



Functions and Equations



REVISE THIS
TOPIC

1 $f(x) = 2x - 9$ $g(x) = 7x + 1$

1 (a) Solve $fg(x) = 35$ [3 marks]

$$\begin{aligned} fg(x) &= 2(7x + 1) - 9 \\ &= 14x + 2 - 9 \\ &= 14x - 7 \end{aligned}$$

$$14x - 7 = 35$$

$$14x = 42$$

Answer $x = 3$

1 (b) Solve $f^{-1}(x) + g^{-1}(x) = 5$ [4 marks]

$$y = 2x - 9$$

$$y = 7x + 1$$

$$\frac{x+9}{2} + \frac{x-1}{7} = 5$$

$$x = 2y - 9$$

$$x = 7y + 1$$

$$\frac{7(x+9) + 2(x-1)}{14} = 5$$

$$x+9 = 2y$$

$$x-1 = 7y$$

$$\frac{x+9}{2} = y$$

$$\frac{x-1}{7} = y$$

$$7x + 63 + 2x - 2 = 70$$

$$9x + 61 = 70$$

$$9x = 9$$

$$f^{-1}(x) = \frac{x+9}{2}$$

$$g^{-1}(x) = \frac{x-1}{7}$$

$$x = 1$$

Answer $x = 1$





$$2 \quad f(x) = \frac{8}{x} \quad g(x) = x - 3 \quad h(x) = x^2$$

2 (a) Solve $f(x) + g(x) = 3$ [3 marks]

$$\frac{8}{x} + x - 3 = 3 \quad x^2 - 6x + 8 = 0$$

$$(x-2)(x-4) = 0$$

$$x = 2 \quad x = 4$$

$$xx \left(\frac{8}{x} + x = 6 \right) xx \quad 8 + x^2 = 6x$$

Answer $x = 2 \quad x = 4$

2 (b) Solve $h(x) = g(4x)$ [3 marks]

$$x^2 = 4x - 3$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \quad x = 1$$

Answer $x = 3 \quad x = 1$

2 (c) $h^{-1}(100) + g^{-1}(3) = f(k)$ where k is a constant. [4 marks]
Work out the value of k

$$h^{-1}(x) = \sqrt{x} \quad h^{-1}(100) + g^{-1}(3) = f(k)$$

$$g^{-1}(x) = x + 3 \quad 10 + 6 = \frac{8}{k}$$

$$16 = \frac{8}{k}$$

$$k = \frac{8}{16}$$

$k = \frac{1}{2}$





3

$$f(x) = \frac{36}{x^2}$$

$$g(x) = \sin(x)$$

$$h(x) = 3x$$

3 (a) Show that $f^{-1}(3) \times g(60)$ is an integer

[4 marks]

$$y = \frac{36}{x^2}$$

$$y = \frac{\sqrt{36}}{\sqrt{x}}$$

$$f^{-1}(3) \times g(60) = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$

$$x = \frac{36}{y^2}$$

$$y = \frac{6}{\sqrt{x}}$$

$$= \frac{6\sqrt{3}}{2\sqrt{3}}$$

$$y^2 = \frac{36}{x}$$

$$f^{-1}(x) = \frac{6}{\sqrt{x}}$$

$$= \frac{6}{2}$$

$$y = \sqrt{\frac{36}{x}}$$

$$= 3 \text{ (integer)}$$

3 (b) Solve $hf(x) - fh(x) = 26$

[4 marks]

$$hf(x) = 3 \times \frac{36}{x^2}$$

$$\frac{108}{x^2} - \frac{4}{x^2} = 26$$

$$= \frac{108}{x^2}$$

$$\frac{104}{x^2} = 26$$

$$fh(x) = \frac{36}{(3x)^2}$$

$$\frac{104}{26} = x^2$$

$$= \frac{36}{9x^2}$$

$$4 = x^2$$

$$= \frac{4}{x^2}$$

$$x = \pm\sqrt{4}$$

Answer

$$x = 2$$

$$x = -2$$





4

$f(x) = x^2$

$g(x) = x + 4$

$h(x) = x + 2$

4 (a) Show that $fg(x) - fh(x) = 2g(x) + 2h(x)$

[5 marks]

$$\begin{aligned} fg(x) &= (x+4)^2 \\ &= x^2 + 8x + 16 \end{aligned}$$

$$\begin{aligned} fh(x) &= (x+2)^2 \\ &= x^2 + 4x + 4 \end{aligned}$$

$$\begin{aligned} fg(x) - fh(x) &= x^2 + 8x + 16 - (x^2 + 4x + 4) \\ &= x^2 + 8x + 16 - x^2 - 4x - 4 \\ &= 4x + 12 \\ &= 2(2x + 6) \\ &= 2(x + 4 + x + 2) \\ &= 2(g(x) + h(x)) \\ &= 2g(x) + 2h(x) \end{aligned}$$

4 (b) Solve $gf^{-1}(x) = 9$

[3 marks]

$f^{-1}(x) = \sqrt{x}$

$\sqrt{x} + 4 = 9$

$gf^{-1}(x) = \sqrt{x} + 4$

$\sqrt{x} = 5$

$x = 25$

Answer

$x = 25$





5 $f(x) = x^2$ $g(x) = \frac{x+8}{11}$ $h(x) = ax + b$ where a and b are integers.

5 (a) Solve $f(x+2) = g^{-1}(x)$ [4 marks]

$$f(x+2) = (x+2)^2$$

$$= x^2 + 4x + 4$$

$$x^2 + 4x + 4 = 11x - 8$$

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$

$$g^{-1}(x) = 11x - 8$$

$$x = 3 \quad x = 4$$

Answer $x = 3 \quad x = 4$

5 (b) $h(3) = 7$
 $h^{-1}(55) = 15$

Work out the values of a and b .

[5 marks]

$$y = ax + b$$

$$h(3) = 7$$

$$15a + b = 55$$

$$x = ay + b$$

$$3a + b = 7$$

$$- \quad 3a + b = 7$$

$$x - b = ay$$

$$12a = 48$$

$$\frac{x-b}{a} = y$$

$$h^{-1}(55) = 15$$

$$a = 4$$

$$\frac{55-b}{a} = 15$$

$$h^{-1}(x) = \frac{x-b}{a}$$

$$3a + b = 7$$

$$55 - b = 15a$$

$$12 + b = 7$$

$$55 = 15a + b$$

$$b = -5$$

$$a = 4$$

$$b = -5$$

