Circle Theorems

1 *A*, *B*, *C* and *D* are points on the circumference of a circle with centre *O*.

Angle *ABD* = 81°
Angle *BDC* = 42°

(a) Work out the size of angle *CAB*.

42 °
(1)

(b) Work out the size of angle *ACD*.

81 °
(1)

(c) Work out the size of angle *AEB*.
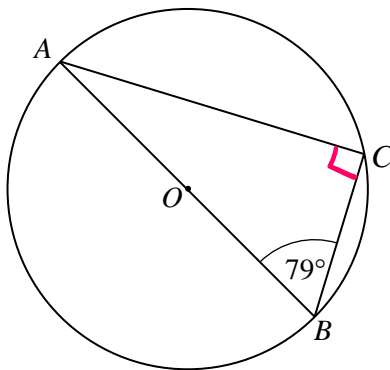
57 °
(1)

(d) Work out the size of angle *BEC*.

123 °
(1)

(Total for Question 1 is 4 marks)

1st

1

2 A, B and C are points on the circumference of a circle with centre O.

Angle ABC = 79°

(a) Work out the size of angle CAB.
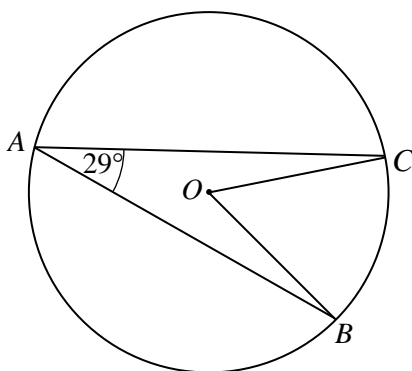
_____11_____ °

(1)

(b) Give a reason for your answer to part (a)

The angle in a semicircle is 90°
Angles in a triangle add to 180°

(1)

(Total for Question 2 is 2 marks)

3 A, B and C are points on the circumference of a circle with centre O.

Angle CAB = 29°

(a) Work out the size of angle COB.

_____58_____ °

(1)

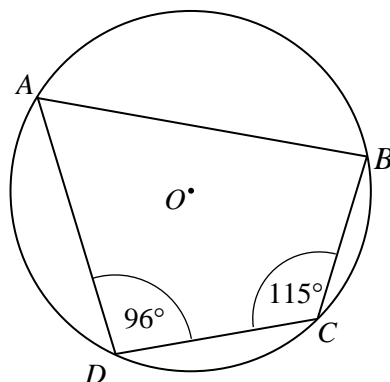(b) Give a reason for your answer to part (a)

The angle at the centre is twice the angle at the circumference.

(1)

(Total for Question 3 is 2 marks)

2

4 *A*, *B*, *C* and *D* are points on the circumference of a circle with centre *O*.

Angle *ADC* = 96°
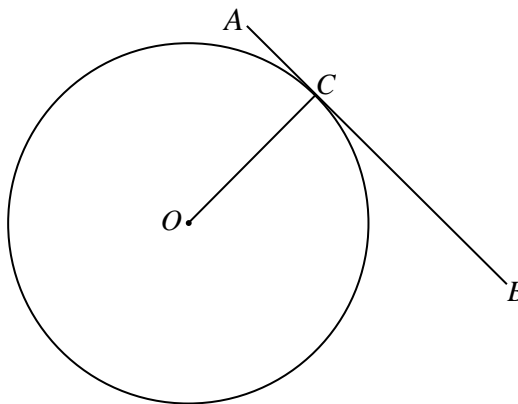Angle *BCD* = 115°

(a) Work out the size of angle *ABC*.

94 °

(1)

(b) Give a reason for your answer to part (a)

Opposite angles in a cyclic quadrilateral add to 180°

(1)

(Total for Question 4 is 2 marks)

5 *A*, *B,* and *C* are points on the circumference of a circle with centre *O*.
AB is the tangent to the circle at point *C*.

(a) Write down the size of angle *OCB*.

90 °

(1)

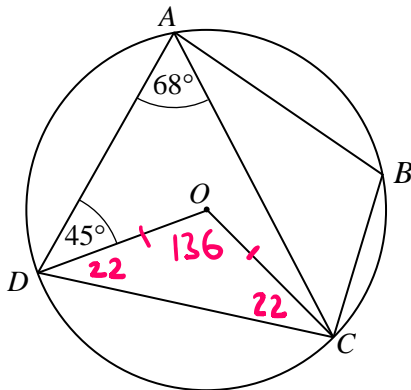(b) Give a reason for your answer to part (a)

A tangent meets a radius at 90°

(1)

(Total for Question 5 is 2 marks)

1st

3

6 A, B, C and D are points on the circumference of a circle with centre O.

Angle $DAC = 68°$
Angle $ADO = 45°$

Work out the size of angle ABC.
Give reasons for each stage of your working.

Angle $DOC = 136°$
The angle at the centre is twice the angle at the circumference.

Angle ODC = Angle $OCD = 22°$
Base angles in an isosceles triangle are equal

Angle $ADC = 22 + 45$
$= 67$

Angle $ABC = 180 - 67$
$= 113$
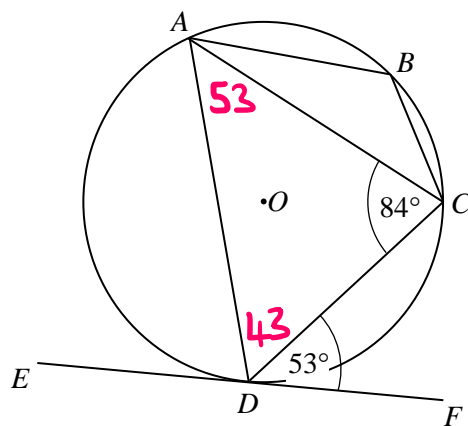Opposite angles in a cyclic quadrilateral add to $180°$

                                                        113 °

(Total for Question 6 is 4 marks)

4

7 *A, B, C* and *D* are points on the circumference of a circle with centre *O*.
EF is the tangent to the circle at point *D*.

Angle *ACD* = 84°
Angle *CDF* = 53°

Work out the size of angle *ABC*.
Give reasons for each stage of your working.

Angle DAC = 53°
Alternate segment theorem

Angle ADC = 43°
Angles in a triangle add to 180°

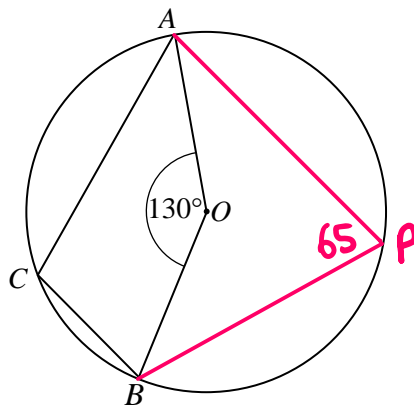Angle ABC = 180 − 43
= 137°
Opposite angles in a cyclic quadrilateral
add to 180°

137 °

(Total for Question 7 is 4 marks)

5

8 *A, B,* and *C* are points on the circumference of a circle with centre *O*.

Angle *AOB* = 130°

Work out the size of angle *ACB*.
Give reasons for each stage of your working.

Angle APB = 65°
The angle at the centre is twice the
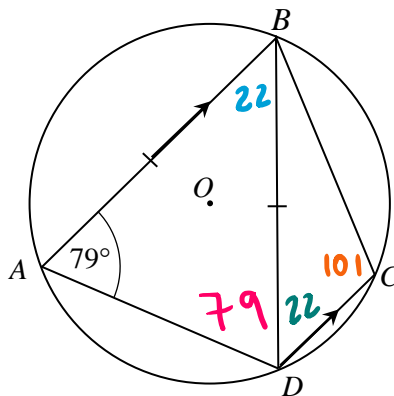an le at the circumference.

Angle ABC = 180 − 65
          = 115
Opposite angles in a cyclic quadrilateral
add to   180°

115 °

(Total for Question 8 is 3 marks)

6

9 *A*, *B*, *C* and *D* are points on the circumference of a circle with centre *O*.

B

22

O.

A 79°

101 *C*

79 22

D

Angle *BAD* = 79°
BA = BD
Lines *AB* and *DC* are parallel.

Work out the size of angle *DBC*.
Give reasons for each stage of your working.

Angle BAD = Angle BDA = 79°

Base angles in an isosceles triangle are equal

Angle ABD = 22°
Angles in a triangle add to 180°

Angle BDC = Angle ABD
Alternate angles are equal

Angle ABC = 180 − 43
       = 137°

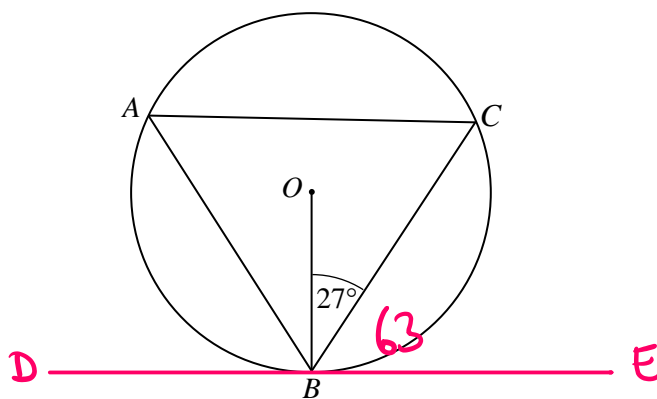Opposite angles in a cyclic quadrilateral add to 180°

Angle DBC = 57°

Angles in a triangle add to 180°

57 °

(Total for Question 9 is 5 marks)

1st

7

10 *A*, *B* and *C* are points on the circumference of a circle with centre *O*.

Angle *CBO* = 27°

Work out the size of angle *BAC*.
Give reasons for each stage of your working.

Angle CBE = 90 - 27
        = 63°
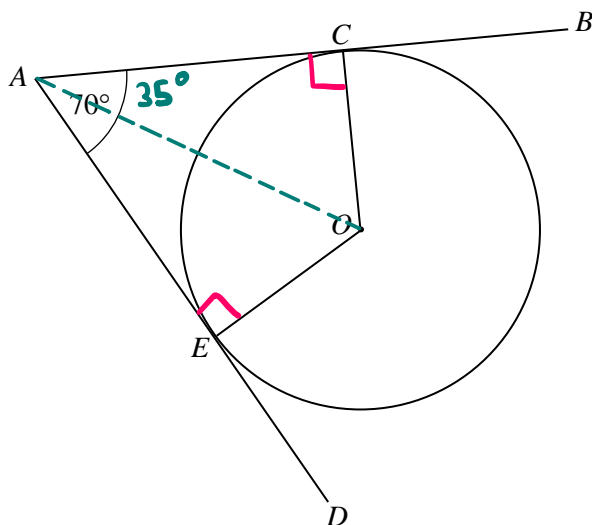
A tangent meets a radius at 90°

Angle BAC = 63°
Alternate segment theorem

63°

(Total for Question 10 is 3 marks)

8

11 *C* and *E* are points on the circumference of a circle with centre *O*.

AB and *AD* are tangents to the circle at *C* and *E*.
Angle *EAC* = 70°

(a) Work out the size of angle *COE*.

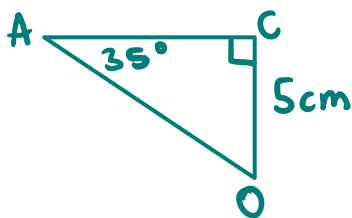Angle AEO = Angle ACO = 90°

A tangent meets a radius at 90°

Angle COE = 110° Angles in a quadrilateral add to 360°

OC = 5 cm

_____**110**_____ °

(2)

(b) Work out the length of *CA*.
   Give your answer to 1 decimal place.
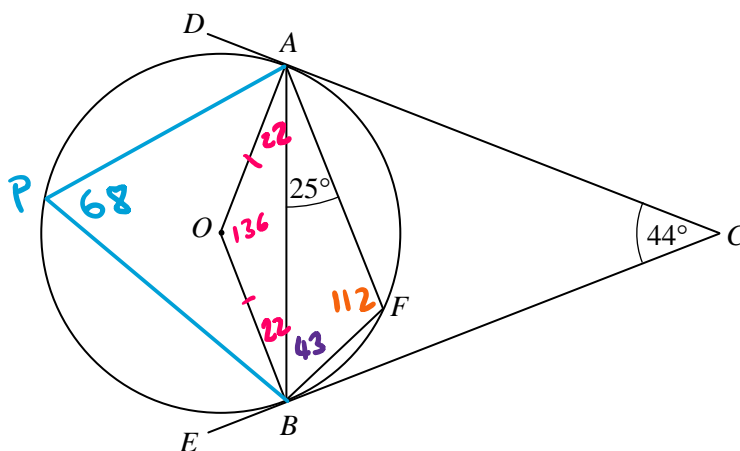
Angle CAO = 35°

$\tan(35) = \dfrac{5}{CA}$

$CA = \dfrac{5}{\tan(35)}$

CA = 7.1407..

_____**7.1**_____ cm

(2)

(Total for Question 11 is 4 marks)

9

12 *A* and *B* are points on the circumference of a circle with centre *O*.

DC and *EC* are tangents to the circle at *A* and *B*.
Angle *ACB* = 44°
Angle *BAF* = 25°

[Reasons not required in this Q]

Work out the size of angle *FBC*.

Angle AOB = 136° A tangent meets a radius at 90°
and angles in a quadrilateral add to 360°
Angle ABO = Angle BAO = 22°
Base angles in an isosceles triangle are equal
Angle APB = 68°
The angle at the centre is twice the
angle at the circumference.
Angle AFB = 112°
Opposite angles in a cyclic quadrilateral
add to 180°
Angle ABF = 43° Angles in a triangle add to 180°

Angle FBC = 25°
A tangent meets a radius at 90°
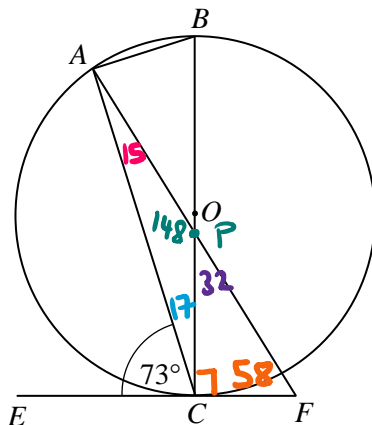
25°

(Total for Question 12 is 4 marks)

10

13 *A*, *B* and *C* are points on the circumference of a circle with centre *O*.

EF is the tangent to the circle at *C*.
Angle *ACE* = 73°
Angle *FAB* = 5 × Angle *CAF*

Work out the size of angle *AFC*.

[Reasons not required in this Q]

Angle ACB = 90°   The angle in a semicircle is 90°
90 ÷ 6 = 15
Angle CAF = 15°
Angle ECB = 90°   Angle ACB = 17°
A tangent meets a radius at 90°
Angle APC = 148°   Angles in a triangle add to 180°
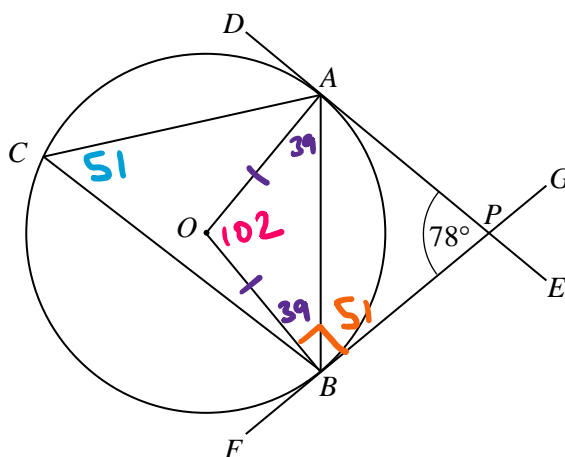Angle CPF = 32°   Angles on a straight line add to 180°

Angle AFC = 58°   Angles in a triangle add to 180°

58 °

(Total for Question 13 is 4 marks)

11

14 *A, B* and *C* are points on the circumference of a circle with centre *O*.

DE and *FG* are tangents to the circle at *A* and *B* that intersect at the point *P*.
Angle *APB* = 78°

[Reasons not required in this Q]

(a) Work out the size of angle *ACB*.

Angle AOB = 102° A tangent meets a radius at 90°
and angles in a quadrilateral add to 360°
Angle ACB = 51° Angle at the circumference is
half the angle at the centre

51 °

(2)

(b) Work out the size of angle *ABP*.

Angle ABO = Angle BAO = 39°
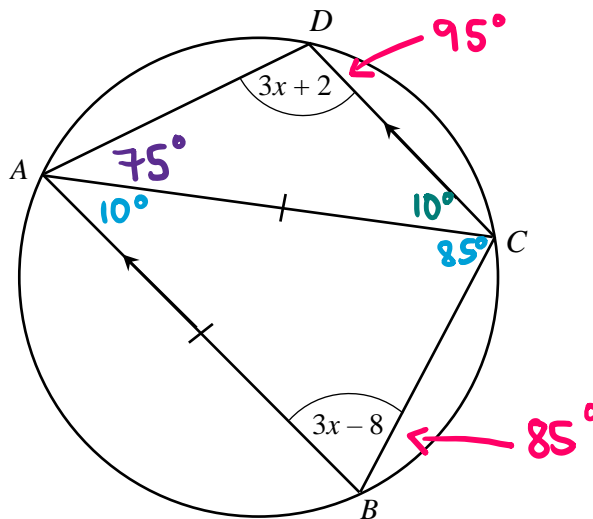Base angles in an isosceles triangle are equal

Angle ABP = 90 - 39
          = 51

A tangent meets a radius at 90°

51 °

(2)

(Total for Question 14 is 4 marks)

1st

15

A, B, C and D are points on the circumference of a circle.
$ABCD$ is a trapezium with AB parallel to DC.

$AB = AC$

Work out the size of angle DAC.
You must show all your working.

[Reasons not required in this Q]

$3x + 2 + 3x - 8 = 180°$

Opposite angles in a cyclic quadrilateral add to $180°$

$6x - 6 = 180$ $3(31) + 2 = 95$
$\quad\quad 6x = 186$ $3(31) - 8 = 85$
$\quad\quad\quad x = 31°$

Angle ACB = Angle ABC = 85

Angle CAB = $10°$ Angles in a triangle add to $180°$

Angle DCA = Angle CAB alternate angles are equal

Angle DAC = $75°$ Angles in a triangle add to $180°$

75 °

(Total for Question 15 is 5 marks)

13

16

$17+7=24$

$\downarrow$

$x+7$ $12°$

$\frac{24}{2}=12$

$5x-7$

A D E C B O
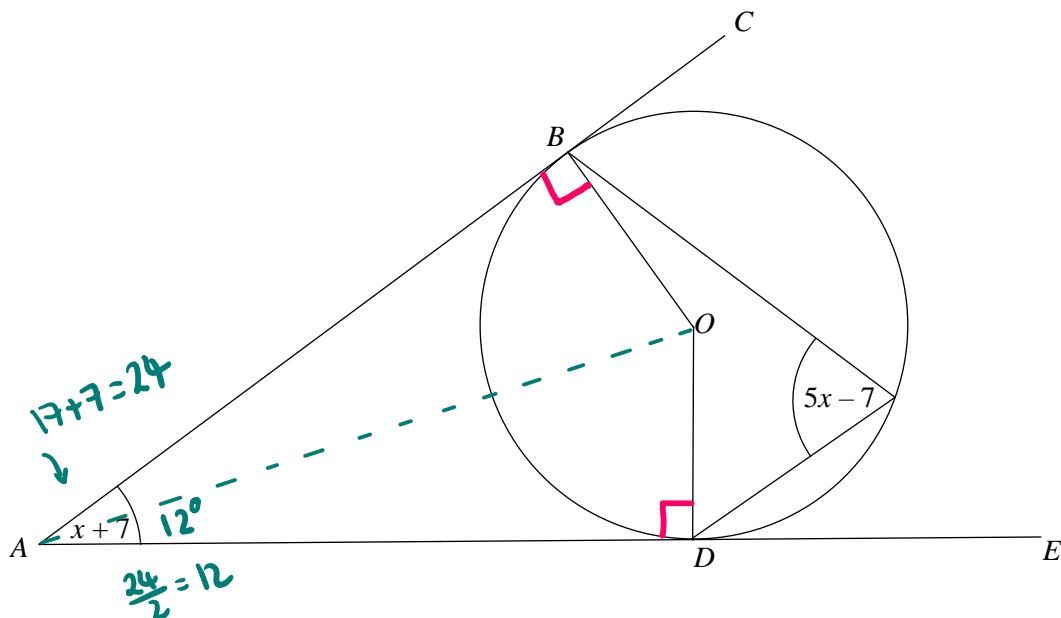
B and *D* are points on the circumference of a circle, centre *O*.
ABC and *ADE* are tangents to the circle.

[Reasons not required in this Q]

(a) Work out value of *x*.
    You must show all your working.

Angle BOD = 2(5x-7)

(twice angle at circumference)

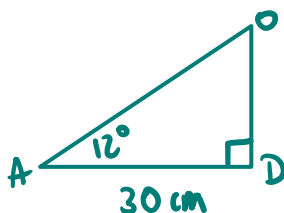Angle BOD = 180 - (x+7)

(angles in quadrilateral add to 360°)

$10x - 14 = 180 - x - 7$
$10x - 14 = 173 - x$
$11x = 187$
$x = 17$

$x =$ ....**17**................ °

(3)

AD = 30 cm

(b) Work out the radius of the circle.
    Give your answer to three significant figures.

BAD = 24°
OAD = 12°

$\tan(12) = \dfrac{OD}{30}$

$OD = 30\tan(12)$

................**3·38**................ cm

(3)

(Total for Question 17 is 6 marks)

10