The Binomial Distribution

SCAN ME

REVISE THIS TOPIC

1 The random variable $X \sim \mathrm{B}(30, 0.4)$

Find

(i) $\mathrm{P}(X = 18)$ **(1)**

(ii) $\mathrm{P}(X \leq 10)$ **(1)**

(iii) $\mathrm{P}(X < 10)$ **(1)**

(iv) $\mathrm{P}(X \geq 13)$ **(2)**

(v) $\mathrm{P}(X > 13)$ **(2)**

(i) $\mathrm{P}(X = 18) = 0.01294$

(ii) $\mathrm{P}(X \leq 10) = 0.2915$

(iii) $\mathrm{P}(X < 10) = \mathrm{P}(X \leq 9) = 0.1763$

(iv) $\mathrm{P}(X \geq 13) = 1 - \mathrm{P}(X \leq 12) = 1 - 0.5784\ldots = 0.4215$

(v) $\mathrm{P}(X > 13) = 1 - \mathrm{P}(X \leq 13) = 1 - 0.7145\ldots = 0.2855$

(Total for Question 1 is 7 marks)

2 The random variable $Y \sim \mathrm{B}(12, 0.65)$

Find

(i) $\mathrm{P}(Y = 7.8)$ **(1)**

(ii) $\mathrm{P}(Y < 7.8)$ **(1)**

(iii) $\mathrm{P}(5 < Y < 11)$ **(2)**

(iv) $\mathrm{P}(3 \leq Y < 9)$ **(2)**

(v) $\mathrm{P}(4 \leq Y \leq 6)$ **(2)**

(i) $\mathrm{P}(Y = 7.8) = 0$

(ii) $\mathrm{P}(Y < 7.8) = \mathrm{P}(X \leq 7) = 0.4167$

(iii) $\mathrm{P}(5 < Y < 11) = \mathrm{P}(X \leq 10) - \mathrm{P}(X \leq 5) = 0.9575\ldots - 0.08463\ldots = 0.8729$

(iv) $\mathrm{P}(3 \leq Y < 9) = \mathrm{P}(X \leq 8) - \mathrm{P}(X \leq 2) = 0.6533\ldots - 0.0008479\ldots = 0.6525$

(v) $\mathrm{P}(4 \leq Y \leq 6) = \mathrm{P}(X \leq 6) - \mathrm{P}(X \leq 3) = 0.2127\ldots - 0.005609\ldots = 0.2071$

(Total for Question 2 is 8 marks)

3 The random variable $X \sim B(60, 0.6)$

k is the smallest integer for which $P(X \leq k) > 0.9$

Find $P(X = k)$ **(3)**

$P(X \leq 40) = 0.8830\ldots$

$P(X \leq 41) = 0.9281\ldots$

$P(X \leq 42) = 0.9587\ldots$

so $41 \leq k \leq 60$

Smallest value of $k = 41$

(Total for Question 3 is 3 marks)

4 Bill flips a coin until he gets three heads in a row.
Let X be the number of times that Bill flips the coin.

(a) Explain why X cannot be modelled with a binomial distribution. **(1)**

Jack selects 5 cards at random from a deck of 52 cards.
Let Y be the number of cards in Jack's hand that are spades.

(b) Explain why Y cannot be modelled with a binomial distribution. **(1)**

(a) The number of trials is not fixed. X could be any number that is 3 or greater.

(b) The probability of selecting a spade on the second card depends on what was drawn previously. Therefore, the trials are not independent.

(Total for Question 4 is 2 marks)

5 Penny has a biased coin that can land on either head or tails.

For Penny's coin, P(Heads) = 0.78

Penny flips the coin 15 times and records the number of times it lands on heads.

(a) Find the probability that the coin lands on heads
(i) exactly 11 times **(1)**
(ii) less than 8 times **(1)**
(iii) more than 12 times **(2)**

Franc also has a biased coin that can land on either heads or tails.

For Franc's coin, P(Heads) = 0.6

Franc flips the coin n times and records the number of times it lands on heads.

0.1 < P(Franc throws less than 10 heads) < 0.4

(b) Find a set of possible values for n **(4)**

(a) Let X be the number of heads recorded by Penny then $X \sim B(15, 0.78)$

(i) $P(X = 11) = 0.2079$

(ii) $P(Y < 8) = P(X \leq 7) = 0.007826$

(iii) $P(X > 12) = 1 - P(X \leq 12) = 1 - 0.6730\ldots = 0.3269$

(b) Let Y be the number of heads recorded by Franc then $Y \sim B(n, 0.6)$

If $n = 16$, $P(Y < 10) = P(X \leq 9) = 0.4728\ldots$ ❌

If $n = 17$, $P(Y < 10) = P(X \leq 9) = 0.3594\ldots$ ✅

If $n = 18$, $P(Y < 10) = P(X \leq 9) = 0.2631\ldots$ ✅

If $n = 19$, $P(Y < 10) = P(X \leq 9) = 0.1860\ldots$ ✅

If $n = 20$, $P(Y < 10) = P(X \leq 9) = 0.1275\ldots$ ✅

If $n = 21$, $P(Y < 10) = P(X \leq 9) = 0.08492\ldots$ ❌

$\{n \in \mathbb{Z} : 17 \leq n \leq 20\}$

(Total for Question 5 is 8 marks)

6 Skye and Ava both practice penalty kicks for their football team.

After taking a large amount of penalty kicks, they find that

P(Skye scores a penalty kick) = 0.85
P(Ava scores a penalty kick) = 0.81

During football training on Monday, both players take one penalty kick.

(a) Find the probability that at least one of the players scores their penalty kick. **(2)**

During football training on Tuesday, both players take 10 penalty kicks.

(b) State suitable binomial distributions to model the number of penalty kicks scored by each of Skye and Ava. **(2)**

(c) Find

(i) the probability that Skye scores fewer than 6 of her penalty kicks. **(1)**

(ii) the probability that Ava scores more than 7 of her penalty kicks. **(2)**

(iii) the probability that both Skye and Ava score exactly 8 of their penalty kicks. **(3)**

Skye says:

"When I miss a few penalties in a row, I feel pressure and I am more likely to miss the next one"

(d) Comment on the suitability of using the binomial distribution to model the number of penalties scored by Skye. **(1)**

(a) P(at least one scores) = 1 – P(both miss) = 1 – (0.15 × 0.19) = 0.9715

(b) Let X be the number of goals scored by Skye then $X \sim B(10, 0.85)$

Let Y be the number of goals scored by Ava then $Y \sim B(10, 0.81)$

(c) (i) $P(X < 6) = P(X \leq 5) = 0.009874$

(ii) $P(Y > 7) = 1 - P(Y \leq 7) = 1 - 0.2922\ldots = 0.7078$

(iii) P(both score 8) = $P(X = 8 \text{ and } Y = 8) = 0.2758\ldots \times 0.3010\ldots = 0.08305$

(d) It is no longer suitable as the trials are not independent of each other.

(Total for Question 6 is 11 marks)

7 A fair dice has the numbers 1, 2, 3, 4, 5 and 6.
A game involves rolling five identical fair dice.

A player wins the game if they roll the number 6 on two or more of the dice.

(a) State a suitable binomial distribution to model the number dice showing the number 6 **(1)**

(b) Calculate the probability that a player wins the game. **(2)**

Felix plays the game seven times.

(c) State a suitable binomial distribution to model the number of times Felix that wins the game. **(1)**

(d) Find the probability that Felix wins the game

(i) all seven times **(1)**

(ii) more than once **(2)**

(iii) either three or four times **(2)**

(a) Let X be the number of dice showing a 6 then $X \sim B(5, \frac{1}{6})$

(b) $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.8037\ldots = 0.1962$

(c) Let Y be the number times Felix wins the game then $Y \sim B(7, 0.1962\ldots)$

(d) (i) $P(Y = 7) = 0.00001121$

(ii) $P(Y > 1) = 1 - P(Y \leq 1) = 1 - 0.5870\ldots = 0.4129$

(iii) $P(Y = 3 \text{ or } Y = 4) = P(Y = 3) + P(Y = 4) = 0.1103\ldots + 0.02695\ldots = 0.1374$

(Total for Question 7 is 9 marks)

8 A supermarket runs a promotion for its customers.

When reaching the checkout, customers are asked to pick an integer from 1 to 100.
The cashier then generates a random integer between 1 and 100 using a random number generator.

If the numbers match, then the customer wins and does not have to pay for their shopping.

Between 3pm and 4pm, 350 customers take part in the promotion.

(a) State a suitable binomial distribution to model the number of customers that win and do not have to pay for their shopping between 3pm and 4pm. **(1)**

(b) Find the probability that between 3pm and 4pm
(i) exactly 3 customers win **(1)**
(ii) no more than 5 customers win **(1)**
(iii) more than 7 customers win **(2)**

Between the hours of 7pm and 8pm the store manager changes the promotion, so customers only need to select a number between 1 and 50.

Between 7pm and 8pm 630 customers take part in the promotion.

The store manager says *"I am 99% sure that no more than 20 customers will win"*

(c) Comment, with calculation, on this claim. **(3)**

(a) Let X be the number of customers who win between 3pm and 4pm then $X \sim B(350, 0.01)$

(b) (i) $P(X = 3) = 0.2166$

(ii) $P(X \leq 5) = 0.8586$

(iii) $P(X > 7) = 1 - P(X \leq 7) = 1 - 0.9739\ldots = 0.02606$

(c) Let Y be the number of customers who win between 7pm and 8pm then $Y \sim B(630, 0.02)$

$P(Y \leq 20) = 0.9823\ldots < 0.99$

The store manager can only claim to be 98.23% sure that no more than 20 customers will win.

(Total for Question 8 is 8 marks)

9 A biased dice can land on the numbers 1, 2, 3, 4 or 5.
The random variable X represents the number that the dice lands on.

X has the following probability distribution, where k is a constant.

x	1	2	3	4	5
$P(X = x)$	k	$3k$	$k + 0.1$	$2k$	$k + 0.02$

(a) Find $P(X < 4)$ **(3)**

The dice is rolled 40 times.
The random variable Y represents the number of times the dice roll is a number 2.

(b) State a suitable binomial distribution to model Y. **(1)**

(c) Find
(i) $P(Y = 13)$ **(1)**
(ii) $P(Y < 11)$ **(1)**
(iii) $P(Y \geq 17)$ **(2)**

(a) $k + 3k + k + 0.1 + 2k + k + 0.02 = 1$

$8k + 0.12 = 1$

$8k = 0.88$

$k = 0.11$

x	1	2	3	4	5
$P(X = x)$	0.11	0.33	0.21	0.22	0.13

$P(X < 4) = P(X \leq 3) = 0.11 + 0.33 + 0.21 = 0.65$

(b) Let Y be the number of times the dice roll is a number 2 then $Y \sim B(40, 0.33)$

(c) (i) $P(Y = 13) = 0.1334$

(ii) $P(Y < 11) = P(Y \leq 10) = 0.1829$

(iii) $P(Y \geq 17) = 1 - P(Y \leq 16) = 1 - 0.8657\ldots = 0.1343$

(Total for Question 9 is 8 marks)

10 To be able to drive, learners need to pass a theory test and then a practical test.

Data shows that 46% of attempts at the theory test result in pass and that 49% of attempts at the practical test result in a pass.

On Monday 60 learners take a theory test, and 28 learners take a practical test.

(a) State suitable binomial distributions to model the number of learners who pass each of the tests. **(2)**

(b) Use your models to find

(i) the probability that the number passing the theory test is exactly 26 **(1)**

(ii) the probability that the number passing the theory test is less than 30 **(1)**

(iii) the probability that the pass rate for the practical test was more than 75% **(2)**

On Tuesday 52 learners take the theory test.

The random variable A represents the number of learners who pass the theory test on Tuesday.

$P(A \leq k) < 0.15$

$P(A \geq k) < 0.99$

(c) Find a set of possible values for k. **(4)**

(a) Let X be the number of learners who pass the theory test on Monday then $X \sim B(60, 0.46)$

Let Y be the number of learners who pass the practical test on Monday then $Y \sim B(28, 0.49)$

(b) (i) $P(X = 26) = 0.09495$

(ii) $P(X < 30) = P(X \leq 29) = 0.6894$

(iii) $0.75 \times 28 = 21$

$P(Y > 21) = 1 - P(Y \leq 20) = 1 - 0.9953\ldots = 0.004642$

(c) Let A be the number of learners who pass the theory test on Tuesday then $A \sim B(52, 0.46)$

$P(A \leq 16) = 0.01837\ldots$ ✅ $P(A \geq 17) = 1 - P(A \leq 15) = 1 - 0.008693\ldots = 0.9913\ldots$ ❌

$P(A \leq 17) = 0.03583\ldots$ ✅ $P(A \geq 17) = 1 - P(A \leq 16) = 1 - 0.01837\ldots = 0.9816\ldots$ ✅

$P(A \leq 18) = 0.06474\ldots$ ✅ $P(A \geq 18) = 1 - P(A \leq 17) = 1 - 0.03583\ldots = 0.9641\ldots$ ✅

$P(A \leq 19) = 0.1088\ldots$ ✅ $P(A \geq 19) = 1 - P(A \leq 18) = 1 - 0.06474\ldots = 0.9352\ldots$ ✅

$P(A \leq 20) = 0.1707\ldots$ ❌ $P(A \geq 20) = 1 - P(A \leq 19) = 1 - 0.1088\ldots = 0.8911\ldots$ ✅

$\{k \in \mathbb{Z} : 17 \leq k \leq 19\}$

(Total for Question 10 is 8 marks)

11 A gameshow has 12 identical boxes.
The word "win" is written on the inside of 7 of the boxes and the word "lose" is written on the rest.

A contestant must select one of the boxes and open it.
All 12 boxes are then mixed up and the contestant repeats this process 4 more times.

X is the random variable representing the number of times that contestants open boxes that say "win".

(a) Write down a suitable distribution for X. **(1)**

(b) Find

(i) $P(X = 3)$ **(1)**

(ii) $P(X < 2)$ **(1)**

To win the gameshow the contestant must open at least 4 boxes that say "win".

(c) Find the probability that a contestant wins the game. **(2)**

During a season, the gameshow has 32 contestants.

(d) Find the probability that at least 10 of the contests win the game. **(3)**

(a) Let X be the number of times a contestant opens boxes saying "win" then $X \sim B(5, \frac{7}{12})$

(b) (i) $P(X = 3) = 0.3446$

(ii) $P(X < 2) = P(X \leq 1) = 0.1004$

(c) $P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.6912\ldots = 0.3087\ldots$

(d) Let Y be the number of contestants who win then $Y \sim B(32, 0.3087\ldots)$

$P(Y \geq 10) = 1 - P(Y \leq 9) = 1 - 0.4518\ldots = 0.5481$

(Total for Question 11 is 8 marks)

==================== BONUS AQA ONLY QUESTIONS ====================

12 Allan is attempting to complete a level in a computer game as fast as possible.

One level has a difficult jump to perform, so Allan practices this jump everyday. During a daily practice session, Allan always attempts this jump 80 times.

On each attempt, Allan successfully performs the jump with probability 0.72

The number of successful jumps in a daily practice session can be modelled using a binomial distribution.

12 (a) Find the mean of the number of times that Allan successfully performs the jump in a daily practice session. **[1 mark]**

Let X be the number of successful jumps during practice then $X \sim B(80, 0.72)$

Mean = $n \times p = 80 \times 0.72 = 57.6$

12 (b) Find the standard deviation of the number of times that Allan successfully performs the jump in a daily practice session. **[2 marks]**

Variance = $n \times p \times (1 - p) = 80 \times 0.72 \times 0.28 = 16.128$

Standard deviation = $\sqrt{16.128} = 4.016$

12 (c) Alan practices the jump 80 times per day, for 5 days in a row.

Find the probability that on each of these 5 days Allan performs the jump successfully at least 55 times. **[3 marks]**

$P(X \geq 55) = 1 - P(X \leq 54) = 1 - 0.2179\ldots = 0.7820\ldots$

Let Y be the number of days with at least 55 successful jumps

then $Y \sim B(5, 0.7820\ldots)$

$P(Y = 5) = 0.2926$

==================== BONUS AQA ONLY QUESTIONS ====================

13 The discrete random variables X and Y can be modelled by the distributions

$X \sim \mathrm{B}(a, 0.18)$
$Y \sim \mathrm{B}(b, 0.81)$

13 (a) Ross calculates the mean of X to be exactly 9.45
Explain why Ross must be incorrect. **[1 mark]**

Mean = $n \times p$

$9.45 = a \times 0.18$

$a = 9.45 \div 0.18 = 52.5$ a must be an integer so Ross cannot be correct.

13 (b) The standard deviation of Y is 3.42

Find $\mathrm{P}(Y \geq 64)$ **[5 marks]**

Variance = (Standard deviation)^2 = 3.42^2 = 11.6964

Variance = $n \times p \times (1 - p)$

$11.6964 = b \times 0.81 \times (1 - 0.81)$

$11.6964 = 0.1539b$

$b = 76$

$Y \sim \mathrm{B}(76, 0.81)$

$\mathrm{P}(Y \geq 64) = 1 - \mathrm{P}(Y \leq 63) = 1 - 0.7074\ldots = 0.2926$

===================== BONUS AQA ONLY QUESTIONS =====================

14 The discrete random variables X and Y can be modelled by the distributions

$X \sim B(n, 0.48)$ where $n > 0$
$Y \sim B(120, p)$ where $p > 0$

mean of X = 3 × mean of Y
variance of X = 2 × variance of Y

Find the values of n and p. **[6 marks]**

$n \times 0.48 = 3 \times 120 \times p$

$0.48n = 360p$

$n = 750p$

$n \times 0.48 \times (1 - 0.48) = 2 \times 120 \times p \times (1 - p)$

$750p \times 0.48 \times (1 - 0.48) = 2 \times 120 \times p \times (1 - p)$

$187.2p = 240p - 240p^2$

$240p^2 + 187.2p - 240p = 0$

$240p^2 - 52.8p = 0$

$p(240p - 52.8) = 0$

$p = 0$ $\quad$ $240p - 52.8 = 0$

but $p > 0$ so disregard $\quad$ $p = 52.8 \div 240$

$\underline{p = 0.22}$

$n = 750p$

$n = 750 \times 0.22$

$\underline{n = 165}$