

Answer $\underline{\quad x = \dfrac{11}{2} \qquad x = -1 \quad}$

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7 Solve $\dfrac{3x+1}{x+1} - \dfrac{1}{x+3} = 4$ **[5 marks]**

$$\frac{(3x+1)(x+3) - (x+1)}{(x+1)(x+3)} = 4$$

$$3x^2 + 10x + 3 - x - 1 = 4(x^2 + 4x + 3)$$
$$3x^2 + 9x + 2 = 4x^2 + 16x + 12$$
$$0 = x^2 + 7x + 10$$
$$0 = (x+5)(x+2)$$

Answer $\underline{\quad x = -5 \qquad x = -2 \quad}$

8 Solve $\dfrac{7}{3x+1} - \dfrac{2}{x-3} = 3$ **[5 marks]**

$$\frac{7(x-3) - 2(3x+1)}{(3x+1)(x-3)} = 3$$

$$7x - 21 - 6x - 2 = 3(3x^2 - 8x - 3)$$
$$x - 23 = 9x^2 - 24x - 9$$
$$0 = 9x^2 - 25x + 14$$
$$0 = (9x - 7)(x - 2)$$

Answer $\underline{\quad x = \dfrac{7}{9} \qquad x = 2 \quad}$

9 Solve $\dfrac{6}{x+7} + \dfrac{2}{x-5} = \dfrac{2}{3}$ **[5 marks]**

$$\frac{6(x-5)+2(x+7)}{(x+7)(x-5)} = \frac{2}{3}$$

$$6x-30+2x+14 = \frac{2}{3}(x^2+2x-35)$$

$$3(8x-16) = 2x^2+4x-70$$

$$24x-48 = 2x^2+4x-70$$

$$0 = 2x^2-20x-22$$

$$0 = x^2-10x-11$$

$$0 = (x-11)(x+1)$$

Answer $\quad x=11 \qquad x=-1$

10 Solve $\dfrac{5x+2}{x+1} - \dfrac{x+8}{x+3} = 2$ **[5 marks]**

$$\frac{(5x+2)(x+3)-(x+8)(x+1)}{(x+1)(x+3)} = 2$$

$$5x^2+17x+6-x^2-9x-8 = 2(x^2+4x+3)$$

$$4x^2+8x-2 = 2x^2+8x+6$$

$$2x^2-8 = 0$$

$$x^2-4 = 0$$

$$(x+2)(x-2) = 0$$

Answer $\quad x=-2 \qquad x=2$

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11   Solve $\dfrac{x}{2x-1} + \dfrac{x-3}{2-x} = \dfrac{1}{4}$ **[5 marks]**

$$\frac{x(2-x) + (x-3)(2x-1)}{(2x-1)(2-x)} = \frac{1}{4}$$

$$2x - x^2 + 2x^2 - 7x + 3 = \tfrac{1}{4}(5x - 2 - 2x^2)$$
$$4(x^2 - 5x + 3) = 5x - 2 - 2x^2$$
$$4x^2 - 20x + 12 = 5x - 2 - 2x^2$$
$$6x^2 - 25x + 14 = 0$$
$$(3x-2)(2x-7) = 0$$

Answer $x = \dfrac{2}{3}$ $x = \dfrac{7}{2}$

12 Solve $\dfrac{1}{x-2} + \dfrac{x}{x+1} = -2$ giving your answer in the form $\dfrac{a \pm \sqrt{b}}{c}$

where a, b and c are integers. **[6 marks]**

$$\frac{x+1 + x(x-2)}{(x-2)(x+1)} = -2$$

$$x+1 + x^2 - 2x = -2(x^2 - x - 2)$$
$$x^2 - x + 1 = -2x^2 + 2x + 4$$
$$3x^2 - 3x - 3 = 0$$
$$a = 3 \quad b = -3 \quad c = -3$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 3 \times -3}}{6}$$

$$x = \frac{3 \pm \sqrt{45}}{6} \qquad \sqrt{45} = \sqrt{9} \times \sqrt{5}$$
$$= 3\sqrt{5}$$

$$x = \frac{3 \pm 3\sqrt{5}}{6}$$

Answer $\quad x = \dfrac{1 \pm \sqrt{5}}{2}$

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