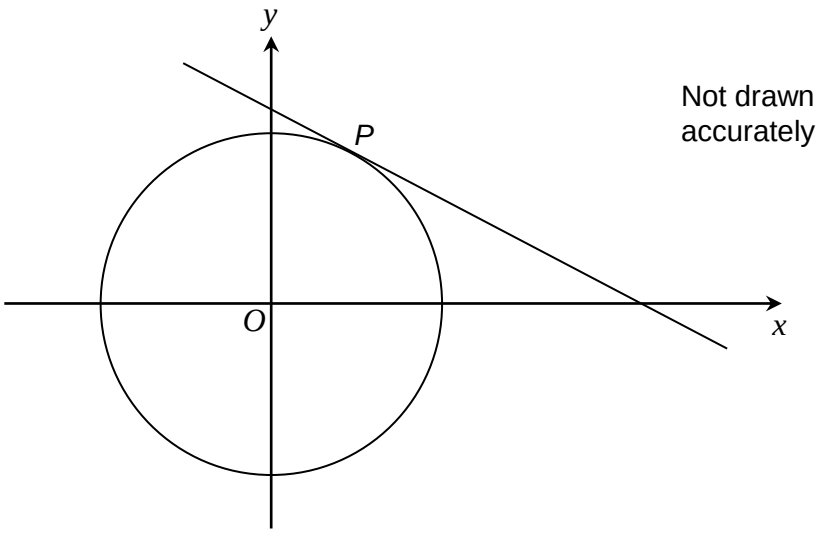
Equation of a Tangent

SCAN ME

REVISE THIS TOPIC

1 P (2, 4) is a point on a circle, centre O.



Not drawn accurately

Work out the equation of the tangent to the circle at P.
Give your answer in the form $y = mx + c$

[4 marks]

gradient of OP $= \frac{4}{2} = 2$

gradient of tangent $= -\frac{1}{2}$

$y = -\frac{1}{2}x + c$

$4 = -\frac{1}{2}(2) + c$

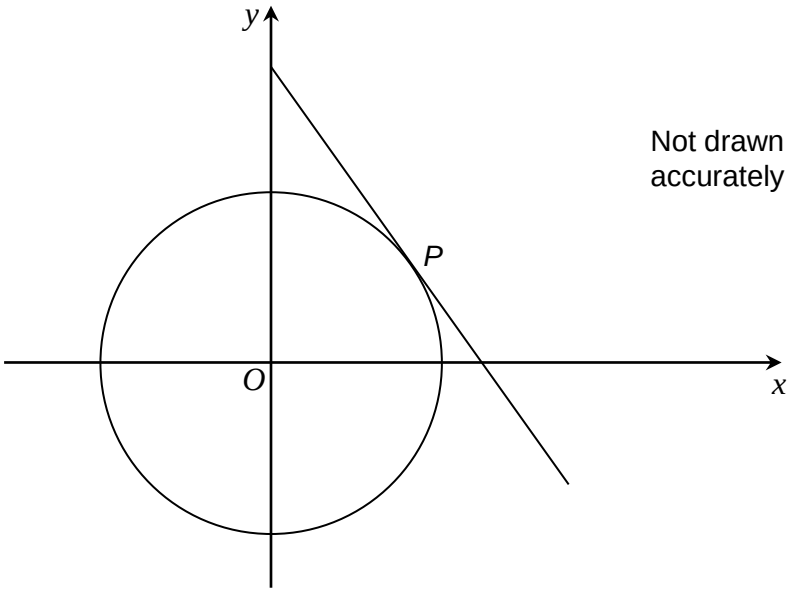
$4 = -1 + c$

$c = 5$

Answer $y = -\frac{1}{2}x + 5$

4

2 P (9, 3) is a point on a circle, centre O.



Not drawn accurately

Work out the equation of the tangent to the circle at P.
Give your answer in the form $y = mx + c$

[4 marks]

gradient of OP = $\frac{3}{9}$ = $\frac{1}{3}$

gradient of tangent = -3

$y = -3x + c$

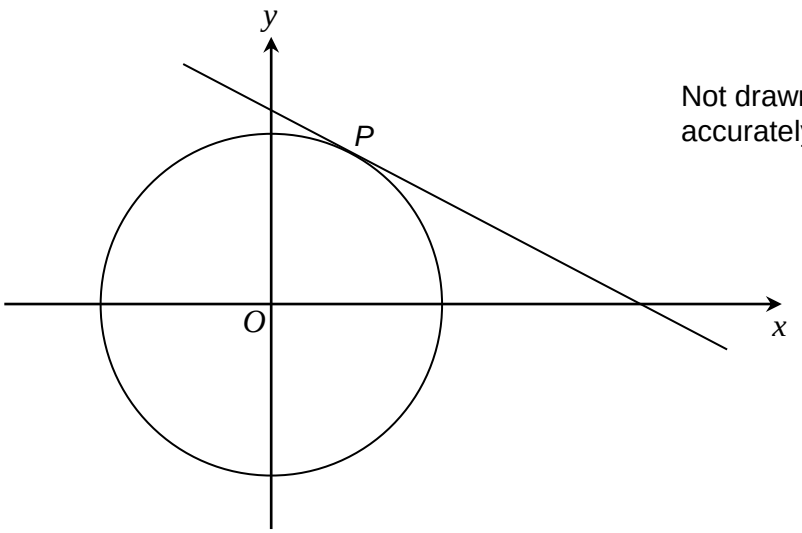
$3 = -3(9) + c$

$3 = -27 + c$

$c = 30$

Answer $y = -3x + 30$

3 P (2, 5) is a point on a circle, centre O.



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Work out the equation of the tangent to the circle at P.
Give your answer in the form $y = mx + c$

[4 marks]

gradient of OP = $\frac{5}{2}$

gradient of tangent = $-\frac{2}{5}$

$y = -\frac{2}{5}x + c$

$5 = -\frac{2}{5}(2) + c$

$5 = -\frac{4}{5} + c$

$c = 5 + \frac{4}{5}$

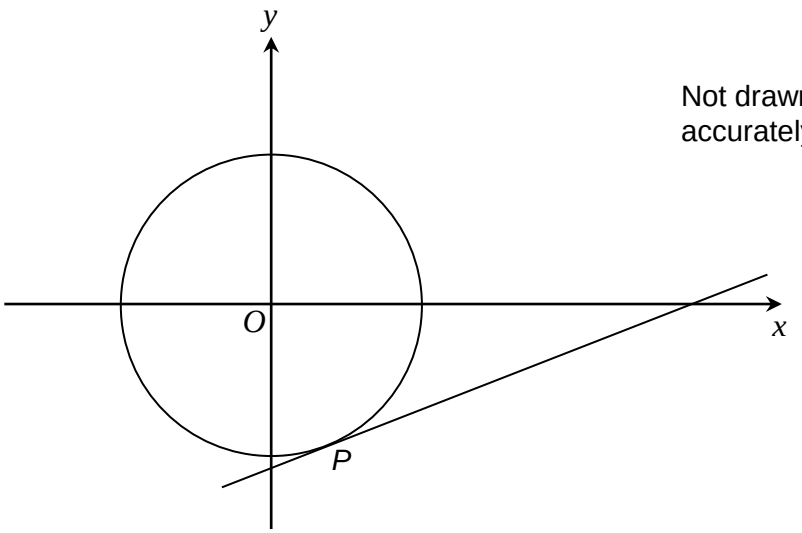
$c = \frac{25}{5} + \frac{4}{5}$

$c = \frac{29}{5}$

Answer $y = -\frac{2}{5}x + \frac{29}{5}$

8

4 P (1, -4) is a point on a circle, centre O.



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Work out the equation of the tangent to the circle at P.
Give your answer in the form $y = mx + c$

[4 marks]

gradient of OP = $\frac{-4}{1}$ = -4

gradient of tangent = $\frac{1}{4}$

$y = \frac{1}{4}x + c$

$-4 = \frac{1}{4}(1) + c$

$-4 = \frac{1}{4} + c$

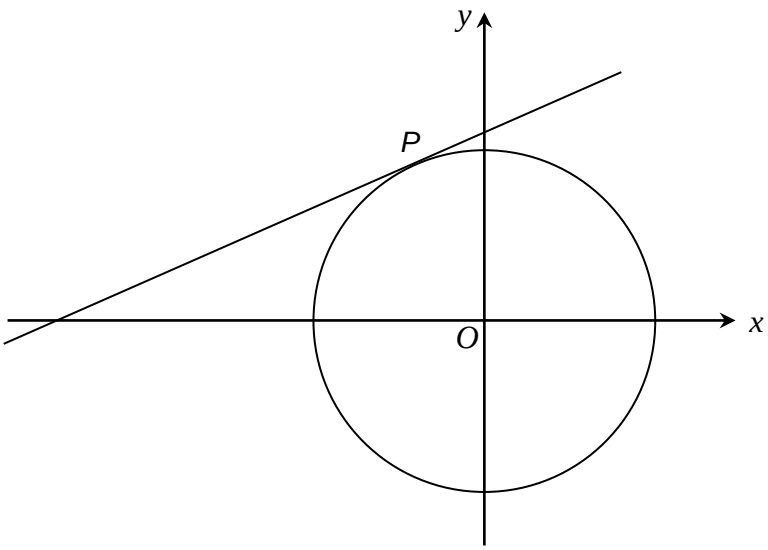
$c = -4 - \frac{1}{4}$

$c = -\frac{16}{4} - \frac{1}{4}$

$c = -\frac{17}{4}$

Answer $y = \frac{1}{4}x - \frac{17}{4}$

5 P (-3, 5) is a point on a circle, centre O.



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Work out the equation of the tangent to the circle at P.
Give your answer in the form $y = mx + c$

[4 marks]

gradient of OP $= -\frac{5}{3}$ gradient of tangent $= \frac{3}{5}$

$y = \frac{3}{5}x + c$

$5 = \frac{3}{5}(-3) + c$

$5 = -\frac{9}{5} + c$

$c = 5 + \frac{9}{5}$

$c = \frac{25}{5} + \frac{9}{5}$

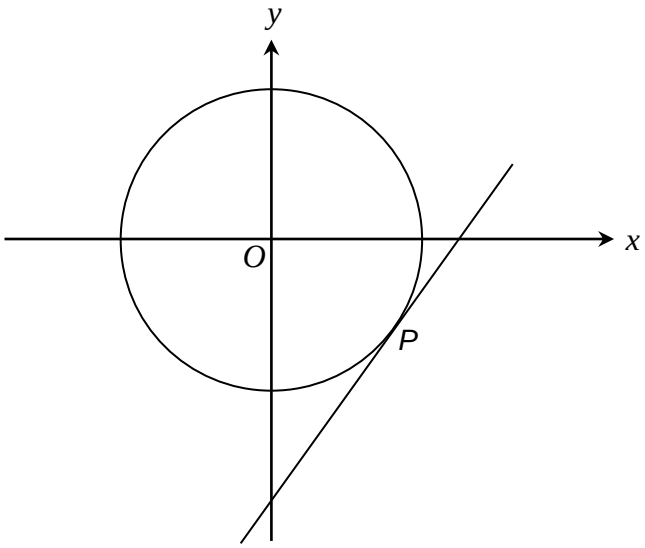
$c = \frac{34}{5}$

Answer $y = \frac{3}{5}x + \frac{34}{5}$

8

6 P is a point on the circle with equation $x^2 + y^2 = 65$

P has coordinates $(7, k)$, where $k < 0$



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Work out the equation of the tangent to the circle at P.

Give your answer in the form $y = mx + c$

[5 marks]

$7^2 + k^2 = 65$

$49 + k^2 = 65$

$k^2 = 16$

$k = \pm 4$

as $k < 0$, $k = 4$

gradient of OP $= -\frac{4}{7}$

gradient of tangent $= \frac{7}{4}$

$y = \frac{7}{4}x + c$

$-4 = \frac{7}{4}(7) + c$

$-4 = \frac{49}{4} + c$

$c = -4 - \frac{49}{4}$

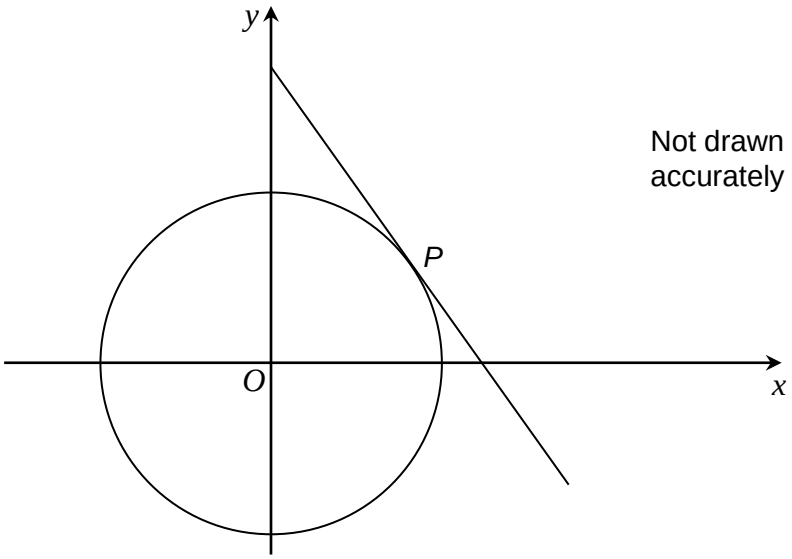
$c = \frac{-16}{4} - \frac{49}{4}$

$c = -\frac{65}{4}$

Answer $y = \frac{7}{4}x - \frac{65}{4}$

7 P is a point on the circle with equation $x^2 + y^2 = 117$

P has coordinates $(9, k)$, where $k > 0$



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Work out the equation of the tangent to the circle at P.

Give your answer in the form $y = mx + c$

[5 marks]

$9^2 + k^2 = 117$

$81 + k^2 = 117$

$k^2 = 36$

$k = \pm 6$

as $k > 0$, $k = 6$

gradient of OP $= \frac{6}{9} = \frac{2}{3}$

gradient of tangent $= -\frac{3}{2}$

$y = -\frac{3}{2}x + c$

$6 = -\frac{3}{2}(9) + c$

$6 = -\frac{27}{2} + c$

$c = 6 + \frac{27}{2}$

$c = \frac{12}{2} + \frac{27}{2}$

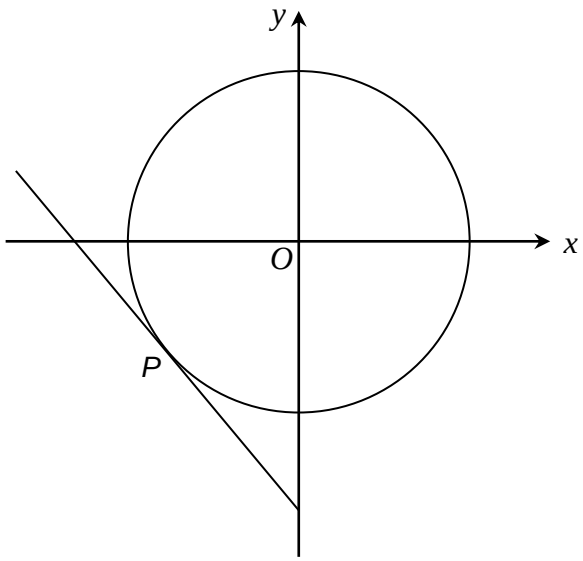
$c = \frac{39}{2}$

Answer $y = -\frac{3}{2}x + \frac{39}{2}$

10

8 P is a point on the circle with equation $x^2 + y^2 = 22.25$

P has coordinates $(-4, k)$, where $k < 0$



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Work out the equation of the tangent to the circle at P.

Give your answer in the form $ay + bx + c = 0$ where a, b and c are integers.

[6 marks]

$(-4)^2 + k^2 = 22.25$

$16 + k^2 = 22.25$

$k^2 = 6.25$

$k = \pm 2.5$

as $k < 0$, $k = -2.5$

gradient of OP $= \frac{-2.5}{-4} = \frac{2.5}{4} = \frac{5}{8}$

gradient of tangent $= -\frac{8}{5}$

$y = -\frac{8}{5}x + c$

$-\frac{5}{2} = -\frac{8}{5}(-4) + c$

$-\frac{5}{2} = \frac{32}{5} + c$

$c = -\frac{5}{2} - \frac{32}{5}$

$c = -\frac{25}{10} - \frac{64}{10}$

$c = -\frac{89}{10}$

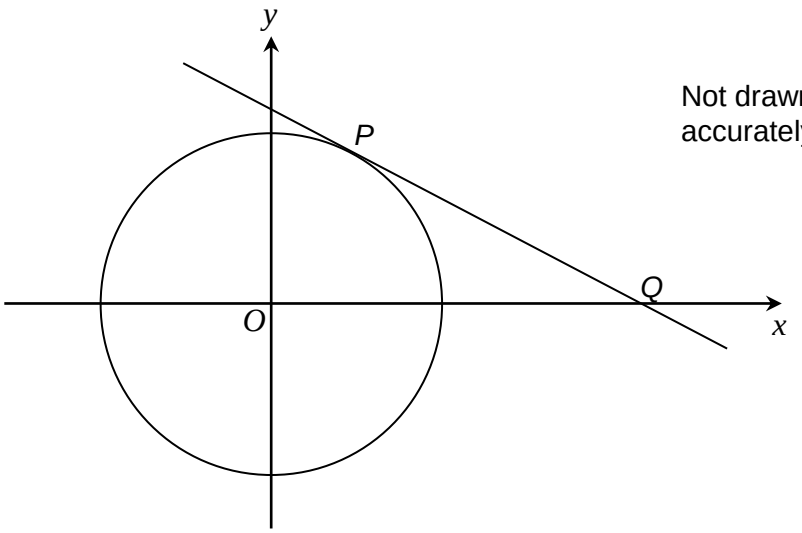
$y = -\frac{8}{5}x - \frac{89}{10}$ (×10)

$10y = -16x - 89$

$10y + 16x + 89 = 0$

Answer $10y + 16x + 89 = 0$

9 P (2, 3) is a point on a circle, centre O.
The tangent at P intersects the x-axis at Q



Not drawn accurately

Work out the coordinates of the point Q. **[5 marks]**

gradient of OP = $\frac{3}{2}$

gradient of tangent = $-\frac{2}{3}$

$y = -\frac{2}{3}x + c$

$3 = -\frac{2}{3}(2) + c$

$3 = -\frac{4}{3} + c$

$c = 3 + \frac{4}{3}$

$c = \frac{9}{3} + \frac{4}{3}$

$c = \frac{13}{3}$

$y = -\frac{2}{3}x + \frac{13}{3}$

At Q, $y = 0$

$0 = -\frac{2}{3}x + \frac{13}{3}$

$\frac{2}{3}x = \frac{13}{3}$

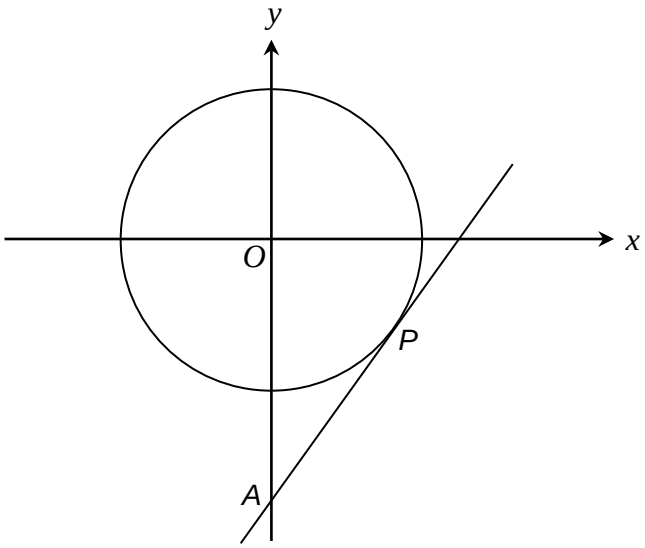
$2x = 13$

$x = \frac{13}{2}$

$x = 6.5$

Answer (6.5 , 0)

10 P (14, -4) is a point on a circle, centre O.
The tangent at P intersects the y-axis at A



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Work out the coordinates of the point A. **[5 marks]**

gradient of OP $= -\frac{4}{14}$
$= -\frac{2}{7}$

gradient of tangent $= \frac{7}{2}$

$y = \frac{7}{2}x + c$

$-4 = \frac{7}{2}(14) + c$

$-4 = 49 + c$

$c = -4 - 49$

$c = -53$

$y = \frac{7}{2}x - 53$

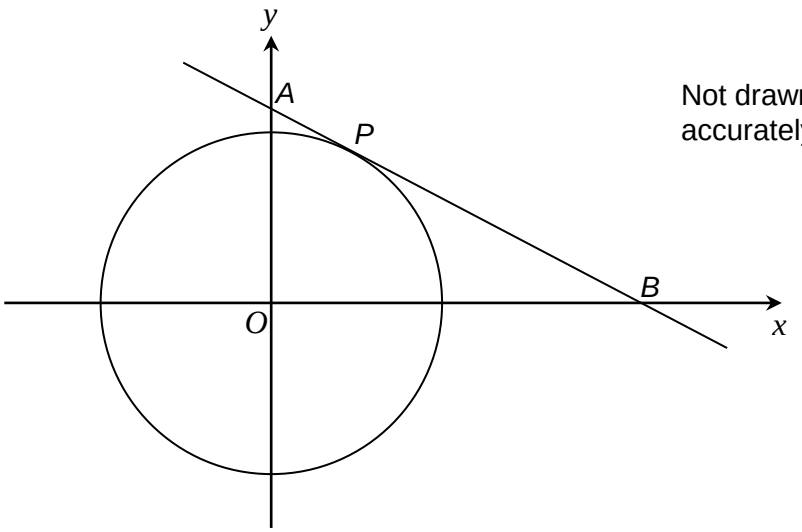
At A, $x = 0$

$y = \frac{7}{2}(0) - 53$

$y = -53$

Answer (0 , -53)

11 P (3, 9) is a point on a circle, centre O.
The tangent at P intersects the axes at points A and B.



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Work out the area of triangle AOB. **[6 marks]**

gradient of OP = $\frac{9}{3}$ = 3

gradient of tangent = $-\frac{1}{3}$

$y = -\frac{1}{3}x + c$

$9 = -\frac{1}{3}(3) + c$

$9 = -1 + c$

$c = 10$

$y = -\frac{1}{3}x + 10$

At A, $x = 0$

$y = -\frac{1}{3}(0) + 10$

$y = 10$

A = (0, 10)

At B, $y = 0$

$0 = -\frac{1}{3}x + 10$

$\frac{1}{3}x = 10$

$x = 30$

B = (30, 0)

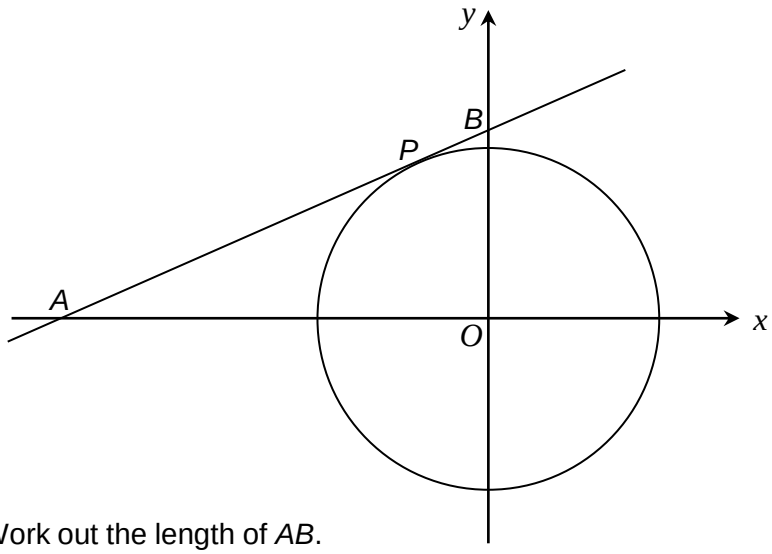
Area = $\frac{1}{2} \times 30 \times 10$

= 150

Answer 150 units^2

11

12 P (-8, 16) is a point on a circle, centre O.
The tangent at P intersects the axes at points A and B.



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Work out the length of AB.
Give your answer in the form $a\sqrt{5}$ where a is an integer. **[6 marks]**

gradient of OP $= -\frac{16}{8}$
$= -2$

gradient of tangent $= \frac{1}{2}$

$y = \frac{1}{2}x + c$

$16 = \frac{1}{2}(-8) + c$

$16 = -4 + c$

$c = 16 + 4$

$c = 20$

$y = \frac{1}{2}x + 20$

At B, $x = 0$

$y = \frac{1}{2}(0) + 20$

$y = 20$

$B = (0, 20)$

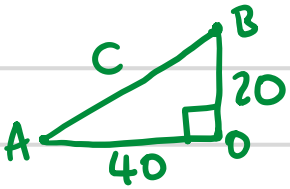
At A, $y = 0$

$0 = \frac{1}{2}x + 20$

$-\frac{1}{2}x = 20$

$x = -40$

$A = (-40, 0)$



$C^2 = 20^2 + 40^2$

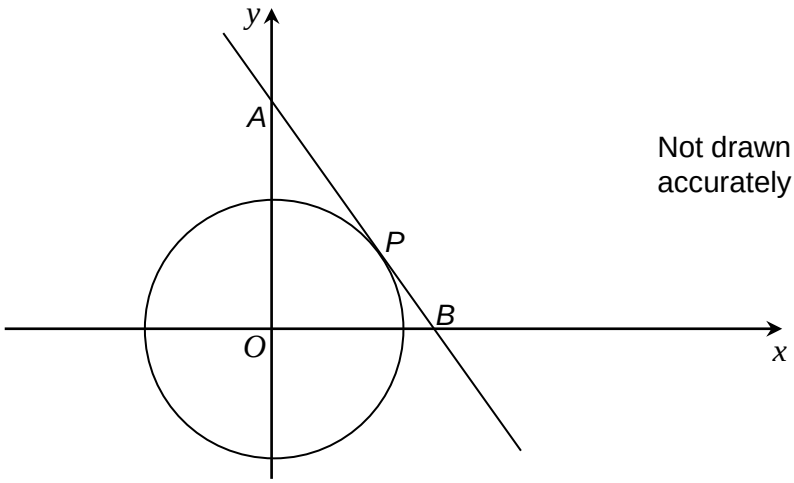
$C^2 = 2000$

$C = \sqrt{2000}$

$C = \sqrt{400} \times \sqrt{5}$

Answer $20\sqrt{5}$ units

13 P (5, 2) is a point on a circle, centre O.
The tangent at P intersects the axes at points A and B.



Not drawn accurately

Work out the length of AB.
Give your answer to 4 significant figures. **[6 marks]**

gradient of OP = $\frac{2}{5}$

gradient of tangent = $-\frac{5}{2}$

$y = -\frac{5}{2}x + c$

$2 = -\frac{5}{2}(5) + c$

$2 = -\frac{25}{2} + c$

$c = 2 + \frac{25}{2}$

$c = \frac{4}{2} + \frac{25}{2}$

$c = \frac{29}{2}$

$y = -\frac{5}{2}x + \frac{29}{2}$

At A, $x = 0$

$y = -\frac{5}{2}(0) + \frac{29}{2}$

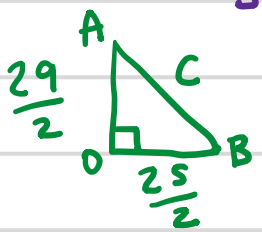
$A = (0, \frac{29}{2})$

At B, $y = 0$

$0 = -\frac{5}{2}x + \frac{29}{2}$

$\frac{5}{2}x = \frac{29}{2}$

$x = \frac{29}{5}$ $\quad B = (\frac{29}{5}, 0)$



$C^2 = (\frac{29}{2})^2 + (\frac{29}{5})^2$

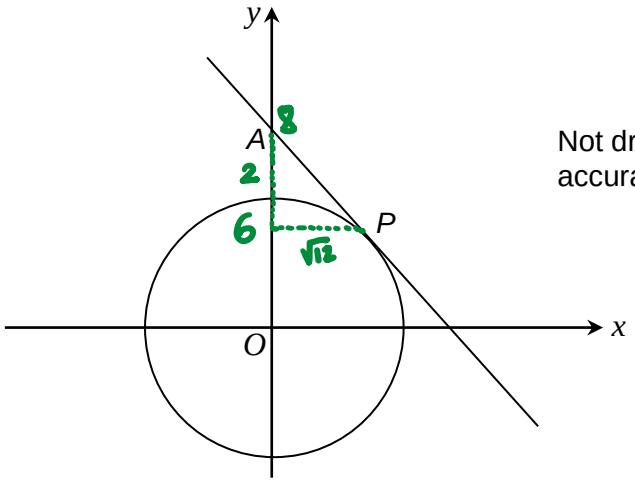
$C^2 = 243.89$

$C = \sqrt{243.89}$

$C = 15.6169...$

Answer 15.62 units

14 $P\ (\sqrt{12}, 6)$ is a point on a circle, centre O.
The tangent at P intersects the y-axis at point A.



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Show that the length of AP is an integer. **[6 marks]**

gradient of $OP = \frac{6}{\sqrt{12}}$

gradient of tangent $= -\frac{\sqrt{12}}{6}$

$y = -\frac{\sqrt{12}}{6}x + c$

$6 = -\frac{\sqrt{12}}{6}(\sqrt{12}) + c$

$6 = -\frac{12}{6} + c$

$6 = -2 + c$

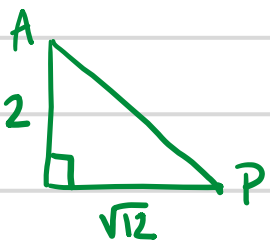
$c = 8$

$y = \frac{\sqrt{12}}{6}x + 8$

At A, $x = 0$

$y = \frac{\sqrt{12}}{6}(0) + 8$

$A = (0, 8)$



$AP^2 = 2^2 + (\sqrt{12})^2$

$AP^2 = 4 + 12$

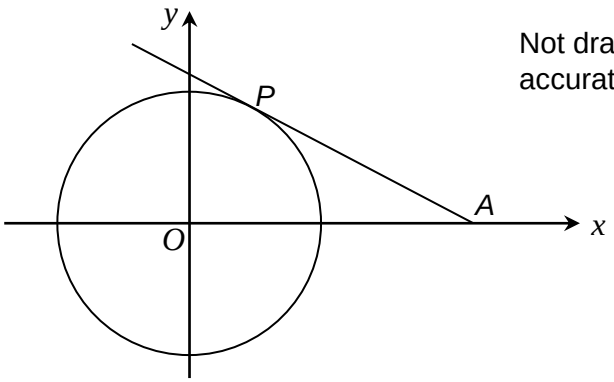
$AP^2 = 16$

$AP = \sqrt{16}$

$AP = 4$

4 is an integer

15 $P\ (\sqrt{5}, \sqrt{20})$ is a point on a circle, centre O.
The tangent at P intersects the x-axis at point A.



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Work out the area of triangle AOP. **[6 marks]**

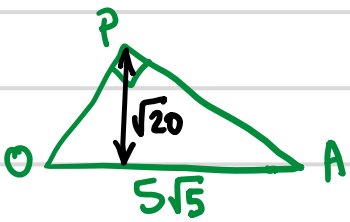
gradient of $OP = \frac{\sqrt{20}}{\sqrt{5}}$
$= \sqrt{4}$
$= 2$

gradient of tangent $= -\frac{1}{2}$

$y = -\frac{1}{2}x + c$
$\sqrt{20} = -\frac{1}{2}(\sqrt{5}) + c$
$c = \sqrt{20} + \frac{\sqrt{5}}{2}$
$c = 2\sqrt{5} + \frac{\sqrt{5}}{2}$
$c = \frac{5\sqrt{5}}{2}$
$y = -\frac{1}{2}x + \frac{5\sqrt{5}}{2}$

At A, $y = 0$

$0 = -\frac{1}{2}x + \frac{5\sqrt{5}}{2}$
$\frac{1}{2}x = \frac{5\sqrt{5}}{2}$
$x = 5\sqrt{5}$
$A = (5\sqrt{5}, 0)$



Area $= \frac{1}{2} \times 5\sqrt{5} \times \sqrt{20}$
$= \frac{1}{2} \times 5\sqrt{100}$
$= \frac{1}{2} \times 5 \times 10$
$= \frac{1}{2} \times 50$

Answer 25 units^2 12