



Vertical Motion Under Gravity



REVISE THIS TOPIC

1 A stone is dropped from rest from a height of X m above horizontal ground.

The stone is modelled as a particle moving freely under gravity.

The stone hits the ground with a speed of 11.2 ms^{-1}

Using the model,

- (a) Find the value of X (2)
- (b) Find the time taken for the stone to first hit the ground. (2)
- (c) Find the time taken for the stone to fall half of the distance from the point where it was dropped to the ground. (3)

| | |
|---------------|------------------------------------|
| (a) $s = X$ | $v^2 = u^2 + 2as$ |
| $u = 0$ | $11.2^2 = 0^2 + 2(9.8)(X)$ |
| $v = 11.2$ | $X = \frac{11.2^2}{2(9.8)}$ |
| $a = 9.8$ | |
| $t = t$ | $X = 6.4 \text{ m}$ |
| | |
| (b) $s = 6.4$ | $v = u + at$ |
| $u = 0$ | $11.2 = 0 + 9.8t$ |
| $v = 11.2$ | $t = \frac{11.2}{9.8}$ |
| $a = 9.8$ | |
| $t = t$ | $t = 1.1 \text{ seconds}$ |
| | |
| (c) $s = 3.2$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 0$ | $3.2 = 0(t) + \frac{1}{2}(9.8)t^2$ |
| $v = v$ | $3.2 = 4.9t^2$ |
| $a = 9.8$ | $t^2 = \frac{3.2}{4.9}$ |
| $t = t$ | $t = \sqrt{\frac{3.2}{4.9}}$ |
| | $t = 0.81 \text{ seconds}$ |

(Total for Question 1 is 7 marks)



2 A ball is dropped from rest from a height of 30 m above horizontal ground.

The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) Find distance that the ball falls during the first 2 seconds. (2)

(b) Find T , time taken for the ball to first hit the ground. (2)

(c) Find V , the speed with which the ball first hits the ground. (2)

The model is refined to include air resistance and new values of T and V are calculated.

(d) Explain how the new values of T and V calculated with this refinement would compare to the values calculated in part (b) and part (c). (2)

| | |
|-----------|------------------------------------|
| $s = s$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 0$ | $s = 0(2) + \frac{1}{2}(9.8)(2)^2$ |
| $v = v$ | $s = \mathbf{19.6 \text{ m}}$ |
| $a = 9.8$ | |
| $t = 2$ | |

| | |
|-----------|------------------------------------|
| $s = 30$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 0$ | $30 = (0)t + \frac{1}{2}(9.8)t^2$ |
| $v = v$ | $30 = 4.9t^2$ |
| $a = 9.8$ | $3.2 = 4.9t^2$ |
| $t = t$ | $t^2 = \frac{30}{4.9}$ |
| | $t = \sqrt{\frac{30}{4.9}}$ |
| | $t = \mathbf{2.5 \text{ seconds}}$ |

| | |
|-----------|-------------------------------------|
| $s = 30$ | $v^2 = u^2 + 2as$ |
| $u = 0$ | $v^2 = 0^2 + 2(9.8)(30)$ |
| $v = v$ | $v^2 = 588$ |
| $a = 9.8$ | $v = \sqrt{588}$ |
| $t = t$ | $v = \mathbf{24.2 \text{ ms}^{-1}}$ |

(d) Air resistance acts in the opposite direction to the motion of the ball so the new value of T would be greater (it takes longer) and the new value of V would be smaller (it hits the ground with a lower speed)

(Total for Question 2 is 8 marks)



3 At time $t = 0$, a small stone is projected vertically upwards with speed 4.9 ms^{-1} from point A , which is 8.4 m above horizontal ground.

The stone is modelled as a particle moving freely under gravity.

Using the model,

- (a) Find the time from when the stone is projected until it first comes to instantaneous rest. (2)
- (b) Find the total time from projection until the stone hits the ground. (3)
- (c) Find the speed with which the stone hits the ground. (2)
- (d) Suggest **two** refinements to the model, apart from air resistance, to make the model more realistic. (2)

(a) $s = s$ $v = u + at$

$u = 4.9$ $0 = 4.9 - 9.8t$

$v = 0$ $4.9 = 9.8t$

$a = -9.8$ $t = \frac{4.9}{9.8}$

$t = 0.5$ 0.5

$t = 0.5 \text{ seconds}$

(b) $s = -8.4$ $s = ut + \frac{1}{2}at^2$

$u = 4.9$ $-8.4 = 4.9t + \frac{1}{2}(-9.8)t^2$

$v = v$ $-8.4 = 4.9t - 4.9t^2$

$a = -9.8$ $4.9t^2 - 4.9t - 8.4 = 0$

$t = t$ $t = 2$ $t = -\frac{6}{7}$ (disregard as negative time)

$t = 2 \text{ seconds}$

(c) $s = -8.4$ $v = u + at$

$u = 4.9$ $v = 4.9 + (-9.8)(2)$

$v = v$ $v = -14.7$

$a = -9.8$

$t = 2$ **speed = 14.7 ms^{-1}**

(d) Allow for the effect of wind

Allow for the spin of the stone

Include the dimensions of the stone

(Total for Question 3 is 9 marks)



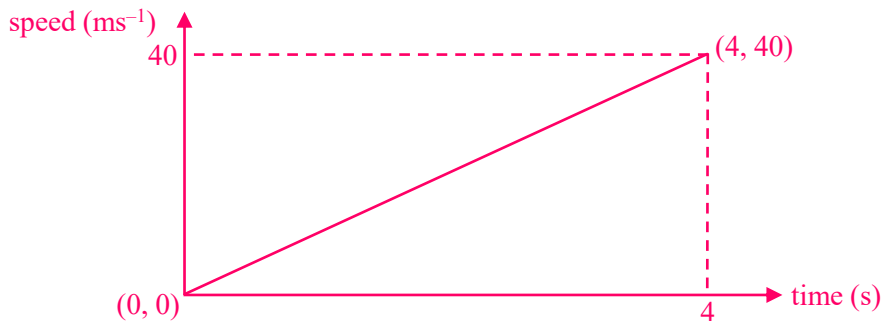
4 A ball is dropped vertically from rest from the top of a cliff that is 80 m above sea level.

The ball is modelled as a particle moving freely under gravity with $g = 10 \text{ ms}^{-2}$

- (a) Find the speed of the ball just before it hits the sea. (2)
- (b) Find the total time taken for the ball to reach the sea. (2)
- (c) Sketch a speed-time graph for the motion of the ball from the time when the ball is dropped to the moment that it hits the sea. You must state the coordinates of the start and end points of your graph. (2)

| | |
|--------------|--------------------------|
| (a) $s = 80$ | $v^2 = u^2 + 2as$ |
| $u = 0$ | $v^2 = 0^2 + 2(10)(80)$ |
| $v = v$ | $v^2 = 1600$ |
| $a = 10$ | $v = \sqrt{1600}$ |
| $t = t$ | $v = 40 \text{ ms}^{-1}$ |

| | |
|--------------|-------------------------|
| (b) $s = 80$ | $v = u + at$ |
| $u = 0$ | $40 = 0 + 10t$ |
| $v = 40$ | $40 = 10t$ |
| $a = 10$ | $t = 4 \text{ seconds}$ |
| $t = t$ | |



The model is refined to use a more accurate value of g .

- (d) State, with a reason, the effect this would have on your sketch in part (c) (1)

(d) A more accurate value of g (e.g. 9.8 or 9.81) would mean a lower acceleration.

This means the gradient of the line drawn would be lower (less steep)



- 5 At $t = 0$ a stone is projected vertically upwards from point O ,
- The point O is x m above horizontal ground.
 - The initial speed of the stone is 23.4 ms^{-1}
 - The stone hits the ground when $t = 5$
 - The stone is modelled as a particle moving freely under gravity

Using the model find,

- (a) the value of x . (3)
- (b) the speed of the stone as it hits the ground. (2)
- (c) the maximum height above horizontal ground reached by the stone. (3)
- (d) the total distance travelled by the stone before it hits the ground. (1)

| | |
|--------------|---|
| (a) $s = -x$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 23.4$ | $-x = (23.4)(5) + \frac{1}{2}(-9.8)(5)^2$ |
| $v = v$ | $-x = -5.5$ |
| $a = -9.8$ | $x = \mathbf{5.5 \text{ m}}$ |
| $t = 5$ | |

| | |
|----------------|--|
| (b) $s = -5.5$ | $v = u + at$ |
| $u = 23.4$ | $v = 23.4 + (-9.8)(5)$ |
| $v = v$ | $v = -25.6$ |
| $a = -9.8$ | |
| $t = 5$ | $\text{speed} = \mathbf{25.6 \text{ ms}^{-1}}$ |

| | |
|-------------|----------------------------|
| (c) $s = s$ | $v^2 = u^2 + 2as$ |
| $u = 23.4$ | $0^2 = 23.4^2 + 2(-9.8)s$ |
| $v = 0$ | $19.6s = 23.4^2$ |
| $a = -9.8$ | $s = \frac{23.4^2}{19.6}$ |
| $t = t$ | $s = 27.9367... \text{ m}$ |

(d) Total distance = $27.9367... + 33.4367... = \mathbf{61.4 \text{ m}}$

Max height = $5.5 + 27.9367... = \mathbf{33.4 \text{ m}}$

(Total for Question 5 is 9 marks)



6 A small stone is projected vertically upwards with speed $U \text{ ms}^{-1}$ from point A , which is 30 m above horizontal ground.

The stone hits the ground 5 seconds after it is projected.

The stone is modelled as a particle moving freely under gravity with $g = 10 \text{ ms}^{-2}$

Using the model,

(a) Find the value of U . (3)

(b) Find the total time for which the stone is more than 44 m above the ground. (4)

The model is refined and uses a more accurate value of g . The value of U is then recalculated.

(c) State, with a reason, how this new value of U would compare to the value found in part (a) using the unrefined model. (1)

| | |
|-----------|------------------------------------|
| $s = -30$ | $s = ut + \frac{1}{2}at^2$ |
| $u = U$ | $-30 = 5U + \frac{1}{2}(-10)(5)^2$ |
| $v = v$ | $-30 = 5U - 125$ |
| $a = -10$ | $5U = 95$ |
| $t = 5$ | $U = 19 \text{ ms}^{-1}$ |

(b) $44 - 30 = 14 \text{ m}$

| | |
|-----------|---------------------------------|
| $s = 14$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 19$ | $14 = 19t + \frac{1}{2}(10)t^2$ |
| $v = v$ | $14 = 19t + 5t^2$ |
| $a = -10$ | $5t^2 - 19t + 14 = 0$ |
| $t = t$ | $t = 1 \quad t = 2.8$ |

$2.8 - 1 = 1.8 \text{ seconds}$

(c) A more accurate value of g would mean a lower downwards acceleration.

This means a smaller initial velocity is needed for the stone to still hit the ground at 5 seconds.

So the new value of U would be lower.

(Total for Question 6 is 8 marks)



7 At time $t = 0$ a skydiver jumps from a helicopter hovering at a height of X m above horizontal ground.

The skydiver is modelled as a particle moving freely under gravity.

At time $t = 20$ the skydiver is 1840 m above horizontal ground.

Using the model,

(a) Find the value of X . (3)

The skydiver opens their parachute at time T , when they are 1000 m above horizontal ground.

(b) Find the value of T . (3)

A refinement of the model includes air resistance. A new value of T is calculated using this refinement.

(c) State, with a reason, how this new value of T would compare to the value found in part (b) using the unrefined model. (1)

| | |
|--------------------------|---|
| $(a) \quad s = X - 1840$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 0$ | $X - 1840 = (0)(20) + \frac{1}{2}(9.8)(20)^2$ |
| $v = v$ | $X - 1840 = 1960$ |
| $a = 9.8$ | $X = 3800 \text{ m}$ |
| $t = 20$ | |

| | |
|----------------------|-------------------------------------|
| $(b) \quad s = 2800$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 0$ | $2800 = (0)t + \frac{1}{2}(9.8)t^2$ |
| $v = v$ | $2800 = 4.9t^2$ |
| $a = 9.8$ | $t^2 = \frac{2800}{4.9}$ |
| $t = T$ | $t = 23.9 \text{ seconds}$ |
| | $t = \sqrt{\frac{2800}{4.9}}$ |

(c) Air resistance acts in the opposite direction to the motion of the skydiver so they will fall more slowly. This means it takes longer to reach the same height so the new value of T will be higher.



8 At time $t = 0$ a ball is projected vertically upwards from a point A , which is 35 m above horizontal ground.

The ball,

- is projected with an initial speed $U \text{ ms}^{-1}$
- reaches a maximum height of 51.9 m
- hits the ground when $t = T$ with speed $V \text{ ms}^{-1}$
- is modelled as a particle moving freely under gravity

Using the model,

- (a) find the value of U . (2)
- (b) find the value of T . (3)
- (c) find the value of V . (2)
- (d) find the total time for which the ball is more than 41.5 m above horizontal ground. (4)

| | | | |
|-----------------------|-------------------------------------|----------------------|--------------------------------------|
| (a) $s = 16.9$ | $v^2 = u^2 + 2as$ | (b) $s = -35$ | $s = ut + \frac{1}{2}at^2$ |
| $u = U$ | $0^2 = U^2 + 2(-9.8)(16.9)$ | $u = 18.2$ | $-35 = 18.2t + \frac{1}{2}(-9.8)t^2$ |
| $v = 0$ | $0 = U^2 - 331.24$ | $v = v$ | $-35 = 18.2t + \frac{1}{2}(-9.8)t^2$ |
| $a = -9.8$ | $U^2 = 331.24$ | $a = -9.8$ | $-35 = 18.2t - 4.9t^2$ |
| $t = t$ | $U = \sqrt{331.24}$ | $t = t$ | $4.9t^2 - 18.2t - 35 = 0$ |
| | $U = \mathbf{18.2 \text{ ms}^{-1}}$ | | $t = 5.11 \quad t = -1.40$ |
| | | | $t = \mathbf{5.11 \text{ seconds}}$ |

| | | | |
|----------------------|-------------------------------------|--|---|
| (c) $s = -35$ | $v^2 = u^2 + 2as$ | (d) $41.5 - 35 = 6.5 \text{ m}$ | $s = ut + \frac{1}{2}at^2$ |
| $u = 18.2$ | $v^2 = 18.2^2 + 2(-9.8)(-35)$ | $s = 6.5$ | $6.5 = 18.2t + \frac{1}{2}(-9.8)t^2$ |
| $v = v$ | $v^2 = 1017.24$ | $u = 18.2$ | $6.5 = 18.2t - 4.9t^2$ |
| $a = -9.8$ | $v = \sqrt{1017.24}$ | $v = v$ | $4.9t^2 - 18.2t + 6.5 = 0$ |
| $t = 5.11\dots$ | $v = \mathbf{31.9 \text{ ms}^{-1}}$ | $a = -9.8$ | $t = 0.40\dots \quad t = 3.31\dots$ |
| | | $t = t$ | |
| | | | $3.31\dots - 0.40\dots = \mathbf{2.91 \text{ seconds}}$ |



9 At time $t = 0$, ball is thrown vertically downwards with speed $U \text{ ms}^{-1}$ from the top of a tower, 166 m above horizontal ground.

The ball is modelled as a particle moving freely under gravity, with $g = 10 \text{ ms}^{-2}$

When $t = 4$ the ball is 60 m above horizontal ground.

- (a) Find the value of U . (3)
- (b) Find the distance that the ball falls during the first second. (2)
- (c) Find the time when the ball first hits the ground. (3)
- (d) Suggest one refinements to the model, apart from air resistance, to make the model more realistic. (1)

| | | | |
|--------------------------------|-----------------------------------|-------------|-------------------------------------|
| (a) $166 - 60 = 106 \text{ m}$ | $s = ut + \frac{1}{2}at^2$ | (b) $s = s$ | $s = ut + \frac{1}{2}at^2$ |
| $s = 106$ | $106 = 4U + \frac{1}{2}(10)(4)^2$ | $u = 6.5$ | $s = 6.5(1) + \frac{1}{2}(10)(1)^2$ |
| $u = U$ | $106 = 4U + 80$ | $v = v$ | $s = 6.5 + 5$ |
| $v = v$ | $4U = 26$ | $a = 10$ | $s = 11.5 \text{ m}$ |
| $a = 10$ | $U = 6.5 \text{ ms}^{-1}$ | $t = 1$ | |
| $t = 4$ | | | |

| | |
|---------------|---|
| (c) $s = 166$ | $166 = 6.5t + \frac{1}{2}(10)t^2$ |
| $u = 6.5$ | $166 = 6.5t + 5t^2$ |
| $v = v$ | $5t^2 + 6.5t - 166 = 0$ |
| $a = 10$ | $t = 5.15 \quad t = -6.45$ (disregard as negative time) |
| $t = t$ | $t = 5.15 \text{ seconds}$ |

- (d) Allow for the effect of wind
- Allow for the spin of the stone
- Include the dimensions of the stone
- Use a more accurate value for g

(Total for Question 9 is 9 marks)



10 At time $t = 0$, ball is thrown vertically downwards with speed $U \text{ ms}^{-1}$ from the top of a tower, 150 m above horizontal ground.

The ball is modelled as a particle moving freely under gravity, with $g = 10 \text{ ms}^{-2}$

Between $t = 2$ and $t = 3$ the ball travels a distance of 37 m.

(a) Find the value of U . (3)

(b) Find the time when the ball first hits the ground. (3)

(c) Find the speed of the ball when it first hits the ground. (2)

It is found experimentally that the ball actually hits the ground with a speed of 20 ms^{-1}

(d) Suggest one reason, with reference to the model, why this is lower than your answer to part (c). (1)

| | | | |
|-------------|---------------------------------|---------------------------------|------------------------------|
| (a) $s = s$ | At $t = 2$ | At $t = 3$ | $(3U + 45) - (2U + 20) = 37$ |
| $u = U$ | $s = ut + \frac{1}{2}at^2$ | $s = ut + \frac{1}{2}at^2$ | $3U + 45 - 2U - 20 = 37$ |
| $v = v$ | $s = 2U + \frac{1}{2}(10)(2)^2$ | $s = 3U + \frac{1}{2}(10)(3)^2$ | $U + 25 = 37$ |
| $a = 10$ | $s = 2U + 20$ | $s = 3U + 45$ | $U = 12 \text{ ms}^{-1}$ |
| $t = t$ | | | |

| | | | |
|---------------|----------------------------------|------------------|----------------------------|
| (b) $s = 150$ | $s = ut + \frac{1}{2}at^2$ | (c) $s = 150$ | $v^2 = u^2 + 2as$ |
| $u = 12$ | $150 = 12t + \frac{1}{2}(10)t^2$ | $u = 12$ | $v^2 = 12^2 + 2(10)(150)$ |
| $v = v$ | $150 = 12t + 5t^2$ | $v = v$ | $v^2 = 3144$ |
| $a = 10$ | $5t^2 + 12t - 150 = 0$ | $a = 10$ | $v = \sqrt{3144}$ |
| $t = t$ | $t = 4.41 \quad t = -6.81$ | $t = 4.41 \dots$ | $v = 56.1 \text{ ms}^{-1}$ |
| | $t = 4.41 \text{ seconds}$ | | |

(d) The model assumes the ball falls freely under gravity.

In reality the ball will fall slower due to air resistance acting in the upwards direction as the ball falls.

(Total for Question 10 is 9 marks)



11 At time $t = 0$, a ball A is projected vertically upwards with speed 18 ms^{-1} from a point on horizontal ground.

At the same instant, a second ball B is thrown vertically downwards with speed 8 ms^{-1} from a point 52 m vertically above horizontal ground.

Both balls are modelled as particles moving freely under gravity with $g = 10 \text{ ms}^{-2}$

At time $t = T$ the two balls collide.

- (a) Find the value of T . (3)
- (b) Find the height above the ground at which the balls collide. (2)
- (c) Find the speed of ball A at the instant when the two balls collide. (2)
- (d) State, with a reason whether balls A is moving upwards or downwards at instant when the two balls collide. (1)

| (a) Ball A | Ball B | $s = ut + \frac{1}{2}at^2$ | $s = uT + \frac{1}{2}aT^2$ |
|--------------|-----------|-----------------------------------|---|
| $s = s_A$ | $s = s_B$ | $s_A = 18T + \frac{1}{2}(-10)T^2$ | $s_B = 8T + \frac{1}{2}(10)T^2$ |
| $u = 18$ | $u = 8$ | $s_A = 18T - 5T^2$ | $s_B = 8T + 5T^2$ |
| $v = v_A$ | $v = v_B$ | | |
| $a = -10$ | $a = 10$ | | $s_A + s_B = 52$ |
| $t = T$ | $t = T$ | $18T - 5T^2 + 8T + 5T^2 = 52$ | |
| | | | $26T = 52$ |
| | | | $T = 2 \text{ seconds}$ |

(b) $s_A = 18T - 5T^2$

$s_A = 18(2) - 5(2)^2$

$s_A = 36 - 20$

$s_A = 16 \text{ m}$

(c) $v = u + at$

$v_A = 18 + (-10)(2)$

$v = -2$

speed = 2 ms^{-1}

(d) Ball A is moving downwards.

When calculating the velocity I took upwards as the positive direction but the velocity was negative.



(Total for Question 11 is 8 marks)

12 Point A is X m above horizontal ground and point B is Y m above horizontal ground.

A stone, modelled as a particle moving freely under gravity, is dropped from rest from point A .

After T seconds the stone passes point B .

After $3T$ seconds of the stone hits the ground.

Express X in terms of Y .

| | | |
|--------------------------------------|-------------------------------------|--|
| | | Substituting $4.9T^2 = X - Y$ into $X = 9 \times 4.9T^2$ |
| $s = X - Y$ | $s = X$ | gives $X = 9 \times (X - Y)$ |
| $u = 0$ | $u = 0$ | $X = 9X - 9Y$ |
| $v = v$ | $v = v$ | $8X = 9Y$ |
| $a = 9.8$ | $a = 9.8$ | $X = \frac{9}{8}Y$ |
| $t = T$ | $t = 3T$ | |
| $s = ut + \frac{1}{2}at^2$ | $s = ut + \frac{1}{2}at^2$ | |
| $X - Y = (0)T + \frac{1}{2}(9.8)T^2$ | $X = (0)T + \frac{1}{2}(9.8)(3T)^2$ | |
| $X - Y = 4.9T^2$ | $X = 9 \times 4.9T^2$ | |

(Total for Question 12 is 4 marks)

13 At time $t = 0$, a ball is projected vertically upwards with initial speed U ms⁻¹, from point A, which is 20 m above horizontal ground.

The ball is modelled as a particle moving freely under gravity.

The ball is above a height of 30 m for exactly 3 seconds.

Find the value of U .

| | | |
|------------|--|--|
| | | $t_2 - t_1 = 3$ |
| $s = 10$ | $s = ut + \frac{1}{2}at^2$ | $\frac{U + \sqrt{U^2 - 196}}{9.8} - \frac{U - \sqrt{U^2 - 196}}{9.8} = 3$ |
| $u = U$ | $10 = Ut + \frac{1}{2}(-9.8)t^2$ | |
| $v = v$ | $10 = Ut - 4.9t^2$ | $\frac{U + \sqrt{U^2 - 196}}{9.8} - \frac{U - \sqrt{U^2 - 196}}{9.8} = 3$ |
| $a = -9.8$ | $4.9t^2 - Ut + 10 = 0$ | |
| $t = T$ | $t_1 = \frac{U - \sqrt{U^2 - 196}}{9.8}$ | $\frac{U + \sqrt{U^2 - 196}}{9.8} - \frac{U - \sqrt{U^2 - 196}}{9.8} = 29.4$ |
| | | $2\sqrt{U^2 - 196} = 29.4$ |
| | $t_2 = \frac{U + \sqrt{U^2 - 196}}{9.8}$ | $\sqrt{U^2 - 196} = 14.7$ |
| | | $U^2 - 196 = 216.09$ |
| | | $U^2 = 412.09$ |
| | | $U = \pm 20.3 \quad U = 20.3 \text{ ms}^{-1}$ |

(Total for Question 13 is 5 marks)

