Measures of Central Tendency

SCAN ME

REVISE THIS TOPIC

1 A school gives all 120 Year 7 students and all 130 Year 8 students the same maths test.

The maths department wishes to compare the test scores of students in Year 7 (x%) with the test scores of students in Year 8 (y%).

$\sum x = 8160$ $\sum y = 8190$

(a) By calculating the mean test score for each year group, compare the test scores of students in Year 7 with those in Year 8. **(2)**

It is found that some of the Year 7 students cheated in the test and each had scored 100%

(b) (i) Suggest another measure of central tendency that the maths department could use to compare the test scores in this case. **(1)**

(ii) Explain why the measure you have suggested would be more suitable than the mean. **(1)**

The school also tests Year 9 students.
The following table summarises the test scores, T%, for all of the Year 9 students.

Test Score, T%	$0 \leq T < 20$	$20 \leq T < 40$	$40 \leq T < 60$	$60 \leq T < 80$	$80 \leq T < 100$
Number of students, f	1	17	52	66	44

(c) Work out an estimate for the mean test score for Year 9 students. **(2)**

(a) $\bar{x} = \frac{8160}{120} = 68\%$ $\bar{y} = \frac{8190}{130} = 63\%$

Year 7 students performed better on average than the Year 8 students.

(b) (i) The median.

(ii) It is not affected by outliers/extreme values.

(c) $\bar{T} = \frac{(10 \times 1) + (30 \times 17) + (50 \times 52) + (70 \times 66) + (90 \times 44)}{1 + 17 + 52 + 66 + 44}$

$\bar{T} = 65\%$

(Total for Question 1 is 6 marks)

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2 The following table summarises heights, y cm to the nearest centimetre, of the female athletes at an athletics club.

Height (y cm)	$160 \leq y < 170$	$170 \leq y < 180$	$180 \leq y < 190$	$190 \leq y < 200$
Number of athletes (f)	3	7	7	8

(a) State the modal class for the heights of the female athletes. **(1)**

(b) Work out the class interval containing the median female height. **(1)**

(c) Work out an estimate for the mean height of the female athletes. **(2)**

For the heights, x cm to the nearest centimetre, of the male athletes at the club

$\sum x = 6615$ $n = 35$

(d) Use this data, and your answer to part (c) to estimate the mean height of all of the male and female athletes at the athletics club. **(2)**

(a) $190 \leq y < 200$

(b) $3 + 7 + 7 + 8 = 25$

$\frac{25 + 1}{2} = 13^{th}$ value Answer: $180 \leq y < 190$

(c) $\bar{y} = \frac{(165 \times 3) + (175 \times 7) + (185 \times 7) + (195 \times 8)}{3 + 7 + 7 + 8}$

$\bar{y} = 183$ cm

(d) $183 \times 25 = 4575$ (total height of all female athletes)

$4575 + 6615 = 11190$ (total height of all male and female athletes)

$11190 \div 60 = 186.5$ cm

(Total for Question 2 is 6 marks)

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3 The following table summarises the annual salaries, £S to the nearest pound, of staff at a multi-academy trust.

Annual Salary (£S)	Frequency (f)
$30{,}000 \leq S < 50{,}000$	84
$50{,}000 \leq S < 70{,}000$	18
$70{,}000 \leq S < 90{,}000$	9
$90{,}000 \leq S < 110{,}000$	3
$110{,}000 \leq S < 200{,}000$	1

(a) Work out the class interval containing the median annual salary. **(1)**

(b) Work out an estimate, to the nearest pound, for the mean annual salary of the staff. **(1)**

(c) Explain why your answer to part (b) is an estimate. **(1)**

It is found that the CEO of the multi-academy trust has an annual salary of £195,000 (yes really)

(d) Explain what effect, if any, will this information have on

(i) your answer to part (a)
(ii) your answer to part (b) **(2)**

(a) $84 + 18 + 9 + 3 + 1 = 115$

$\frac{115 + 1}{2} = 58^{th}$ value Answer: $30{,}000 \leq S < 50{,}000$

(b) $\overline{S} = \frac{(40000 \times 84) + (60000 \times 18) + (80000 \times 9) + (100000 \times 3) + (155000 \times 1)}{84 + 18 + 9 + 3 + 1}$

$\overline{S} = £48{,}826$

(c) We do not know any of the actual salaries.

We have assumed each of the salaries in each class is the midpoint of that class.

(d) (i) It will have no effect on the interval containing the median.

The median is not affected by extreme values

(d) (ii) It will increase the estimate of the mean.

In the original calculation £155,000 was used at the midpoint for the final class interval

but we know this salary is actually much more than this at £195,000.

(Total for Question 3 is 5 marks)

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4 On Monday 12 students were late to school.
The number of minutes late L, to the nearest minute, of the first 8 students are shown below.

Student	A	B	C	D	E	F	G	H
Lateness (L minutes)	1	2	3	3	4	6	8	9

(a) State the modal number of minutes late for the first 8 students. **(1)**

(b) Work out the median number of minutes late for the first 8 students. **(1)**

(c) Work out the mean number of minutes late for the first 8 students. **(1)**

Any student who is 10 or more minutes late receives a detention.
On Monday 4 students received a detention for being late.

The mean number of minutes late for all 12 students on Monday was 10.5 minutes.
(d) Work out the mean number of minutes late for the 4 students who received a detention on Monday. **(2)**

(a) 3 minutes

(b) 3.5 minutes (the 4.5th value)

(c) $\frac{\sum L}{n} = \frac{1+2+3+3+4+6+8+9}{8}$

$= 4.5$ minutes

(d) $12 \times 10.5 = 126$ (total minutes late for all 12 students)

$8 \times 4.5 = 36$ (total minutes late for first 8 students)

$126 - 36 = 90$ (total minutes late for students with a detention)

$90 \div 4 = 22.5$ minutes

(Total for Question 4 is 5 marks)

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5 The table below summarises the number of fillings of 20 patients randomly sampled at a dentist.

Number of Fillings	0	1	2	3	4	5	6
Frequency (f)	7	4	3	3	0	2	1

(a) State the modal number of fillings. **(1)**

(b) Work out the median number of fillings. **(1)**

(c) Work out the mean number of fillings. **(2)**

Another patient is added to the sample. They have 2 fillings.

(d) State what effect, if any, this will have on

(i) your answer to part (a)
(ii) your answer to part (b)
(iii) your answer to part (c) **(3)**

(a) 0 fillings.

(b) 1 filling (the 10.5^{th} value)

(c) $\bar{x} = \dfrac{(0 \times 7) + (1 \times 4) + (2 \times 3) + (3 \times 3) + (4 \times 0) + (5 \times 2) + (6 \times 1)}{7 + 4 + 3 + 3 + 0 + 2 + 1}$

$\bar{x} = 1.75$ fillings

(d) (i) The mode will remain unchanged.

(ii) The median will now be the 11^{th} value, which will still be 1 filling. (So median is unchanged)

(iii) The mean will increase as $2 > 1.75$

(Total for Question 5 is 7 marks)

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6 Mark's internet provider claims that the average internet speed that he will receive will be at least 100 Mbps.

Mark tests his internet speed S, to the nearest Mbps, at 12 different times during the day. The table below summarises the results.

Internet Speed (S Mbps)	90-99	100-109	110-119	120-129
Frequency (f)	4	6	1	1

(a) Work out an estimate of the mean internet speed.
Give your answer to 4 significant figures. **(2)**

(b) Explain why your answer to part (a) is only an estimate. **(1)**

(c) Does your answer to part (a) support the claim made by the internet provider? **(1)**

Mark works out the exact value for the mean using his recorded values.

(d) Is it possible that Mark's mean internet speed is below 100 Mbps?
Give a reason for your answer. **(2)**

(a) $\overline{S} = \dfrac{(94.5 \times 4) + (104.5 \times 6) + (114.5 \times 1) + (124.5 \times 1)}{4 + 6 + 1 + 1}$

$\overline{S} = 103.7$ Mbps

(b) We do not know any of the actual internet speeds.
We have assumed each of the internet speeds in each class is the midpoint of that class.

(c) Yes as $103.7 > 100$

(d) Yes. Since we don't know the actual values the speeds in each class could be as low as 89.5 Mbps for (90-99), 99.5 Mbps for (100-109), 109.5 Mbps for (110-119), 119.5 Mbps for (120-129)

If we assume all values in each class are the lowest possible value then recalculate the mean we get

$\overline{S} = \dfrac{(89.5 \times 4) + (99.5 \times 6) + (109.5 \times 1) + (119.5 \times 1)}{4 + 6 + 1 + 1}$

$\overline{S} = 98.7$ Mbps < 100 Mbps

(Total for Question 6 is 6 marks)

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7 The speeds, S mph to the nearest mph, of some vehicles on a motorway are recorded.

Lane of Motorway	Lane 1	Lane 2	Lane 3
Number of Vehicles	124	148	165
Mean Speed (mph)	55.2	67.8	71.3

Work out the mean speed of all vehicles across all lanes of the motorway.
Give your answer to 3 significant figures. **(2)**

$124 \times 55.2 = 6844.8$

$148 \times 67.8 = 10034.4$

$165 \times 71.3 = 11764.5$

$6844.8 + 10034.4 + 11764.5 = 28643.7$

$124 + 148 + 165 = 437$

$28643.7 \div 437 = 65.5$ mph

(Total for Question 7 is 2 marks)

8 The following table summarises the ages, A years, of people taking a driving test at a test centre.

Age (A years)	10-19	20-29	30-39	40-49	50-59	60-69
Frequency (f)	21	4	1	2	1	1

(a) Work out the class interval containing the median age. **(1)**

(b) Work out an estimate for the mean age of the people taking a driving test at the test centre. **(2)**

The minimum age for taking a driving test is 17 years old.

(c) Comment on whether the true mean age of the drivers at the test centre is likely to be different to your answer to part (b). Give a reason for your answer. **(1)**

(a) 10-19 (the 15.5th value)

(b) $\overline{A} = \dfrac{(14.5 \times 21) + (24.5 \times 4) + (34.5 \times 1) + (44.5 \times 2) + (54.5 \times 1) + (64.5 \times 1)}{21 + 4 + 1 + 2 + 1 + 1}$

$\overline{A} = 21.5$ years

(c) In our estimate we assumed all drivers in the 10-19 category were 14.5 years old.
Since they must be at least 17 the true mean is likely to be higher than our answer in part (b)

(Total for Question 8 is 4 marks)

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