



Upper and Lower Bounds



REVISE THIS TOPIC



- 1 To the nearest pound, Eric has £8.00
To the nearest 10p, Nicky has £1.60

(a) Work out the maximum possible total amount of money.

$$7.50 \leq E < 8.50$$

$$1.55 \leq N < 1.65$$

$$£8.49 + £1.64 = 10.13$$

$$£ \underline{10.13}$$

(3)

- (b) Eric buys a new phone case.
The phone case costs £2.50 (to the nearest 50p).

Work out the maximum amount of money that Eric could have left after buying the phone case.

$$2.25 \leq P < 2.75$$

$$£8.49 - £2.25 = £6.24$$

$$£ \underline{6.24}$$

(3)

(Total for Question 1 is 6 marks)



2 To 2 significant figures, the capacity of a can of drink is 330 ml
A multipack contains 24 cans of drink.

(a) Work out the upper bound for the capacity of the multipack of cans.

$$325 \leq c < 335$$

$$335 \times 24 = 8040$$

$$\begin{array}{r} 8040 \\ \hline \end{array} \text{ml}$$

(2)

(b) Work out the lower bound for the capacity of the multipack of cans.

$$325 \times 24 = 7800$$

$$\begin{array}{r} 7800 \\ \hline \end{array} \text{ml}$$

(2)

(c) Arya opens one of the cans of drink.
She drinks 72 ml (to the nearest ml) of the drink.

Work out the lower bound for the amount of drink that could be left in the can.

$$325 \leq c < 335$$

$$71.5 \leq A < 72.5$$

$$325 - 72.5 = 252.5$$

$$\begin{array}{r} 252.5 \\ \hline \end{array} \text{ml}$$

(3)

(Total for Question 2 is 7 marks)



- 3 A stadium contains 32000 fans (to 2 significant figures).
On average, each fan spends £3.50 (to the nearest 50p) at the stadium.

(a) Work out the upper bound for the total amount of money spent.

$$31500 \leq F < 32500$$

$$3.25 \leq S < 3.75$$

$$32500 \times \pounds 3.75 = \pounds 121875$$

$$\pounds \frac{121875}{(3)}$$

(b) Work out the lower bound for the total amount of money spent.

$$31500 \times \pounds 3.25$$

$$= \pounds 102375$$

$$\pounds \frac{102375}{(2)}$$

(c) At half time 30% (to the nearest 10%) of the fans leave the stadium.

Work out the lower bound for the number of fans that leave the stadium.

$$25\% \leq P < 35\%$$

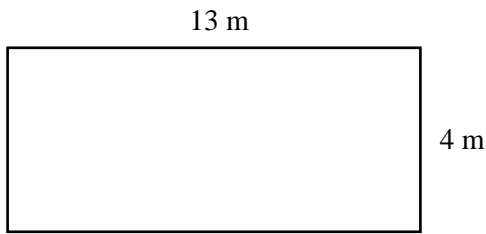
$$31500 \times 0.25 = 7875$$

$$\frac{7875}{(2)}$$

(Total for Question 3 is 7 marks)



4 The dimensions of a rectangle are shown to the nearest metre.



(a) Work out the upper bound for the **area** of the rectangle.

$$12.5 \leq l < 13.5$$

$$3.5 \leq w < 4.5$$

$$13.5 \times 4.5 = 60.75$$

$$\underline{\hspace{1cm} 60.75 \hspace{1cm}} \text{ cm}^2$$

(3)

(b) Work out the lower bound for the **perimeter** of the rectangle.

$$12.5 + 3.5 + 12.5 + 3.5$$

$$\underline{\hspace{1cm} 32 \hspace{1cm}} \text{ cm}$$

(2)

(Total for Question 4 is 5 marks)



5 To 1 decimal place, the radius of a circle is 6.5 cm.

(a) Work out the lower bound for the area of the circle.

$$6.45 \leq r < 6.55$$

$$\pi \times 6.45^2 = 130.6981084$$

$$\underline{\underline{130.7}} \text{ cm}^2$$

(2)

(b) Work out the upper bound for the circumference of the circle.

$$2 \times \pi \times 6.55 = 41.15486376$$

$$\underline{\underline{41.2}} \text{ cm}$$

(2)

(Total for Question 5 is 4 marks)

6 $x = 700$ (to 1 significant figure)
 $y = 84$ (to the nearest integer)

Work out the upper bound for $2x + y$

$$650 \leq x < 750$$

$$83.5 \leq y < 84.5$$

$$2 \times 750 + 84.5 = 1584.5$$

$$\underline{\underline{1584.5}}$$

(Total for Question 6 is 2 marks)



- 7 Jacob invest £600 (to 1 significant figure) in a bank for 4 years. The bank pays compound interest at 3.2% (to 1 decimal place).

Work out the upper and lower bound for the total amount of money that Jacob has in his account after 4 years.

$$650 \times 1.0325^4 = 738.71$$

$$550 \times 1.0315^4 = 622.64$$

Upper Bound £ 738.71

Lower Bound £ 622.64

(Total for Question 7 is 4 marks)

- 8 The interior angle of a regular polygon is 150° (correct to 2 significant figures).

Work out the maximum and minimum number of sides of the regular polygon.

$$145^\circ \leq \text{interior} < 155^\circ$$

$$25^\circ < \text{exterior} \leq 35^\circ$$

$$360 \div 25 = 14.4$$

$$360 \div 35 = 10.28571429$$

Maximum 14

Minimum 11

(Total for Question 8 is 4 marks)



- 9 Box A has a mass of 800 kg (to the nearest 100 kg)
 Box B has a mass of 600 kg (to the nearest 100 kg)
 Box C has a mass of 1500 kg (to the nearest 100 kg)

A lorry can safely carry a load of 3 tonnes. [1 tonne = 1000 kg]

The lorry driver says:

“I can be sure that I can carry all three boxes safely as $800 \text{ kg} + 600 \text{ kg} + 1500 \text{ kg} = 2900 \text{ kg}$ ”

Is the lorry driver correct? Give reasons for your answer.

$$\begin{array}{ll}
 750 \leq A < 850 & 750 + 550 + 1450 \\
 550 \leq B < 650 & = 2750 \text{ kg} \\
 1450 \leq C < 1550 & 850 + 650 + 1550 \\
 & = 3050 \text{ kg}
 \end{array}$$

No - the total mass may exceed 3000 kg

(Total for Question 9 is 4 marks)

- 10 $p = 3.9$ (to 1 decimal place)
 $q = 0.33$ (to 2 decimal places)

Work out the lower bound for $\frac{p^2}{q}$

Give your answer to 6 significant figures.

$$\begin{array}{l}
 3.85 \leq p < 3.95 \\
 0.325 \leq q < 0.335
 \end{array}$$

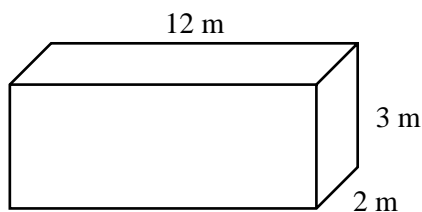
$$\frac{3.85^2}{0.335} = 44.24626866$$

44.2463

(Total for Question 10 is 3 marks)



11 The dimensions of a cuboid are shown to the nearest metre.



$$11.5 \leq l < 12.5$$

$$1.5 \leq w < 2.5$$

$$2.5 \leq h < 3.5$$

$$27.5 \leq p \leq 28.5$$

The outside surfaces of the cuboid are to be painted.
Each tin of paint covers 28 m^2 (to the nearest square metre).

Show clearly that 6 tins of paint may not be enough to paint the outside surfaces.

$$12.5 \times 2.5 = 31.25$$

$$12.5 \times 3.5 = 43.75$$

$$3.5 \times 2.5 = \underline{8.75}$$

$$83.75$$

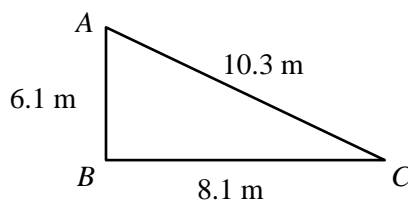
$$83.75 \times 2 = 167.5$$

$$27.5 \times 6 = 165$$

$$167.5 > 165$$

(Total for Question 11 is 4 marks)

12 The dimensions of a triangle are shown to the nearest 0.1 m



Show clearly that angle ABC cannot be a right angle.

$$6.05 \leq AB < 6.15$$

$$8.05 \leq BC < 8.15$$

$$10.25 \leq AC < 10.35$$

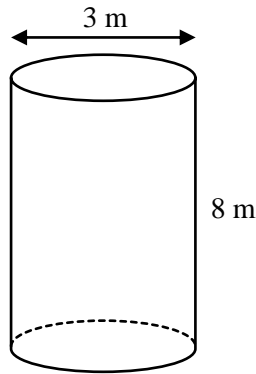
$$UB_{AC} = \sqrt{6.15^2 + 8.15^2} = 10.21004407$$

$$10.21... < 10.25 \text{ so cannot be true.}$$

(Total for Question 12 is 4 marks)



13 The dimensions of a cylinder are shown to the nearest metre.



$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The cylinder exerts a force of 8×10^5 Newtons (to 1 significant figure) onto a floor.

Calculate the lower bound for the pressure between the cylinder and the floor.
Give your answer to 5 significant figures.

$$\begin{aligned}
 750000 &\leq F < 850000 \\
 2.5 &\leq \text{Diameter} < 3.5 \\
 1.25 &\leq \text{Radius} < 1.75
 \end{aligned}$$

$$\begin{aligned}
 \text{Pressure}_{\text{LB}} &= \frac{750000}{\pi \times 1.75^2} \\
 &= 77953.44181
 \end{aligned}$$

$$77953 \dots \text{N/m}^2$$

(Total for Question 13 is 5 marks)



14 $k = \sqrt{\frac{a-b}{c}}$

$a = 430$ (to 2 significant figures)
 $b = 320$ (to 2 significant figures)
 $c = 0.07$ (to 1 significant figure)

$425 \leq a < 435$
 $315 \leq b < 325$
 $0.065 \leq c < 0.075$

By considering bounds, work out the value of k to a suitable degree of accuracy. Give a reason for your answer.

LB = $\sqrt{\frac{425-325}{0.075}}$

UB = $\sqrt{\frac{435-315}{0.065}}$

= 36.5148...

= 42.96689...

Both agree to 1 significant figure

40

(Total for Question 14 is 5 marks)

15 $y = \frac{m+n}{6-h}$

$m = 9.8$ (to 1 decimal place)
 $n = 4.4$ (to 1 decimal place)
 $h = 5.41$ (to 2 decimal places)

$9.75 \leq m < 9.85$
 $4.35 \leq n < 4.45$
 $5.405 \leq h < 5.415$

By considering bounds, work out the value of y to a suitable degree of accuracy. Give a reason for your answer.

LB = $\frac{9.75+4.35}{6-5.405}$

UB = $\frac{9.85+4.45}{6-5.415}$

= 23.69747899

= 24.4

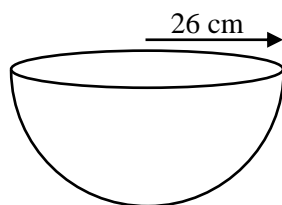
Both agree to 2 significant figures

24

(Total for Question 15 is 5 marks)



- 16 A container is in the shape of a hemisphere
 The radius of the hemisphere is 26 cm (to the nearest centimetre).



Liquid fills the hemisphere at a constant rate.
 The constant rate = 550 ml (to the nearest 50 ml) per minute.

Show that it takes at least 1 hour to fill the hemisphere.

$$\begin{aligned}
 25.5 &\leq r < 26.5 \\
 525 &\leq \text{rate} < 575 \\
 \text{Volume}_{(\text{LB})} &= \frac{2}{3} \times \pi \times 25.5^3 \\
 &= 34727.95059 \text{ cm}^3 \\
 \text{Time}_{(\text{LB})} &= \frac{34727.95\dots}{575} \\
 &= 60.396\dots \text{ mins} \\
 60.396 &> 60 \text{ mins (1 hour)}
 \end{aligned}$$

(Total for Question 16 is 5 marks)

