Measures of Location/Spread

SCAN ME

REVISE THIS TOPIC

1 Four teams took part in a quiz that had seven different rounds.
The table below summarises the results for each of the rounds.

	Team 1	Team 2	Team 3	Team 4
Round 1 Score	8	6	6	8
Round 2 Score	7	6	1	9
Round 3 Score	5	2	8	9
Round 4 Score	6	8	10	9
Round 5 Score	6	8	5	8
Round 6 Score	6	5	5	8
Round 7 Score	5	6	3	5

(a) Work out the median score during round 3. **(1)**

(b) By considering the range, determine which team had the most consistent scores. **(2)**

(c) By considering the interquartile range, determine which team has the most consistent scores. **(2)**

(a) 2 5 8 9 Answer: 6.5

(b) Team 1 = 8 – 5 = 3 Team 2 = 8 – 2 = 6 Team 3 = 10 – 1 = 9 Team 4 = 9 – 5 = 4

Team 1 had the most consistent scores as they had the lowest range.

(c) Team 1 = 7 – 5 = 2 Team 2 = 8 – 5 = 3 Team 3 = 8 – 3 = 5 Team 4 = 9 – 8 = 1

Team 4 had the most consistent scores as they had the lowest interquartile range.

(Total for Question 1 is 5 marks)

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2 There are 70 teachers who work at school A and 50 teachers who work at school B.

A sample of 11 teachers from school A are selected and their ages are shown below.

26 43 50 31 48 x 39 39 30 26 42

The interquartile range of the sample is 14.

(a) Explain why the median age of the sample must be 39. **(1)**

(b) Work out two possible values for x. **(2)**

A sample of 7 teachers from school B are selected and their ages are shown below.

24 29 33 35 36 40 42

(c) Compare the ages of teachers who work at school A with the ages of teachers working at school B. **(2)**

(a) The median will be the 6th data item.

If x is > 39 then the second 39 will be in the 6th position.

If x is < 39 then the first 39 will be in the 6th position.

Either way 39 will be in the 6th position, and therefore the median.

(b) 26 26 30 31 39 39 42 43 44 48 50

26 26 29 30 31 39 39 42 43 48 50

x = 29 or 44

(c) 24 29 33 35 36 40 42

Median for school B = 35 IQR for school B = 40 – 29

= 11

The median age of teachers at school A (39) is higher than that of school B (35)

so they are older on average.

The IQR of ages of teachers at school A (14) is higher than that of school B (11)

so their ages were less consistent/more varied

(Total for Question 2 is 5 marks)

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3 A traffic officer recorded the speeds of 2400 vehicles on a motorway.

They recorded the speeds of cars as they passed under a bridge until they had 800 cars from each of the three lanes.

The table below shows information about the speeds of the vehicles in mph.

	Lane 1	**Lane 2**	**Lane 3**
Lowest Speed	55	57	63
Q_1	58	59	64
Q_2	59	62	65
Q_3	60	64	70
Greatest Speed	61	69	71

(a) Name the type of sampling method used by the traffic officer. **(1)**

(b) Work out the lane for which the speeds of the vehicles were most consistent.
Give a reason for your answer. **(2)**

(c) Write down the percentage of the vehicles in lane 1 that were travelling below 58 mph. **(1)**

(d) Work out how many of the vehicles were travelling more than 64 mph. **(2)**

(a) Quota Sampling

(b) Lane 1 = 60 – 58 = 2 Lane 2 = 64 – 59 = 5 Lane 3 = 70 – 64 = 6

The speeds of vehicles in lane 1 are most consistent as they have the lowest interquartile range.

(c) 25%

(d) 25% of vehicles in lane 2 and 75% of vehicles in lane 3.

= 200 + 600

= 800

(Total for Question 3 is 6 marks)

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4 The table below shows the GCSE grades of Year 12 students taking A-Level maths at a college.

GCSE Grade	Frequency (f)	Cumulative Frequency
6	8	8
7	17	25
8	14	39
9	12	51

(a) Work out the range of the GCSE grades. **(1)**

(b) Work out the median GCSE grade. **(1)**

(c) Work out the lower quartile of the GCSE grades. **(1)**

(d) Work out the upper quartile of the GCSE grades. **(1)**

(a) $9 - 6 = 3$

(b) $n = 51$ Median is in 26th position. Median = grade 8

(c) $n = 51$ Q_1 is in 13th position. Q_1 = grade 7

(d) $n = 51$ Q_3 is in 39th position. Q_3 = grade 8

(Total for Question 4 is 4 marks)

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5 A company makes chocolate bars using two different machines.
The table below summarises information about the masses of 300 chocolate bars made on machine A.

Mass of Chocolate Bar (m grams)	$116 \leq m < 118$	$118 \leq m < 120$	$120 \leq m < 122$	$122 \leq m < 124$
Frequency (Machine A)	44	176	75	5
Cumulative Frequency	44	220	295	300

(a) Use linear interpolation to calculate an estimate for the median mass of the 300 chocolate bars made by machine A. Give your answer to 2 decimal places. **(2)**

(b) Use linear interpolation to calculate an estimate for the interquartile range of the 300 chocolate bars made by machine A. Give your answer to 2 decimal places. **(3)**

The company also records the masses of 300 chocolate bars made on machine B.

For machine B: $Q_1 = 117.9$ g $Q_2 = 120.1$ g $Q_3 = 121.4$ g

The company requires that the chocolate bars produced are close to 120 g.
If the mass is too low, customers may complain. If the mass is too high, chocolate is being wasted.

(c) Using your answers to parts (a) and (b) suggest a reason why the company may prefer to use

(i) machine A
(ii) machine B **(3)**

(a) $\frac{300}{2} = 150^{th}$ position

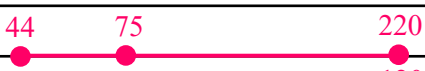


$\frac{150 - 44}{220 - 44} = \frac{x}{120 - 118}$

$x = 1.2045...$

$Q_2 = 119.20$ g

(b) $\frac{300}{4} = 75^{th}$ position

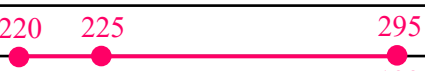


$\frac{75 - 44}{220 - 44} = \frac{x}{120 - 118}$

$x = 0.3522...$

$Q_1 = 118.35$ g

$\frac{3 \times 300}{4} = 225^{th}$ position



$\frac{225 - 220}{295 - 220} = \frac{x}{122 - 120}$

$x = 0.133...$

$Q_3 = 120.13$ g

$Q_3 - Q_1 = 120.13 - 118.35$

$= 1.78$ g

(c) IQR for machine B = 3.5 g therefore machine A has a lower IQR so less variation in mass.

The median for machine B is closer to the desired mass of 120g than the median for machine A.

(Total for Question 5 is 8 marks)

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6 The race times for runners of the Brighton Half Marathon are shown below.

Race Time (t minutes)	Frequency (f)	Cumulative Frequency
$60 \leq t < 90$	437	437
$90 \leq t < 120$	3657	4094
$120 \leq t < 150$	3053	7147
$150 \leq t < 180$	846	7993
$180 \leq t < 210$	152	8145
$210 \leq t < 240$	21	8166

The winner completed the race in 1 hour, 7 minutes and 17 seconds.

(a) Use linear interpolation to calculate an estimate for the median race time giving your answer in hours, minutes and seconds. **(2)**

The actual median time for the race was 1 hour, 59 minutes and 57 seconds.

(b) Give a reason why your answer to part (a) is different to the true median time. **(1)**

(c) Use linear interpolation to calculate an estimate for the 10th to 90th interpercentile range of the race times. Give your answer in hours, minutes and seconds. **(3)**

(a) $\frac{8166}{2} = 4083^{rd}$ position

437		4083	4094
90		$90 + x$	120

$\frac{4083 - 437}{4094 - 437} = \frac{x}{120 - 90}$

$x = 29.9097621$

$Q_2 = 119.9097621$ minutes

$Q_2 = 1$ hour 59 minutes 55 seconds

(b) Linear interpolation assumes that the times are uniformly distributed within the interval.

(c) $\frac{10 \times 8166}{100} = 816.6^{th}$ position

437		816.6		4094
90		$90 + x$		120

$\frac{816.6 - 437}{4094 - 437} = \frac{x}{120 - 90}$

$x = 3.114\ldots$ $P_{10} = 93.114$ mins

$\frac{90 \times 8166}{100} = 7349.4^{th}$ position

7147		7349.4		7993
150		$150 + x$		180

$\frac{7349.4 - 7147}{7993 - 7147} = \frac{x}{180 - 150}$

$x = 7.177\ldots$ $P_{90} = 157.177$ mins

$P_{90} - P_{10} = 157.177 - 93.114$

$= 64.063$

Answer: 1 hour 4 minutes 4 seconds

(Total for Question 6 is 6 marks)

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7 The following table summarises the maximum daily temperatures in a city, T °C to the nearest degree, recorded for the months May to October in a particular year.

Temperature (T °C)	13 – 17	18 – 22	23 – 27	28 – 32
Frequency (f)	25	32	70	57
Cumulative Frequency	25	57	127	184

(a) Use linear interpolation to calculate an estimate for the interquartile range for the temperatures. Give your answer to 2 decimal places. **(3)**

(b) Use linear interpolation to calculate an estimate for the 38th percentile for the temperatures. Give your answer to 2 decimal places. **(2)**

Sunny correctly calculates an estimate for the 85th percentile for the temperatures as 30.1 °C

Sunny claims "This means that 15% of the temperatures were greater than 30.1°C"

(c) Explain why Sunny's claim could be incorrect. **(1)**

(a) $\frac{184}{4} = 46^{th}$ position

25 — 46 — 57

17.5 — 17.5 + x — 22.5

$\frac{46 - 25}{57 - 25} = \frac{x}{22.5 - 17.5}$

$x = 3.28125$

$Q_1 = 20.78125$ °C

$\frac{3 \times 184}{4} = 138^{th}$ position

127 — 138 — 184

27.5 — 27.5 + x — 32.5

$\frac{138 - 127}{184 - 127} = \frac{x}{32.5 - 27.5}$

$x = 0.9649\ldots$

$Q_3 = 28.4649\ldots$ °C

$Q_3 - Q_1 = 28.4649 - 20.78125$

$= \mathbf{7.68}$ **°C**

(b) $\frac{38 \times 184}{100} = 69.92$ position

57 — 69.92 — 127

22.5 — 22.5 + x — 27.5

$\frac{69.92 - 57}{127 - 57} = \frac{x}{27.5 - 22.5}$

$x = 0.922\ldots$ $P_{38} = \mathbf{23.42}$ **°C**

(c) Sunny's answer is only an estimate, which assumes the data is distributed uniformly within the interval

(Total for Question 7 is 6 marks)

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8 The following table summarises the annual salaries, £S to the nearest £1000, of employees at a company in the year 2023.

Salary (£S)	20,000 – 29,000	30,000 – 39,000	40,000 – 49,000	50,000 – 59,000
Frequency (f)	25	15	6	2
Cumulative Frequency	25	40	46	48

(a) Use linear interpolation to calculate an estimate for the 20th to 80th interpercentile range of the annual salaries in 2023. Give your answer to the nearest pound. **(3)**

The company claims that between 2023 and 2024 they increased annual salaries by 10%.

For the 2024 salaries: $Q_1 = £27{,}000$ $Q_2 = £31{,}000$ $Q_3 = £41{,}000$

(b) Use linear interpolation to calculate Q_1, Q_2 and Q_3 for the salaries in 2023. **(3)**

(c) Comment on the company's claim. **(1)**

(a) $\frac{20 \times 48}{100} = 9.6^{th}$ position

0	9.6		25
19500	$19500 + x$		29500

$\frac{9.6 - 0}{25 - 0} = \frac{x}{29500 - 19500}$

$x = 3840$

$P_{20} = 19500 + 3840$

$P_{20} = £23340$

$\frac{80 \times 48}{100} = 38.4^{th}$ position

25		38.4	40
29500		$29500 + x$	39500

$\frac{38.4 - 25}{40 - 25} = \frac{x}{39500 - 29500}$

$x = 8933$

$P_{80} = 29500 + 8933$

$P_{80} = £38433$

$P_{80} - P_{20} = £38433 - £23340$

$= £15093$

(Total for Question 8 is 7 marks)

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Question 8 continued

$\frac{48}{4} = 12^{th}$ position

0 — 12 — 25

19500 — $19500 + x$ — 29500

$\frac{12 - 0}{25 - 0} = \frac{x}{29500 - 19500}$

$x = 4800$

$Q_1 = 19500 + 4800$

$Q_1 = £24300$

$\frac{48}{2} = 24^{th}$ position

0 — 24 — 25

19500 — $19500 + x$ — 29500

$\frac{24 - 0}{25 - 0} = \frac{x}{29500 - 19500}$

$x = 9600$

$Q_2 = 19500 + 9600$

$Q_2 = £29100$

$\frac{3 \times 48}{4} = 36^{th}$ position

25 — 36 — 40

29500 — $29500 + x$ — 39500

$\frac{36 - 25}{40 - 25} = \frac{x}{39500 - 29500}$

$x = 7333$

$Q_3 = 29500 + 7333$

$Q_3 = £36833$

For 2023	$Q_1 = £24{,}300$	$Q_2 = £29{,}100$	$Q_3 = £36{,}833$
Add 10% to each gives	$Q_1 = £26{,}730$	$Q_2 = £32{,}010$	$Q_3 = £40{,}516$
For 2024	$Q_1 = £27{,}000$	$Q_2 = £31{,}000$	$Q_3 = £41{,}000$

The lower quartile and upper quartile have both increased by more than 10%, however the median has not.

So the company's claim may be true for some but not all workers.

(Total for Question 8 is 7 marks)

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