Iteration

SCAN ME

REVISE THIS TOPIC

1 (a) Use the iteration formula $x_{n+1} = \frac{(x_n)^2 + 3}{5}$ to find the values of x_1, x_2 and x_3

Start with $x_0 = \sqrt{3}$

$x_1 = \frac{(\sqrt{3})^2 + 3}{5}$

$= 1.2$

$x_2 = \frac{1.2^2 + 3}{5}$

$= 0.888$

$x_3 = \frac{0.888^2 + 3}{5}$

$= 0.7577088$

$x_1 =$ 1.2

$x_2 =$ 0.888

$x_3 =$ 0.7577088

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^2 - 5x + 3 = 0$

$x = \frac{x^2+3}{5}$ can be rearranged to give $x^2 - 5x + 3 = 0$. This means x_1, x_2 and x_3 are estimates to a solution of the equation $x^2 - 5x + 3 = 0$

(2)

(Total for Question 1 is 5 marks)

2 (a) Use the iteration formula $x_{n+1} = \sqrt{\frac{x_n}{10} + 2}$ to find the values of x_1, x_2 and x_3

Start with $x_0 = 42.5$

$x_1 = \sqrt{\frac{42.5}{10} + 2}$

$= 2.5$

$x_2 = \sqrt{\frac{2.5}{10} + 2}$

$= 1.5$

$x_3 = \sqrt{\frac{1.5}{10} + 2}$

$= 1.46628783$

$x_1 =$ 2.5

$x_2 =$ 1.5

$x_3 =$ 1.46628783

(3)

The values of x_1, x_2 and x_3 found in part (a) are estimates to the solution of an equation in the form $ax^2 - x + b = 0$ where a and b are integers.

(b) Find the values of a and b.

$x = \sqrt{\frac{x}{10} + 2}$

$x^2 = \frac{x}{10} + 2$

$10x^2 = x + 20$

$10x^2 - x + 20 = 0$

$a =$ 10

$b =$ 20

(2)

(Total for Question 2 is 5 marks)

---

3 (a) Use the iteration formula $x_{n+1} = \sqrt{80 - 5x_n}$ to find the values of x_1, x_2 and x_3

Start with $x_0 = 12.8$

$x_1 = \sqrt{80 - 5(12.8)}$

$= 4$

$x_2 = \sqrt{80 - 5(4)}$

$= 7.745966692$

$x_3 = \sqrt{80 - 5(7.74...)}$

$= 6.42418606$

$x_1 =$ 4

$x_2 =$ 7.745966692

$x_3 =$ 6.42418606

(3)

The values of x_1, x_2 and x_3 found in part (a) are estimates to the solution of an equation in the form $x^2 + ax + b = 0$ where a and b are integers.

(b) Find the values of a and b.

$x = \sqrt{80 - 5x}$

$x^2 = 80 - 5x$

$x^2 + 5x - 80 = 0$

$a =$ 5

$b =$ -80

(2)

(Total for Question 3 is 5 marks)

4 (a) Use the iteration formula $x_{n+1} = \sqrt[3]{8 - (x_n)^2}$ to find the values of x_1, x_2 and x_3

Start with $x_0 = 1.8$

$x_1 = \sqrt[3]{8 - 1.8^2}$
$= 1.682166517$

$x_2 = \sqrt[3]{8 - 1.68...^2}$
$= 1.729175302$

$x_3 = \sqrt[3]{8 - 1.72...^2}$
$= 1.711109795$

$x_1 =$ 1.682166517

$x_2 =$ 1.729175302

$x_3 =$ 1.711109795

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 + x^2 - 8 = 0$

$x = \sqrt[3]{8 - x^2}$ can be rearranged to give $x^3 + x^2 - 8 = 0$. This means x_1, x_2 and x_3 are estimates to a solution of the equation $x^3 + x^2 - 8 = 0$

(2)

(Total for Question 4 is 5 marks)

5 (a) Show that the equation $x^3 - x - 4 = 0$ has a solution between $x = 1$ and $x = 2$

$1^3 - 1 - 4 = -4$

$2^3 - 2 - 4 = 2$

change of sign so there must be a solution between $x = 1$ and $x = 2$

(2)

(b) Show that the equation $x^3 - x - 4 = 0$ can be rearranged to give $x = \sqrt[3]{x + 4}$

$x^3 - 4 = x$

$x^3 = x + 4$

$x = \sqrt[3]{x + 4}$

(2)

(c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{x_n + 4}$ three times to find an estimate for the solution of $x^3 - x - 4 = 0$

$x_1 = \sqrt[3]{2 + 4}$
$= 1.817120593$

$x_2 = \sqrt[3]{1.81\ldots + 4}$
$= 1.798467893$

$x_3 = \sqrt[3]{1.79\ldots + 4}$
$= 1.796543562$

1.796543562

(3)

(d) By substituting your answer to part (c) into $x^3 - x - 4$ comment on the accuracy of your estimate for the solution to $x^3 - x - 4 = 0$

$(1.796\ldots)^3 - (1.796\ldots) - 4 = 0.001924331\ldots$

This is close to zero so an accurate solution

(2)

(Total for Question 5 is 9 marks)

6 (a) Show that the equation $x^2 + x - 13 = 0$ has a solution between $x = 3$ and $x = 4$

$3^2 + 3 - 13 = -1$

$4^2 + 4 - 13 = 7$

change of sign so there must be a solution between $x = 3$ and $x = 4$

(2)

(b) Show that the equation $x^2 + x - 13 = 0$ can be rearranged to give $x = \sqrt{13 - x}$

$x^2 + x - 13 = 0$

$x^2 + x = 13$

$x^2 = 13 - x$

$x = \sqrt{13 - x}$

(2)

(c) Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \sqrt{13 - x_n}$ three times to find an estimate for the solution of $x^2 + x - 13 = 0$

$x_1 = \sqrt{13 - 3}$

$= 3.16227766$

$x_2 = \sqrt{13 - 3.16...}$

$= 3.136514361$

$x_3 = \sqrt{13 - 3.14...}$

3.140618671

3.140618671

(3)

(d) By substituting your answer to part (c) into $x^2 + x - 13$ comment on the accuracy of your estimate for the solution to $x^2 + x - 13 = 0$

$(3.14...)^2 + (3.14...) - 13 = 0.004104309809$

This is close to zero so an accurate solution

(2)

(Total for Question 6 is 9 marks)

7 (a) Show that the equation $x^2 - 10x + 6 = 0$ has a solution between $x = 0$ and $x = 1$

$0^2 - 10(0) + 6 = 6$

$1^2 - 10(1) + 6 = -3$

change of sign so there must be a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^2 - 10x + 6 = 0$ can be rearranged to give $x = \frac{x^2 + 6}{10}$

$x^2 - 10x + 6 = 0$

$x^2 + 6 = 10x$

$\frac{x^2 + 6}{10} = x$

$x = \frac{x^2 + 6}{10}$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \frac{(x_n)^2 + 6}{10}$ three times to find an estimate for the solution of $x^2 - 10x + 6 = 0$

$x_1 = \frac{1^2 + 6}{10}$

$= 0.7$

$x_2 = \frac{0.7^2 + 6}{10}$

$= 0.649$

$x_3 = \frac{0.649^2 + 6}{10}$

$= 0.6421201$

0.6421201

(3)

(d) By substituting your answer to part (c) into $x^2 - 10x + 6$ comment on the accuracy of your estimate for the solution to $x^2 - 10x + 6 = 0$

$(0.642...)^2 - 10(0.642...) + 6 = -0.008882777...$

This is close to zero so an accurate solution

(2)

(Total for Question 7 is 9 marks)

8 (a) Show that the equation $x^3 - 20x^2 + 100x - 8 = 0$ has a solution between $x = 10$ and $x = 11$

$10^3 - 20(10)^2 + 100(10) - 8 = -8$ change of sign so there must

$11^3 - 20(11)^2 + 100(11) - 8 = 3$ be a solution between

$x = 10$ and $x = 11$

(2)

(b) Show that the equation $x^3 - 20x^2 + 100x - 8 = 0$ can be rearranged to give $x = \sqrt{\frac{8}{x}} + 10$

$x^3 - 20x^2 + 100x = 8$

$x(x^2 - 20x + 100) = 8$

$x^2 - 20x + 100 = \frac{8}{x}$

$(x-10)(x-10) = \frac{8}{x}$

$(x-10)^2 = \frac{8}{x}$

$x - 10 = \sqrt{\frac{8}{x}}$

$x = \sqrt{\frac{8}{x}} + 10$

(4)

(c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt{\frac{8}{x_n}} + 10$ three times to find an estimate for the solution of $x^3 - 20x^2 + 100x - 8 = 0$

$x_1 = \sqrt{\frac{8}{2}} + 10$

$= 12$

$x_2 = \sqrt{\frac{8}{12}} + 10$

$= 10.81649658$

$x_3 = \sqrt{\frac{8}{10.8...}} + 10$

$= 10.8600064$

10.8600064

(3)

(d) By substituting your answer to part (c) into $x^3 - 20x^2 + 100x - 8$ comment on the accuracy of your estimate for the solution to $x^3 - 20x^2 + 100x - 8 = 0$

$(10.86...)^3 - 20(10.86...)^2 + 100(10.86...) - 8 = 0.0321803437 8$

This is close to zero so an accurate solution

(2)

(Total for Question 8 is 9 marks)