

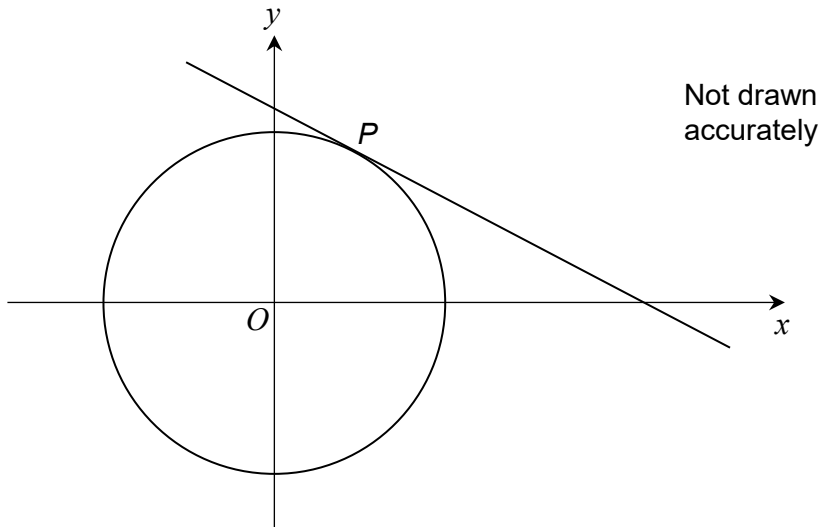


SCAN ME

Equation of a Tangent

← REVISE THIS TOPIC

1 $P(2, 4)$ is a point on a circle, centre O .



Work out the equation of the tangent to the circle at P .
Give your answer in the form $y = mx + c$

[4 marks]

gradient of $OP = \frac{4}{2} = 2$ gradient of tangent = $-\frac{1}{2}$

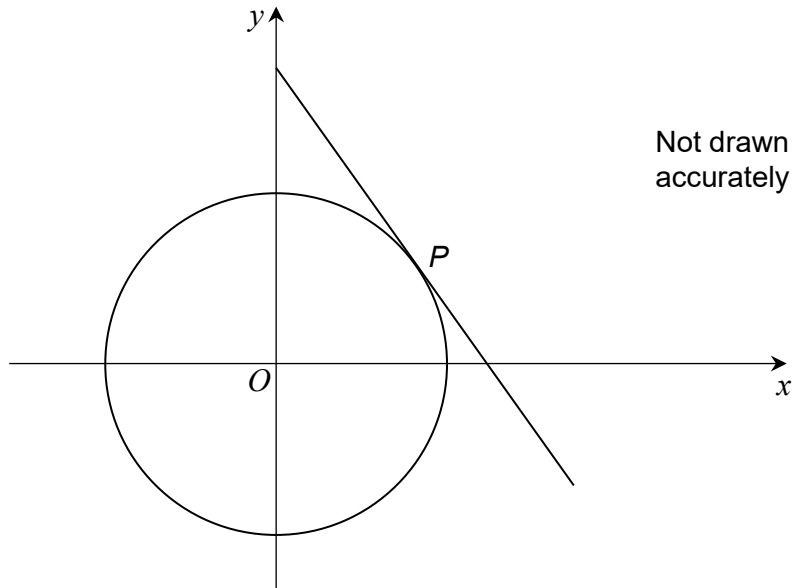
$y = -\frac{1}{2}x + c$
 $4 = -\frac{1}{2}(2) + c$
 $4 = -1 + c$
 $c = 5$

Answer $y = -\frac{1}{2}x + 5$

$\frac{4}{4}$



2

 $P(9, 3)$ is a point on a circle, centre O .Work out the equation of the tangent to the circle at P .Give your answer in the form $y = mx + c$

[4 marks]

$$\text{gradient of } OP = \frac{3}{9} = \frac{1}{3} \quad \text{gradient of tangent} = -3$$

$$y = -3x + c$$

$$3 = -3(9) + c$$

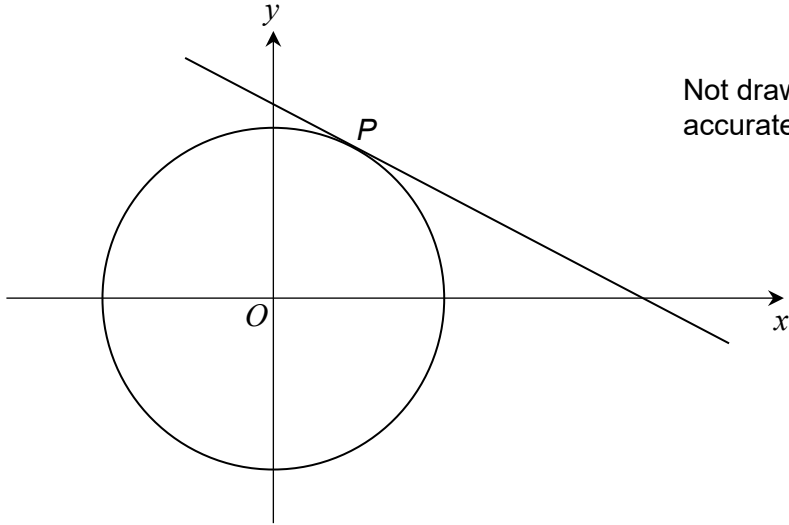
$$3 = -27 + c$$

$$c = 30$$

Answer $y = -3x + 30$ 



3 $P(2, 5)$ is a point on a circle, centre O .



Work out the equation of the tangent to the circle at P .
Give your answer in the form $y = mx + c$

[4 marks]

gradient of $OP = \frac{5}{2}$ gradient of tangent = $-\frac{2}{5}$

$$y = -\frac{2}{5}x + c$$

$$5 = -\frac{2}{5}(2) + c$$

$$5 = -\frac{4}{5} + c$$

$$c = 5 + \frac{4}{5}$$

$$c = \frac{25}{5} + \frac{4}{5}$$

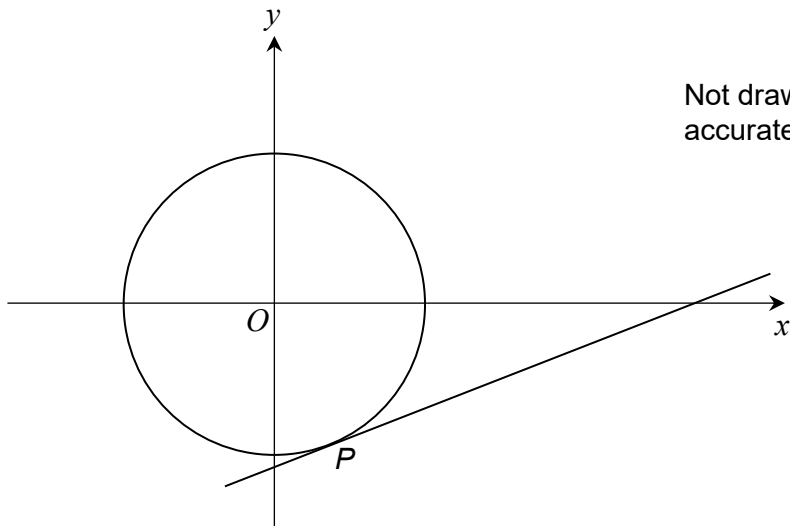
$$c = \frac{29}{5}$$

Answer

$$y = -\frac{2}{5}x + \frac{29}{5}$$



4

 $P(1, -4)$ is a point on a circle, centre O .Work out the equation of the tangent to the circle at P .Give your answer in the form $y = mx + c$

[4 marks]

$$\text{gradient of } OP = \frac{-4}{1} = -4 \quad \text{gradient of tangent} = \frac{1}{4}$$

$$y = \frac{1}{4}x + c$$

$$-4 = \frac{1}{4}(1) + c$$

$$-4 = \frac{1}{4} + c$$

$$c = -4 - \frac{1}{4}$$

$$c = -\frac{16}{4} - \frac{1}{4}$$

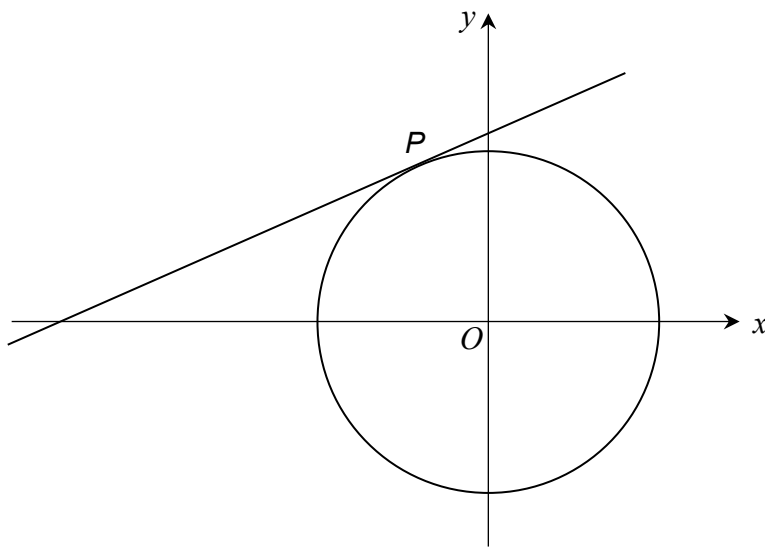
$$c = -\frac{17}{4}$$

Answer

$$y = \frac{1}{4}x - \frac{17}{4}$$



5

 $P(-3, 5)$ is a point on a circle, centre O .Not drawn
accuratelyWork out the equation of the tangent to the circle at P .Give your answer in the form $y = mx + c$

[4 marks]

$$\text{gradient of } OP = -\frac{5}{3} \quad \text{gradient of tangent} = \frac{3}{5}$$

$$y = \frac{3}{5}x + c$$

$$5 = \frac{3}{5}(-3) + c$$

$$5 = -\frac{9}{5} + c$$

$$c = 5 + \frac{9}{5}$$

$$c = \frac{25}{5} + \frac{9}{5}$$

$$c = \frac{34}{5}$$

Answer

$$y = \frac{3}{5}x + \frac{34}{5}$$

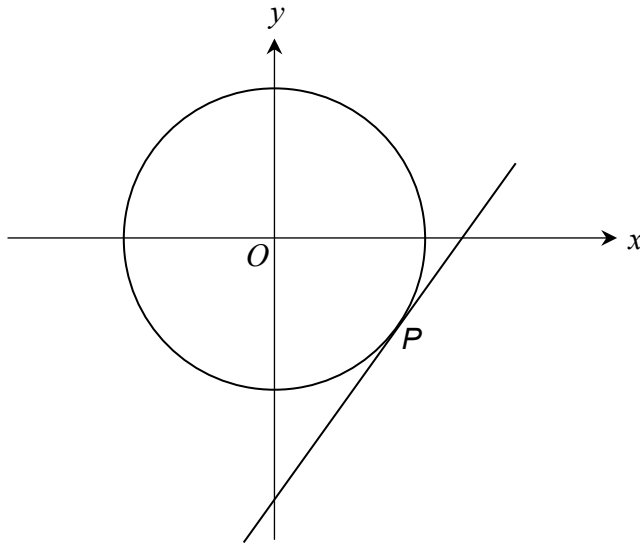
8

Turn over ►





6 P is a point on the circle with equation $x^2 + y^2 = 65$
 P has coordinates $(7, k)$, where $k < 0$



Not drawn accurately

Work out the equation of the tangent to the circle at P .
 Give your answer in the form $y = mx + c$

[5 marks]

$$7^2 + k^2 = 65$$

$$49 + k^2 = 65$$

$$k^2 = 16$$

$$k = \pm 4$$

$$\text{as } k < 0, \underline{\underline{k = -4}}$$

$$\text{gradient of } OP = -\frac{4}{7}$$

$$\text{gradient of tangent} = \frac{7}{4}$$

$$y = \frac{7}{4}x + c$$

$$-4 = \frac{7}{4}(7) + c$$

$$-4 = \frac{49}{4} + c$$

$$c = -4 - \frac{49}{4}$$

$$c = \frac{-16}{4} - \frac{49}{4}$$

$$c = -\frac{65}{4}$$

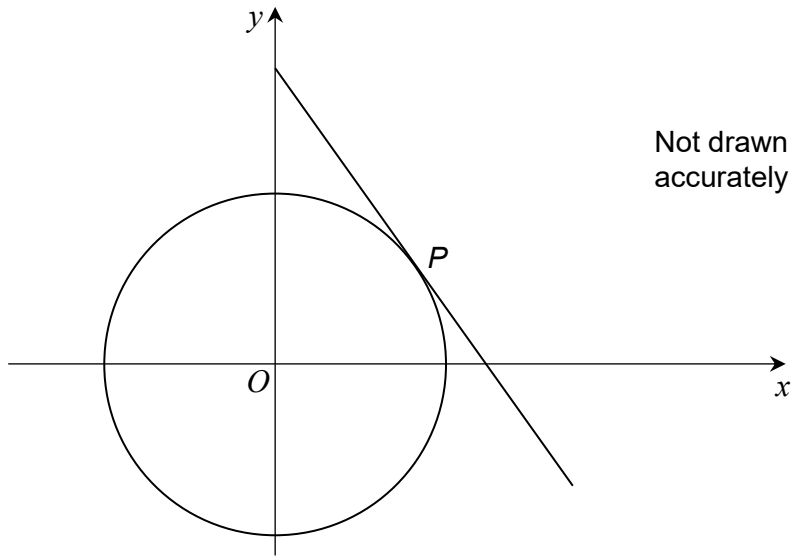
Answer

$$y = \frac{7}{4}x - \frac{65}{4}$$





7 P is a point on the circle with equation $x^2 + y^2 = 117$
 P has coordinates $(9, k)$, where $k > 0$



Work out the equation of the tangent to the circle at P .
 Give your answer in the form $y = mx + c$

[5 marks]

$$9^2 + k^2 = 117$$

$$81 + k^2 = 117$$

$$k^2 = 36$$

$$k = \pm 6$$

$$\text{as } k > 0, \underline{\underline{k=6}}$$

$$\text{gradient of } OP = \frac{6}{9} = \frac{2}{3}$$

$$\text{gradient of tangent} = -\frac{3}{2}$$

$$y = -\frac{3}{2}x + c$$

$$6 = -\frac{3}{2}(9) + c$$

$$6 = -\frac{27}{2} + c$$

$$c = 6 + \frac{27}{2}$$

$$c = \frac{12}{2} + \frac{27}{2}$$

$$c = \frac{39}{2}$$

Answer

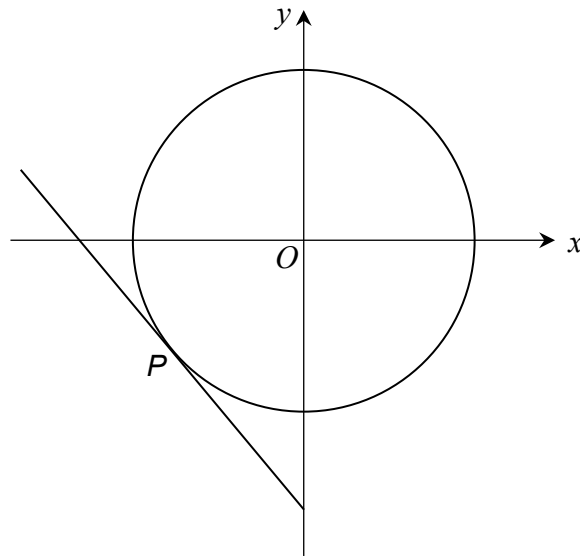
$$y = -\frac{3}{2}x + \frac{39}{2}$$

Turn over ▶





8 P is a point on the circle with equation $x^2 + y^2 = 22.25$
 P has coordinates $(-4, k)$, where $k < 0$



Not drawn accurately

Work out the equation of the tangent to the circle at P .

Give your answer in the form $ay + bx + c = 0$ where a, b and c are integers.

[6 marks]

$$(-4)^2 + k^2 = 22.25$$

$$16 + k^2 = 22.25$$

$$k^2 = 6.25$$

$$k = \pm 2.5$$

$$\text{as } k < 0, \underline{\underline{k = -2.5}}$$

$$\text{gradient of } OP = \frac{-2.5}{-4}$$

$$= \frac{2.5}{4}$$

$$= \frac{5}{8}$$

$$\text{gradient of tangent} = -\frac{8}{5}$$

$$y = -\frac{8}{5}x + c$$

$$-\frac{5}{2} = -\frac{8}{5}(-4) + c$$

$$-\frac{5}{2} = \frac{32}{5} + c$$

$$c = -\frac{5}{2} - \frac{32}{5}$$

$$c = -\frac{25}{10} - \frac{64}{10}$$

$$c = -\frac{89}{10}$$

$$y = -\frac{8}{5}x - \frac{89}{10}$$

$\times 10 \downarrow$

$\downarrow \times 10$

$$10y = -16x - 89$$

$$10y + 16x + 89 = 0$$

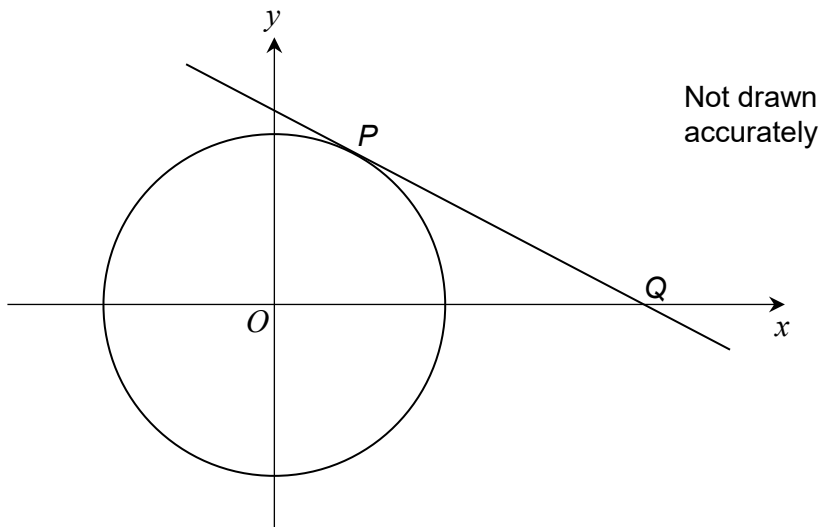
Answer

$$10y + 16x + 89 = 0$$



9

$P(2, 3)$ is a point on a circle, centre O .
The tangent at P intersects the x -axis at Q



Work out the coordinates of the point Q .

[5 marks]

$$\text{gradient of } OP = \frac{3}{2}$$

$$\text{gradient of tangent} = -\frac{2}{3}$$

$$y = -\frac{2}{3}x + c$$

$$3 = -\frac{2}{3}(2) + c$$

$$3 = -\frac{4}{3} + c$$

$$c = 3 + \frac{4}{3}$$

$$c = \frac{9}{3} + \frac{4}{3}$$

$$c = \frac{13}{3}$$

$$y = -\frac{2}{3}x + \frac{13}{3}$$

At Q , $y = 0$

$$0 = -\frac{2}{3}x + \frac{13}{3}$$

$$\frac{2}{3}x = \frac{13}{3}$$

$$2x = 13$$

$$x = \frac{13}{2}$$

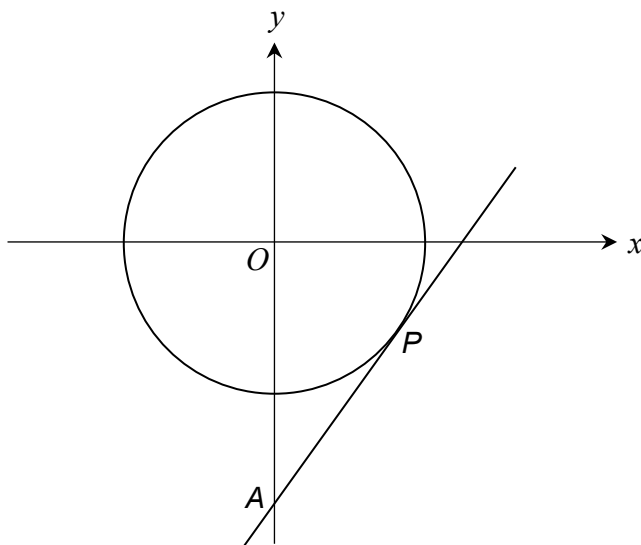
$$x = 6.5$$

Answer (6.5 , 0)



10

$P(14, -4)$ is a point on a circle, centre O .
The tangent at P intersects the y -axis at A



Not drawn
accurately

Work out the coordinates of the point A .

[5 marks]

$$\text{gradient of } OP = -\frac{4}{14} = -\frac{2}{7} \quad \text{gradient of tangent} = \frac{7}{2}$$

$$y = \frac{7}{2}x + c$$

$$-4 = \frac{7}{2}(14) + c$$

$$-4 = 49 + c$$

$$c = -4 - 49$$

$$c = -53$$

$$y = \frac{7}{2}x - 53$$

$$\text{At } A, x = 0$$

$$y = \frac{7}{2}(0) - 53$$

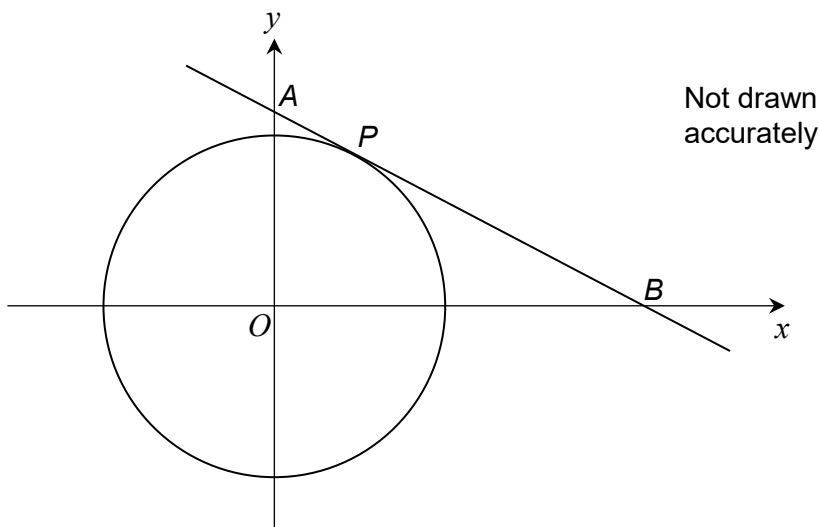
$$y = -53$$

Answer (0 , -53)



11

$P(3, 9)$ is a point on a circle, centre O .
The tangent at P intersects the axes at points A and B .



Work out the area of triangle AOB .

[6 marks]

gradient of $OP = \frac{9}{3}$
 $= 3$ gradient of tangent $= -\frac{1}{3}$

$$y = -\frac{1}{3}x + c$$

$$9 = -\frac{1}{3}(3) + c$$

$$9 = -1 + c$$

$$c = 10$$

$$y = -\frac{1}{3}x + 10$$

At A , $x = 0$

$$y = -\frac{1}{3}(0) + 10$$

$$y = 10$$

$$A = (0, 10)$$

At B , $y = 0$

$$0 = -\frac{1}{3}x + 10$$

$$\frac{1}{3}x = 10$$

$$x = 30$$

$$B = (30, 0)$$

$$\text{Area} = \frac{1}{2} \times 30 \times 10$$
$$= 150$$

Answer

150

units²

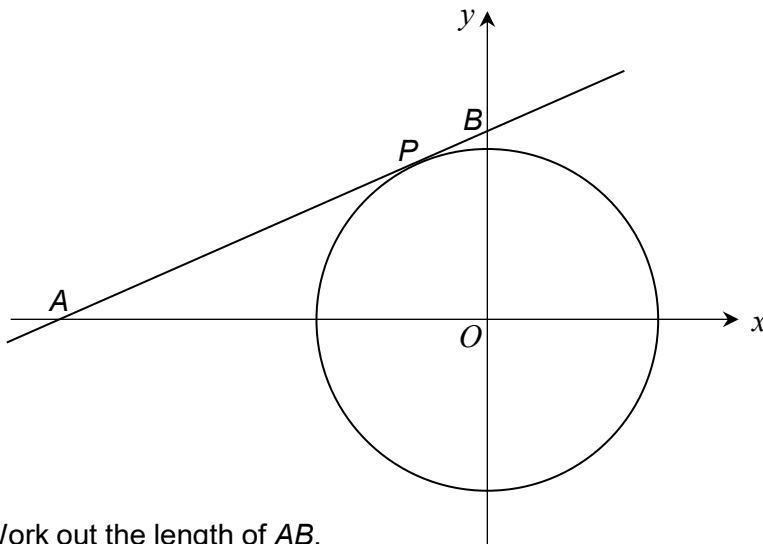
11

Turn over ►



12

$P(-8, 16)$ is a point on a circle, centre O .
The tangent at P intersects the axes at points A and B .



Not drawn accurately

Work out the length of AB .

Give your answer in the form $a\sqrt{5}$ where a is an integer.

[6 marks]

gradient of $OP = -\frac{16}{8} = -2$ gradient of tangent = $\frac{1}{2}$

$$y = \frac{1}{2}x + c$$

$$16 = \frac{1}{2}(-8) + c$$

$$16 = -4 + c$$

$$c = 16 + 4$$

$$c = 20$$

$$y = \frac{1}{2}x + 20$$

At B , $x = 0$

$$y = \frac{1}{2}(0) + 20$$

$$y = 20$$

$$B = (0, 20)$$

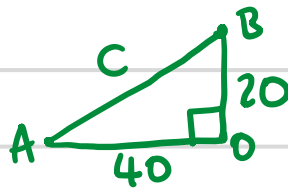
At A , $y = 0$

$$0 = \frac{1}{2}x + 20$$

$$-\frac{1}{2}x = 20$$

$$x = -40$$

$$A = (-40, 0)$$



$$c^2 = 20^2 + 40^2$$

$$c^2 = 2000$$

$$c = \sqrt{2000}$$

$$c = \sqrt{400 \times 5}$$

Answer

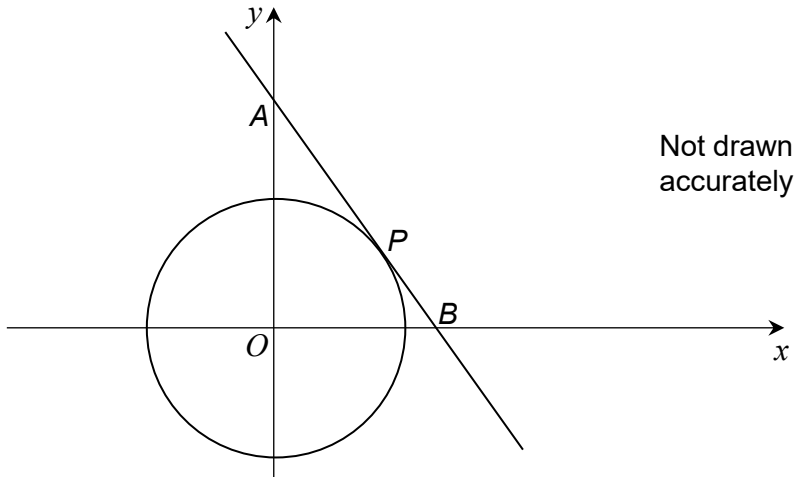
$$20\sqrt{5}$$

units



13

$P(5, 2)$ is a point on a circle, centre O .
The tangent at P intersects the axes at points A and B .



Work out the length of AB .

Give your answer to 4 significant figures.

[6 marks]

gradient of $OP = \frac{2}{5}$ gradient of tangent = $-\frac{5}{2}$

$$y = -\frac{5}{2}x + C$$

$$2 = -\frac{5}{2}(5) + C$$

$$2 = -\frac{25}{2} + C$$

$$C = 2 + \frac{25}{2}$$

$$C = \frac{4}{2} + \frac{25}{2}$$

$$C = \frac{29}{2}$$

$$y = -\frac{5}{2}x + \frac{29}{2}$$

At A , $x = 0$

$$y = -\frac{5}{2}(0) + \frac{29}{2}$$

$$A = (0, \frac{29}{2})$$

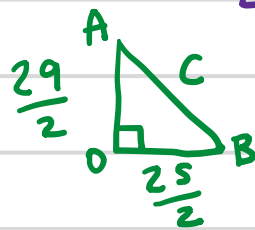
At B , $y = 0$

$$0 = -\frac{5}{2}x + \frac{29}{2}$$

$$\frac{5}{2}x = \frac{29}{2}$$

$$x = \frac{29}{5}$$

$$B = (\frac{29}{5}, 0)$$



$$C^2 = (\frac{29}{2})^2 + (\frac{29}{5})^2$$

$$C^2 = 243.89$$

$$C = \sqrt{243.89}$$

$$C = 15.6169...$$

Answer 15.62

units

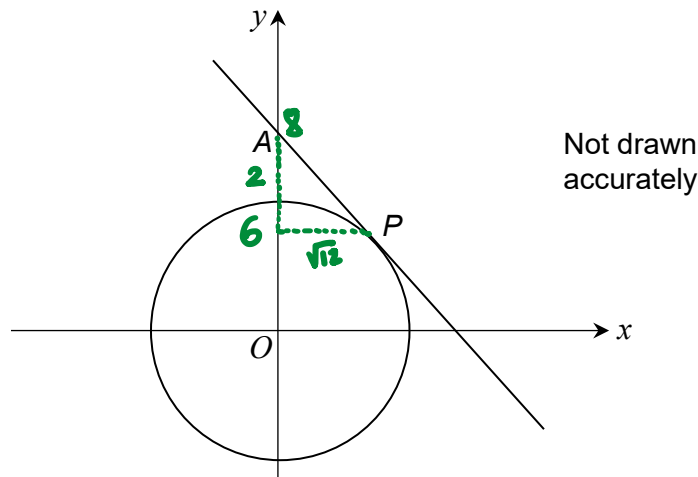
12

Turn over ▶



14

$P(\sqrt{12}, 6)$ is a point on a circle, centre O .
The tangent at P intersects the y -axis at point A .



Show that the length of AP is an integer.

[6 marks]

gradient of $OP = \frac{6}{\sqrt{12}}$ gradient of tangent $= -\frac{\sqrt{12}}{6}$

$$y = -\frac{\sqrt{12}}{6}x + c$$

$$6 = -\frac{\sqrt{12}}{6}(\sqrt{12}) + c$$

$$6 = -\frac{12}{6} + c$$

$$6 = -2 + c$$

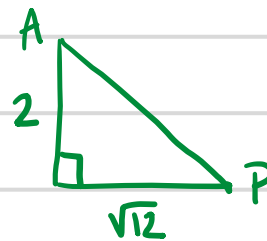
$$c = 8$$

$$y = \frac{\sqrt{12}}{6}x + 8$$

At A , $x = 0$

$$y = \frac{\sqrt{12}}{6}(0) + 8$$

$$A = (0, 8)$$



$$AP^2 = 2^2 + (\sqrt{12})^2$$

$$AP^2 = 4 + 12$$

$$AP^2 = 16$$

$$AP = \sqrt{16}$$

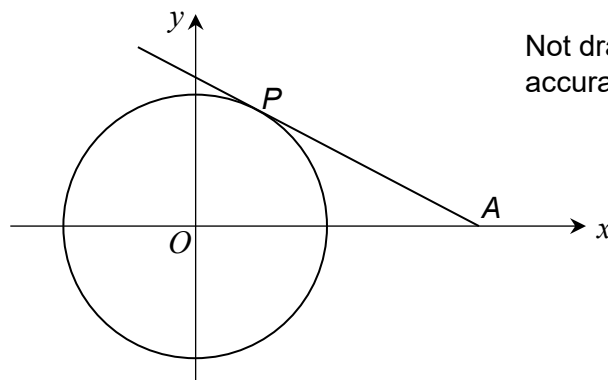
$$AP = 4$$

4 is an integer



15

$P(\sqrt{5}, \sqrt{20})$ is a point on a circle, centre O .
The tangent at P intersects the x -axis at point A .



Not drawn accurately

Work out the area of triangle AOP .

[6 marks]

$$\begin{aligned} \text{gradient of } OP &= \frac{\sqrt{20}}{\sqrt{5}} \\ &= \sqrt{4} \\ &= 2 \end{aligned} \quad \begin{aligned} \text{gradient of} \\ \text{tangent} &= -\frac{1}{2} \end{aligned}$$

$$y = -\frac{1}{2}x + C$$
$$\sqrt{20} = -\frac{1}{2}(\sqrt{5}) + C$$

$$C = \sqrt{20} + \frac{\sqrt{5}}{2}$$

$$C = 2\sqrt{5} + \frac{\sqrt{5}}{2}$$

$$C = \frac{5\sqrt{5}}{2}$$

$$y = -\frac{1}{2}x + \frac{5\sqrt{5}}{2}$$

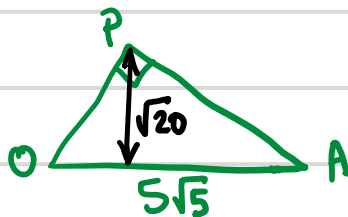
At A , $y = 0$

$$0 = -\frac{1}{2}x + \frac{5\sqrt{5}}{2}$$

$$\frac{1}{2}x = \frac{5\sqrt{5}}{2}$$

$$x = 5\sqrt{5}$$

$$A = (5\sqrt{5}, 0)$$



$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 5\sqrt{5} \times \sqrt{20} \\ &= \frac{1}{2} \times 5\sqrt{100} \\ &= \frac{1}{2} \times 5 \times 10 \\ &= \frac{1}{2} \times 50 \end{aligned}$$

25

Answer

units²

12

