REVISE THIS TOPIC

Vectors

1   *OACB* is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

Write the following vectors in terms of **a** and **b**.

1 (a) $\overrightarrow{AO}$ **[1 mark]**

Answer _____ $-a$ _____

1 (b) $\overrightarrow{BC}$ **[1 mark]**

Answer _____ a _____

1 (c) $\overrightarrow{AB}$ **[1 mark]**

Answer _____ $b - a$ _____

1 (d) $\overrightarrow{CO}$ **[1 mark]**

Answer _____ $-a - b$ _____

4

2   *OXYZ* is a quadrilateral.
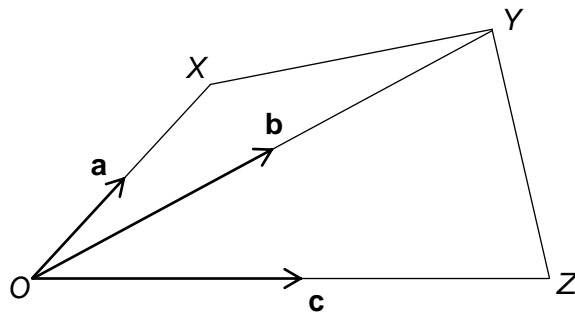
$\overrightarrow{OX} = \mathbf{a}$ $\overrightarrow{OY} = \mathbf{b}$ $\overrightarrow{OZ} = \mathbf{c}$

Write the following vectors in terms of **a, b** and **c**.

2 (a) $\overrightarrow{ZO}$ **[1 mark]**

Answer _____ $-\mathbf{c}$ _____

2 (b) $\overrightarrow{XY}$ **[1 mark]**

Answer _____ $\mathbf{b} - \mathbf{a}$ _____

2 (c) $\overrightarrow{ZY}$ **[1 mark]**

Answer _____ $\mathbf{b} - \mathbf{c}$ _____

2 (d) $\overrightarrow{XZ}$ **[1 mark]**

Answer _____ $\mathbf{c} - \mathbf{a}$ _____

3
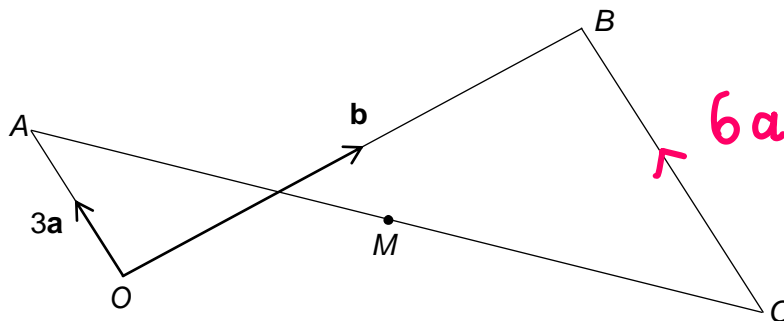
$$\overrightarrow{OA} = 3\mathbf{a} \qquad \overrightarrow{OB} = \mathbf{b} \qquad \overrightarrow{CB} = 2\overrightarrow{OA}$$

Write the following vectors in terms of **a**, **b** and **c**.

3 (a) $\overrightarrow{AB}$

[1 mark]

Answer $\qquad b - 3a \qquad$

3 (b) $\overrightarrow{CA}$

[2 marks]

$$\overrightarrow{CA} = \overrightarrow{CB} + \overrightarrow{BO} + \overrightarrow{OA}$$
$$= 6a - b + 3a$$

Answer $\qquad 9a - b \qquad$

3 (c) M is the midpoint of AC.

Write $\overrightarrow{CM}$ in terms of **a** and **b**.

[2 marks]

$$\overrightarrow{CM} = \tfrac{1}{2}\overrightarrow{CA}$$
$$= \tfrac{1}{2}(9a - b)$$

Answer $\qquad \frac{9}{2}a - \tfrac{1}{2}b \qquad$

9

Turn over ▶

4   *OACB* is a trapezium
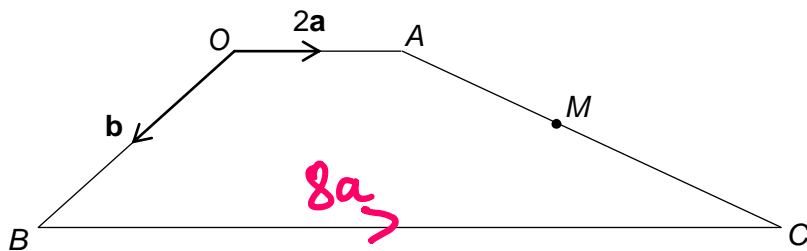
$\overrightarrow{OA} = 2\mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$ $\overrightarrow{BC} = 4\overrightarrow{OA}$

4 (a)   Write $\overrightarrow{AC}$ in term of **a** and **b**.   **[2 marks]**

$$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{BC}$$
$$= -2a + b + 8a$$

Answer ___ $6a + b$ ___

4 (c)   *M* is the midpoint of *AC*.

Write $\overrightarrow{BM}$ in term of **a** and **b**.   **[3 marks]**

$$\overrightarrow{BM} = \overrightarrow{BO} + \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AC}$$
$$= -b + 2a + \tfrac{1}{2}(6a + b)$$
$$= -b + 2a + 3a + \tfrac{1}{2}b$$

Answer ___ $5a - \tfrac{1}{2}b$ ___

5   *OACD* is a parallelogram.
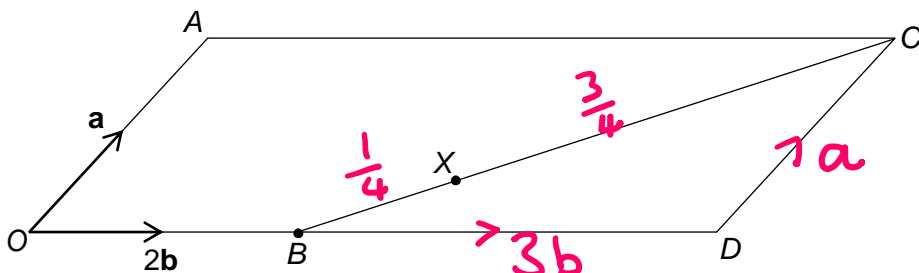
$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = 2\mathbf{b}$ $\overrightarrow{OD} = 2.5\,\overrightarrow{OB}$

5 (a)   Write $\overrightarrow{AD}$ in term of **a** and **b**.   **[2 marks]**

$$\overrightarrow{AD} = \overrightarrow{AO} + \overrightarrow{OD}$$
$$= -a + 5b$$

Answer $5b - a$

5 (b)   Write $\overrightarrow{BC}$ in term of **a** and **b**.   **[2 marks]**

$$\overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{DC}$$
$$= 3b + a$$

Answer $3b + a$

5 (c) $BX : XC = 1 : 3$

Write $\overrightarrow{OX}$ in term of **a** and **b**.   **[2 marks]**

$$\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{BX}$$
$$= 2b + \tfrac{1}{4}\overrightarrow{BC}$$
$$= 2b + \tfrac{1}{4}(3b + a)$$
$$= 2b + \tfrac{3}{4}b + \tfrac{1}{4}a$$

Answer $\tfrac{11}{4}b + \tfrac{1}{4}a$

11

Turn over ▶

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6 *PQRST* is a pentagon.
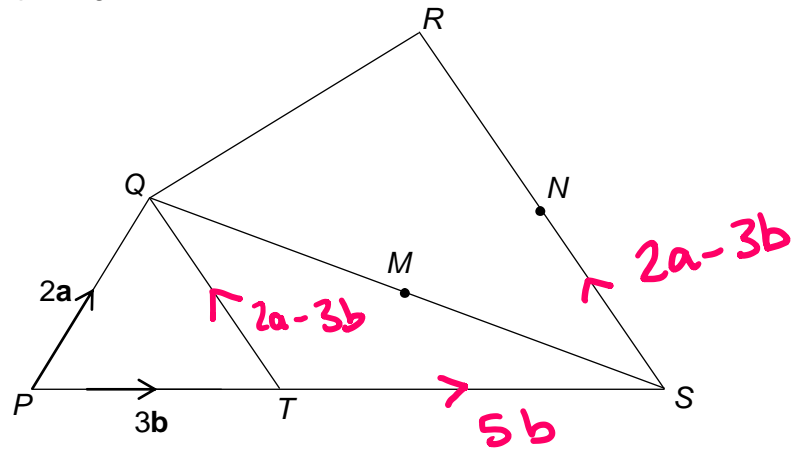
$\overrightarrow{PQ} = 2\mathbf{a}$ $\overrightarrow{PT} = 3\mathbf{b}$ $\overrightarrow{RS} = 2\overrightarrow{QT}$

PTS is a straight line with $PT : TS = 3 : 5$

M is the midpoint of *QS*.
N is the midpoint of *RS*.

Write $\overrightarrow{MN}$ in term of **a** and **b**.   **[4 marks]**

Handwritten working:

$TS = 5b$

$\overrightarrow{QT} = 2a - 3b$ $\overrightarrow{RS} = 2(2a - 3b)$
$\overrightarrow{SN} = 2a - 3b$ $\quad = 4a - 6b$

$\overrightarrow{QS} = \overrightarrow{QT} + \overrightarrow{TS}$
$\qquad = 3b - 2a + 5b$
$\qquad = 8b - 2a$
$\overrightarrow{MS} = \tfrac{1}{2}(8b - 2a)$
$\qquad = 4b - a$
$\overrightarrow{MN} = \overrightarrow{MS} + \overrightarrow{SN}$
$\qquad = 4b - a + 2a - 3b$

Answer $a + b$

Not drawn
accurately

$9\mathbf{b} - \mathbf{a}$

$6\mathbf{b}$

$2\mathbf{a}$
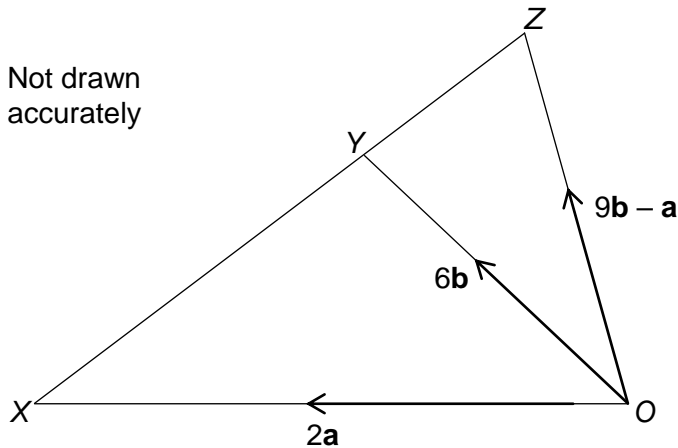
Prove, using vectors, that XYZ is a straight line.

[3 marks]

$$\overrightarrow{XY} = 6b - 2a$$
$$\overrightarrow{XZ} = 9b - a - 2a$$
$$= 9b - 3a$$
$$= \tfrac{3}{2}(6b - 2a)$$

$$\overrightarrow{XZ} = \tfrac{3}{2}XY$$
therefore XYZ is a straight line

Turn over ▶

8 $OXYZ$ is a parallelogram
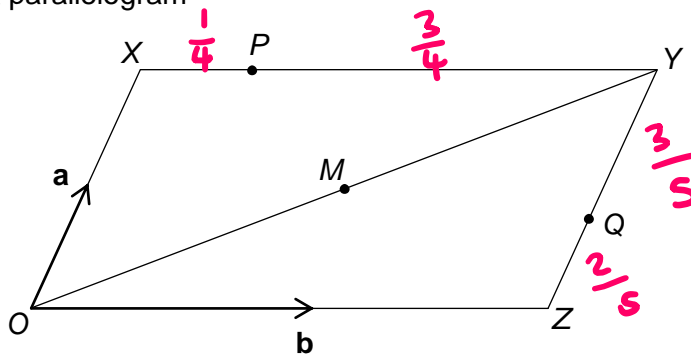
$\overrightarrow{OX}$ = **a** $\overrightarrow{OZ}$ = **b**

$XP : PY = 1 : 3$
$ZQ : QY = 2 : 3$

M is the midpoint of OY

8 (a) Write $\overrightarrow{PQ}$ in term of **a** and **b**.   **[2 marks]**

$$\overrightarrow{PQ} = \overrightarrow{PY} + \overrightarrow{YQ}$$
$$= \frac{3}{4}b - \frac{3}{5}a$$

Answer _____ $\frac{3}{4}b - \frac{3}{5}a$ _____

8 (b) Write $\overrightarrow{MQ}$ in term of **a** and **b**.   **[3 marks]**

$$\overrightarrow{MQ} = \overrightarrow{MY} + \overrightarrow{YQ}$$
$$= \frac{1}{2}(\overrightarrow{OY}) + \overrightarrow{YQ}$$
$$= \frac{1}{2}(a+b) - \frac{3}{5}a$$
$$= \frac{1}{2}a + \frac{1}{2}b - \frac{3}{5}a$$

Answer _____ $\frac{1}{2}b - \frac{1}{10}a$ _____

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9
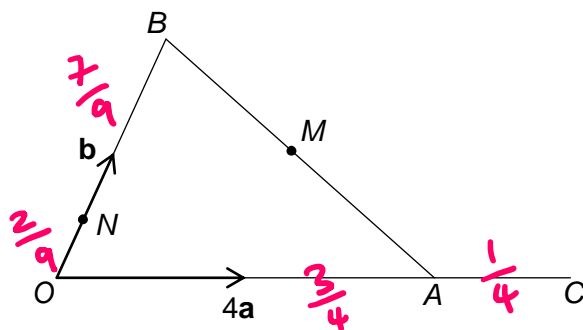
$$\overrightarrow{OA} = 4\mathbf{a} \qquad \overrightarrow{OB} = \mathbf{b}$$

$OA : OC = 3 : 4$

$ON : OB = 2 : 9$

M is the midpoint of AB

9 (a)   Write $\overrightarrow{MC}$ in term of **a** and **b**.

[3 marks]

$$\overrightarrow{MC} = \overrightarrow{MA} + \overrightarrow{AC}$$
$$= \tfrac{1}{2}\overrightarrow{BA} + \overrightarrow{AC}$$
$$= \tfrac{1}{2}(4a - b) + \tfrac{4}{3}a$$
$$= 2a - \tfrac{1}{2}b + \tfrac{4}{3}a$$

Answer $\dfrac{10}{3}a - \dfrac{1}{2}b$

9 (b)   Write $\overrightarrow{NM}$ in term of **a** and **b**.

[2 marks]

$$\overrightarrow{NM} = \overrightarrow{NB} + \overrightarrow{BM}$$
$$= \tfrac{7}{9}b + 2a - \tfrac{1}{2}b$$

Answer $\dfrac{5}{18}b + 2a$

10

Turn over ▶

10   *OXYZ* is a parallelogram

X $\frac{1}{3}$ P $\frac{2}{3}$ Y

M $\frac{2}{7}$

Not drawn accurately

a

$\frac{5}{7}$

Q

O   **b**   Z
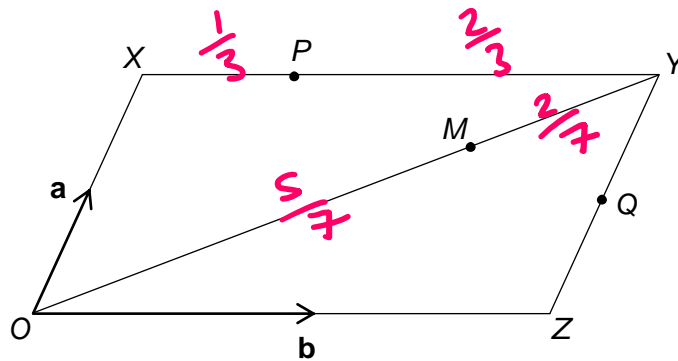
$\overrightarrow{OX} = \mathbf{a}$ $\overrightarrow{OZ} = \mathbf{b}$

ZQ = QY
XP : PY = 1 : 2
OM : MY = 5 : 2

Prove, using vectors, that *PMQ* is a straight line.   **[4 marks]**

$$\overrightarrow{PM} = \overrightarrow{PY} + \overrightarrow{YM}$$
$$= \overrightarrow{PY} + \tfrac{2}{7}(\overrightarrow{YO})$$
$$= \tfrac{2}{3}b + \tfrac{2}{7}(-b - a)$$
$$= \tfrac{2}{3}b - \tfrac{2}{7}b - \tfrac{2}{7}a$$
$$= \tfrac{8}{21}b - \tfrac{2}{7}a$$

$$\overrightarrow{PQ} = \overrightarrow{PY} + \overrightarrow{YQ}$$
$$= \tfrac{2}{3}b - \tfrac{1}{2}a$$

$$\overrightarrow{PQ} = \tfrac{7}{4}\overrightarrow{PM}$$

therefore PMQ is a straight line

11   *OACB* is a trapezium
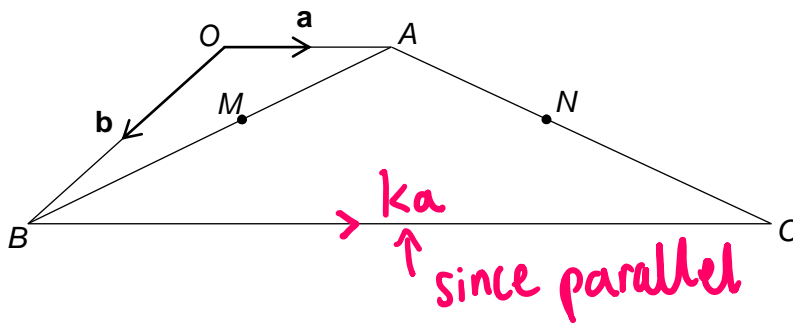
$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

M and *N* are the midpoints of *AB* and *AC*.

Prove, using vectors, that *MN* is parallel to *OA*.   **[4 marks]**

$$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN}$$
$$= \tfrac{1}{2}\overrightarrow{BA} + \tfrac{1}{2}\overrightarrow{AC}$$
$$= \tfrac{1}{2}(a - b) + \tfrac{1}{2}(-a + b + ka)$$
$$= \tfrac{1}{2}a - \tfrac{1}{2}b - \tfrac{1}{2}a + \tfrac{1}{2}b + \tfrac{k}{2}a$$
$$= \tfrac{k}{2}a$$

$$\overrightarrow{MN} = \tfrac{k}{2}\overrightarrow{OA}$$

therefore they are parallel

8

12   *ABCD* and *CDEF* are trapeziums

$\mathbf{4a}$ $\mathbf{b}$ $\mathbf{3a}$ $\mathbf{2a}$

Not drawn accurately
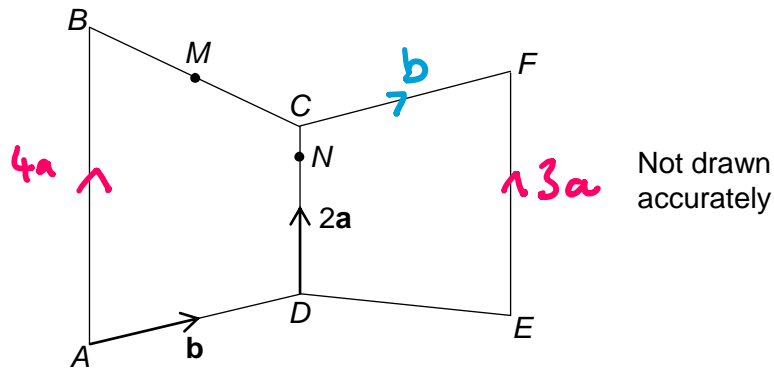
$\overrightarrow{DC} = 2\mathbf{a}$ $\overrightarrow{AD} = \overrightarrow{CF} = \mathbf{b}$

$AB : DC : EF = 4 : 2 : 3$
M is the midpoint of *BC.*
N is on the line *CD.*

MNE is a straight line.

$DN : NC = k : 1$, where k is an integer.

Work out the value of k.   **[5 marks]**

$\overrightarrow{MN} = \overrightarrow{MC} + \overrightarrow{CN}$

$= \tfrac{1}{2}(BC) + \overrightarrow{CN}$

$= \tfrac{1}{2}(-4a + b + 2a) - \alpha(2a)$

$= \tfrac{1}{2}b - a - 2\alpha a$

$= \tfrac{1}{2}b - (2\alpha + 1)a$

$\overrightarrow{NE} = \overrightarrow{NC} + \overrightarrow{CF} + \overrightarrow{FE}$

$= \alpha(2a) + b - 3a$

$= b - (3 - 2\alpha)a$

$\overrightarrow{NE} = 2\overrightarrow{MN}$

$3 - 2\alpha = 2(2\alpha + 1)$

$3 - 2\alpha = 4\alpha + 2$

$\alpha = \tfrac{1}{6}$

so $NC = \tfrac{1}{6}DC$

$DC : NC = 6 : 1$

$DN : NC = 5 : 1$

$k = $ ___5___

13 *POS* is a triangle.

$\overrightarrow{OS} = \overrightarrow{OR} + \overrightarrow{RS}$
$= 3b - 2a + \frac{3}{2}b$
$= \frac{9}{2}b - 2a$
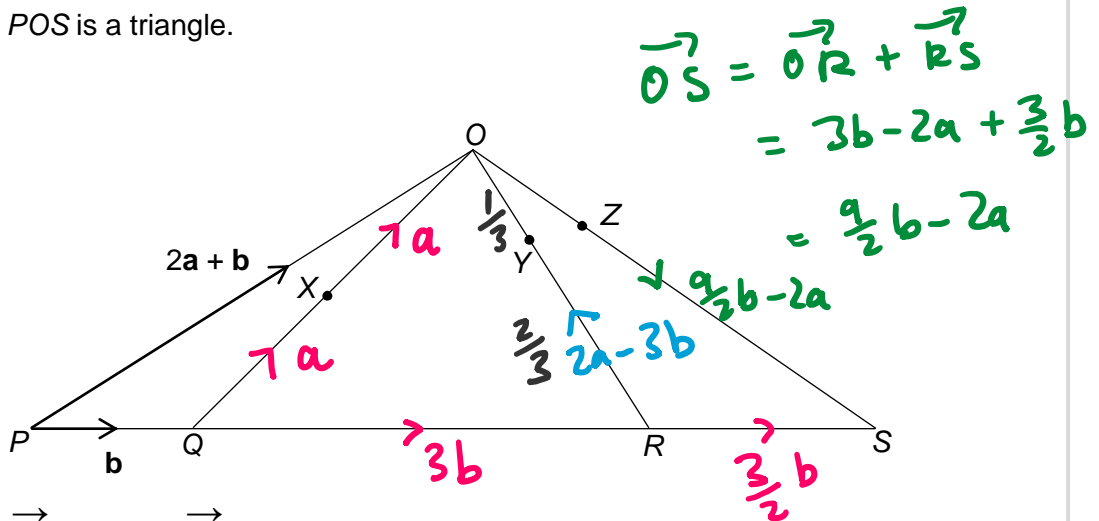
$\overrightarrow{PQ} = \mathbf{b}$ $\qquad$ $\overrightarrow{PO} = 2\mathbf{a} + \mathbf{b}$

X is the midpoint of *QO*
OY : *YR* = 1 : 2
PQ : *QR* : *RS* = 2 : 6 : 3

XYZ is a straight line.

OZ : *OS* = 1 : *k*

$\overrightarrow{RO} = \overrightarrow{RQ} + \overrightarrow{QO}$
$= -3b + 2a$
$= 2a - 3b$

Work out the value of *k*. **[6 marks]**

$\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY}$
$= a - \frac{1}{3}(2a - 3b)$
$= a - \frac{2}{3}a + b$
$= b + \frac{1}{3}a$

$\overrightarrow{XZ} = n(\overrightarrow{XY})$
$= n(b + \frac{1}{3}a)$
$= nb + \frac{n}{3}a$

$1 - \frac{2}{k} = \frac{n}{3}$
$3 - \frac{6}{k} = n$

and $\frac{9}{2k} = n$

$\overrightarrow{XZ} = \overrightarrow{XO} + \overrightarrow{OZ}$
$= a + \frac{1}{k}(\frac{9}{2}b - 2a)$
$= a + \frac{9}{2k}b - \frac{2}{k}a$
$= (1 - \frac{2}{k})a + \frac{9}{2k}b$

equate

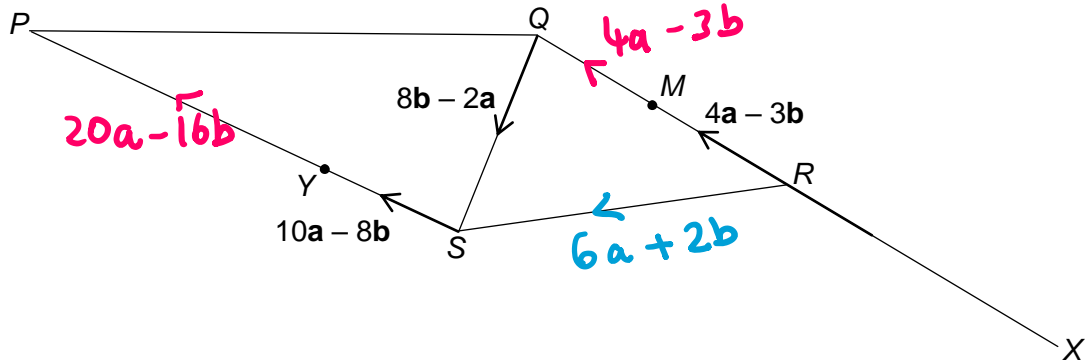$3 - \frac{6}{k} = \frac{9}{2k}$
$3k - 6 = \frac{9}{2}$
$6k - 12 = 9$
$k = \frac{21}{6} = \frac{7}{2}$

$k = \underline{\quad 3.5 \quad}$

11

Turn over ▶

14   *PQRS* is a quadrilateral.

$$\overrightarrow{SY} = 10\mathbf{a} - 8\mathbf{b} \qquad \overrightarrow{QS} = 8\mathbf{b} - 2\mathbf{a} \qquad \overrightarrow{RM} = 4\mathbf{a} - 3\mathbf{b}$$

RM = MQ
SY : YP = 1 : 2
QRX is a straight line.
XS is parallel to *RP*.

Work out *XS* : *RP*
Give your answer in the form n : 1   **[6 marks]**

(diagram annotations):
$4a - 3b$
$20a - 16b$
$6a + 2b$
$8\mathbf{b} - 2\mathbf{a}$
$4\mathbf{a} - 3\mathbf{b}$
$10\mathbf{a} - 8\mathbf{b}$

$$\overrightarrow{RP} = \overrightarrow{RQ} + \overrightarrow{QS} + \overrightarrow{SP}$$
$$= 8a - 6b + 8b - 2a + 30a - 24b$$
$$= 36a - 22b$$

$$\overrightarrow{XS} = \overrightarrow{XR} + \overrightarrow{RS} \qquad\qquad \overrightarrow{XS} = n\,\overrightarrow{RP}$$
$$= K(4a - 3b) + 6a + 2b \qquad = 36n\,a - 22n\,b$$
$$= (4K + 6)a - (3K - 2)b$$

$$4K + 6 = 36n \quad (\times 3)$$
$$3K - 2 = 22n \quad (\times 4) \qquad\qquad n = \frac{26}{20}$$
$$12K + 18 = 108n$$
$$\underline{12K - 8 = 88n} \qquad\qquad n = \frac{13}{10}$$
$$26 = 20n$$

Answer _____ $1.3 : 1$ _____

6