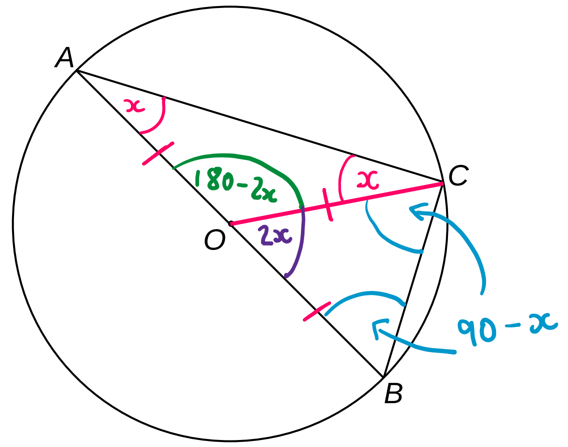
Circle Theorem Proofs

SCAN ME

REVISE THIS TOPIC

1



A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOB is a diameter of the circle.

Prove that angle *ACB* = 90° **[4 marks]**

$OC = OA = OB$ (all radii)

Let angle $OAC = x$

Angle ACO = angle $OAC = x$
(Base angles in an isosceles triangle are equal)

Angle $AOC = 180 - x - x$
$= 180 - 2x$ (angles in a triangle add to 180°)

Angle $BOC = 180 - (180 - 2x)$
$= 2x$ (angles on a straight line add to 180°)

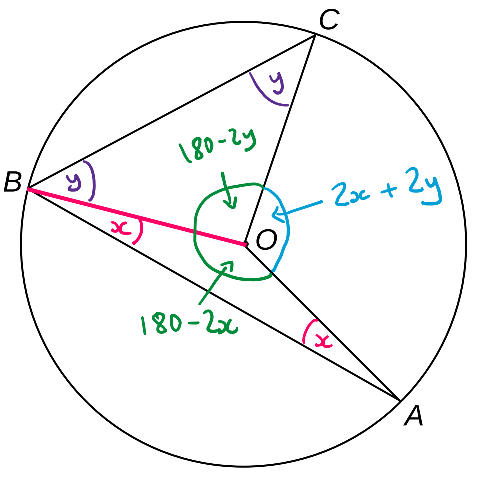
Angle OBC = Angle $OCB = \frac{1}{2}(180 - 2x)$
$= 90 - x$

(Base angles in an isosceles triangle are equal)

Angle ACB = Angle ACO + Angle OCB
$= x + 90 - x$
$= 90°$

$\frac{}{4}$

2



A, *B* and *C* are points on the circumference of a circle, centre *O*.

Prove that angle *AOC* = 2 × angle *ABC* **[4 marks]**

OC = OA = OB (all radii)

Let angle OAB = x Let angle OCB = y

Angle OBA = x Angle OBC = y

Base angles in an isosceles triangle are equal

Angle COB = $180 - 2y$ Angle AOB = $180 - 2x$

Angles in a triangle add to $180°$

Angle ABC = $x + y$

Angle AOC = $360 - (180 - 2y) - (180 - 2x)$

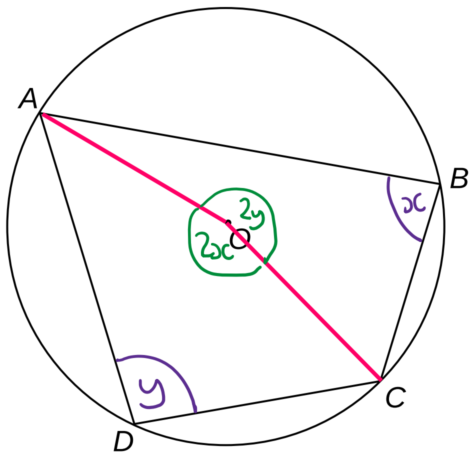
$= 360 - 180 + 2y - 180 + 2x$

$= 2x + 2y$ angles around a point add to $360°$

$= 2(x + y)$

$= 2 \times$ Angle ABC

3



A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.

Prove that angle *ABC* + angle *CDA* = 180° **[4 marks]**

Let angle ABC = x and Angle CDA = y

Minor angle AOC = $2x$ and major angle AOC = $2y$

as angle at the centre is twice the angle at the circumference

$2x + 2y = 360°$ (angles around a point add to 360°)

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$x + y = 180°$

Angle ABC + Angle CDA = 180°

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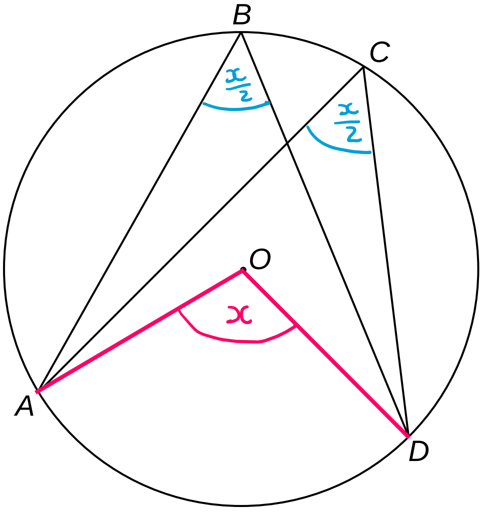
4



A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.

Prove that angle *ABC* = angle *ACD* **[2 marks]**

Let angle AOD $= x$

Angle ABD $= \frac{x}{2}$ } angle at the centre is
Angle ACD $= \frac{x}{2}$ } twice the angle at the circumference

Angle ABD = Angle ACD $= \frac{x}{2}$

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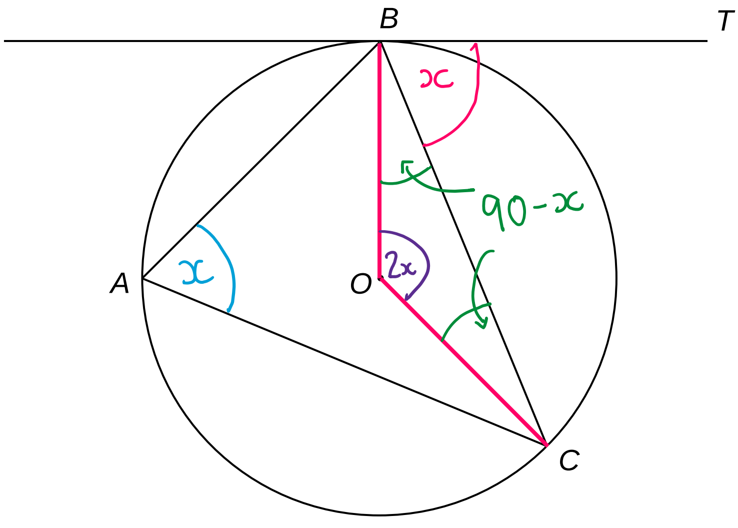

A, B and C are points on the circumference of a circle, centre O.
BT is the tangent to the circle at B.

Prove that angle CAB = angle CBT **[4 marks]**

Let angle CBT = x
Angle OBC = $90 - x$ (a tangent meets a
Angle BCO = $90 - x$ radius at $90°$)
Angle COB = $180 - (90 - x) - (90 - x)$
$= 180 - 90 + x - 90 + x$
$= 2x$
(angles in a triangle add to $180°$)
Angle CAB = x angle at the centre is twice the angle at the circumference

Angle CAB = Angle CBT = x

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