



Rationalising the Denominator



REVISE THIS TOPIC



1 Show that $\frac{10}{\sqrt{5}}$ can be written in the form $a\sqrt{b}$ where a and b are integers.

$$\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5}$$

$$= 2\sqrt{5}$$

(Total for Question 1 is 2 marks)

2 Show that $\frac{18}{\sqrt{6}}$ can be written in the form $a\sqrt{b}$ where a and b are integers.

$$\frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{18\sqrt{6}}{6}$$

$$= 3\sqrt{6}$$

(Total for Question 2 is 2 marks)

3 Show that $\frac{70}{\sqrt{2}}$ can be written in the form $a\sqrt{b}$ where a and b are integers.

$$\frac{70}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{70\sqrt{2}}{2}$$

$$= 35\sqrt{2}$$

(Total for Question 3 is 2 marks)

4 Show that $\frac{20}{\sqrt{10}}$ can be written in the form $a\sqrt{b}$ where a and b are integers.

$$\frac{20}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{20\sqrt{10}}{10}$$

$$= 2\sqrt{10}$$

(Total for Question 4 is 2 marks)

For the entire booklet



5 Show that $\frac{24}{\sqrt{15}}$ can be written in the form $\frac{a\sqrt{15}}{b}$ where a and b are integers.

$$\begin{aligned}\frac{24}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}} &= \frac{24\sqrt{15}}{15} \\ &= \frac{8\sqrt{15}}{5}\end{aligned}$$

(Total for Question 5 is 2 marks)

6 Show that $\frac{35}{4\sqrt{5}}$ can be written in the form $\frac{a\sqrt{5}}{b}$ where a and b are integers.

$$\begin{aligned}\frac{35}{4\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} &= \frac{35\sqrt{5}}{20} \\ &= \frac{7\sqrt{5}}{4}\end{aligned}$$

(Total for Question 6 is 2 marks)

7 Show that $\frac{1}{9\sqrt{2}}$ can be written in the form $\frac{\sqrt{2}}{a}$ where a is an integer.

$$\frac{1}{9\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{18}$$

(Total for Question 7 is 2 marks)

8 Show that $\frac{60}{\sqrt{24}}$ can be written in the form $a\sqrt{b}$ where a and b are integers

$$\begin{aligned}\sqrt{24} &= \sqrt{4} \times \sqrt{6} \\ &= 2\sqrt{6}\end{aligned}\quad \begin{aligned}\frac{60}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} &= \frac{60\sqrt{6}}{12} \\ &= 5\sqrt{6}\end{aligned}$$

(Total for Question 8 is 2 marks)

9 Show that $\frac{24}{\sqrt{45}}$ can be written in the form $\frac{a\sqrt{5}}{b}$ where a and b are integers.

$$\begin{aligned}\sqrt{45} &= \sqrt{9} \times \sqrt{5} \\ &= 3\sqrt{5}\end{aligned}\quad \begin{aligned}\frac{24}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} &= \frac{24\sqrt{5}}{15} \\ &= \frac{8\sqrt{5}}{5}\end{aligned}$$

(Total for Question 9 is 2 marks)



10 Show that $\frac{10 - \sqrt{32}}{\sqrt{2}}$ can be written in the form $a\sqrt{2} - b$ where a and b are integers.

$$\begin{aligned}
 \frac{10 - \sqrt{32}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} &= \frac{\sqrt{2}(10 - \sqrt{32})}{2} \\
 &= \frac{10\sqrt{2} - \sqrt{64}}{2} \\
 &= \frac{10\sqrt{2} - 8}{2} = 5\sqrt{2} - 4
 \end{aligned}$$

(Total for Question 10 is 3 marks)

11 Show that $\frac{\sqrt{12} + 9}{\sqrt{3}}$ can be written in the form $a + b\sqrt{3}$ where a and b are integers.

$$\begin{aligned}
 \frac{\sqrt{12} + 9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} &= \frac{\sqrt{3}(\sqrt{12} + 9)}{3} \\
 &= \frac{\sqrt{36} + 9\sqrt{3}}{3} \\
 &= \frac{6 + 9\sqrt{3}}{3} = 2 + 3\sqrt{3}
 \end{aligned}$$

(Total for Question 11 is 3 marks)

12 Show that $\frac{\sqrt{180} + 40}{\sqrt{20}}$ can be written in the form $a + b\sqrt{5}$ where a and b are integers.

$$\begin{aligned}
 \sqrt{180} &= \sqrt{36} \times \sqrt{5} = 6\sqrt{5} \\
 \sqrt{20} &= \sqrt{4} \times \sqrt{5} = 2\sqrt{5} \\
 \frac{6\sqrt{5} + 40}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} &= \frac{\sqrt{5}(6\sqrt{5} + 40)}{10} \\
 &= \frac{30 + 40\sqrt{5}}{10} \\
 &= 3 + 4\sqrt{5}
 \end{aligned}$$

(Total for Question 12 is 3 marks)



13 Show that $\left(\frac{1}{\sqrt{2}}\right)^5$ can be written in the form $\frac{\sqrt{2}}{a}$ where a is an integer.

$$\frac{1^5}{(\sqrt{2})^5} = \frac{1}{4\sqrt{2}} \qquad \frac{1}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{8}$$

(Total for Question 13 is 3 marks)

14 Show that $\frac{24}{\sqrt{6}} + \sqrt{54}$ can be written in the form $k\sqrt{6}$ where k is an integer.

$$\frac{24}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{24\sqrt{6}}{6} \qquad \sqrt{54} = \sqrt{9 \times 6}$$

$$= 4\sqrt{6} \qquad = 3\sqrt{6}$$

$$4\sqrt{6} + 3\sqrt{6} = 7\sqrt{6}$$

(Total for Question 14 is 3 marks)

15 Show that $\frac{42}{\sqrt{18}} + \sqrt{200}$ can be written in the form $k\sqrt{2}$ where k is an integer.

$$\sqrt{18} = \sqrt{9 \times 2}$$

$$= 3\sqrt{2}$$

$$\sqrt{200} = \sqrt{100 \times 2}$$

$$= 10\sqrt{2}$$

$$\frac{42}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{42\sqrt{2}}{6}$$

$$= 7\sqrt{2}$$

$$7\sqrt{2} + 10\sqrt{2} = 17\sqrt{2}$$

(Total for Question 15 is 4 marks)



16 Show that $\frac{21}{\sqrt{3}} + \frac{12}{\sqrt{48}}$ can be written in the form $k\sqrt{3}$ where k is an integer.

$$\frac{21}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{21\sqrt{3}}{3}$$

$$= 7\sqrt{3}$$

$$\begin{aligned}\sqrt{48} &= \sqrt{16} \times \sqrt{3} \\ &= 4\sqrt{3}\end{aligned}$$

$$\frac{12}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{12}$$

$$= \sqrt{3}$$

$$7\sqrt{3} + \sqrt{3} = 8\sqrt{3}$$

(Total for Question 16 is 3 marks)

17 Show that $20 \times \sqrt{3\frac{1}{5}}$ can be written in the form $k\sqrt{5}$ where k is an integer.

$$20 \times \sqrt{\frac{16}{5}} = 20 \times \frac{\sqrt{16}}{\sqrt{5}}$$

$$= 20 \times \frac{4}{\sqrt{5}}$$

$$= \frac{80}{\sqrt{5}}$$

$$= \frac{80}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{80\sqrt{5}}{5}$$

$$= 16\sqrt{5}$$

(Total for Question 17 is 4 marks)

18 Show that $\frac{\sqrt{3} + \sqrt{5}}{\sqrt{2}} - \frac{5}{\sqrt{10}}$ can be written in the form $\frac{\sqrt{6}}{a}$ where a is an integer.

$$\frac{\sqrt{3} + \sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6} + \sqrt{10}}{2}$$

$$\frac{5}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{5\sqrt{10}}{10}$$

$$= \frac{\sqrt{10}}{2}$$

$$\frac{\sqrt{6} + \sqrt{10}}{2} - \frac{\sqrt{10}}{2}$$

$$= \frac{\sqrt{6} + \sqrt{10} - \sqrt{10}}{2}$$

$$= \frac{\sqrt{6}}{2}$$

(Total for Question 18 is 4 marks)



19 Show that $\frac{1}{3-\sqrt{2}}$ can be written in the form $\frac{a+\sqrt{2}}{b}$ where a and b are integers.

$$\begin{aligned}
 \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} &= \frac{3+\sqrt{2}}{(3-\sqrt{2})(3+\sqrt{2})} \\
 &= \frac{3+\sqrt{2}}{9+3\sqrt{2}-3\sqrt{2}-2} \\
 &= \frac{3+\sqrt{2}}{7}
 \end{aligned}$$

(Total for Question 19 is 3 marks)

20 Show that $\frac{7}{2+\sqrt{3}}$ can be written in the form $a-b\sqrt{3}$ where a and b are integers.

$$\begin{aligned}
 \frac{7}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} &= \frac{7(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} \\
 &= \frac{14-7\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-3} \\
 &= \frac{14-7\sqrt{3}}{1} \\
 &= 14-7\sqrt{3}
 \end{aligned}$$

(Total for Question 20 is 3 marks)

21 Show that $\frac{\sqrt{2}}{\sqrt{6}-2}$ can be written in the form $a\sqrt{3}+b\sqrt{2}$ where a and b are integers.

$$\begin{aligned}
 \frac{\sqrt{2}}{\sqrt{6}-2} \times \frac{\sqrt{6}+2}{\sqrt{6}+2} &= \frac{\sqrt{2}(\sqrt{6}+2)}{(\sqrt{6}-2)(\sqrt{6}+2)} &= \frac{\sqrt{4} \times \sqrt{3} + 2\sqrt{2}}{2} \\
 &= \frac{\sqrt{12} + 2\sqrt{2}}{6+2\sqrt{6}-2\sqrt{6}-4} &= \frac{2\sqrt{3} + 2\sqrt{2}}{2} \\
 &= \frac{\sqrt{12} + 2\sqrt{2}}{2} &= \sqrt{3} + \sqrt{2}
 \end{aligned}$$

(Total for Question 21 is 3 marks)



22 Show that $\frac{\sqrt{5}}{3-2\sqrt{2}}$ can be written in the form $a\sqrt{5} + b\sqrt{10}$ where a and b are integers.

$$\begin{aligned}
 \frac{\sqrt{5}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} &= \frac{\sqrt{5}(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})} \\
 &= \frac{3\sqrt{5} + 2\sqrt{10}}{9 + 6\sqrt{2} - 6\sqrt{2} - 8} \\
 &= \frac{3\sqrt{5} + 2\sqrt{10}}{1} \\
 &= 3\sqrt{5} + 2\sqrt{10}
 \end{aligned}$$

(Total for Question 22 is 4 marks)

23 Show that $\frac{\sqrt{45} + \sqrt{5}}{3 + \sqrt{10}}$ can be written in the form $a\sqrt{2} - b\sqrt{5}$ where a and b are integers.

$$\begin{aligned}
 \sqrt{45} &= \sqrt{9} \times \sqrt{5} \\
 &= 3\sqrt{5} \\
 \sqrt{45} + \sqrt{5} &= 3\sqrt{5} + \sqrt{5} \\
 &= 4\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 \frac{4\sqrt{5}}{3 + \sqrt{10}} \times \frac{3 - \sqrt{10}}{3 - \sqrt{10}} &= \frac{4\sqrt{5}(3 - \sqrt{10})}{(3 + \sqrt{10})(3 - \sqrt{10})} \\
 &= \frac{12\sqrt{5} - 4\sqrt{50}}{9 - 3\sqrt{10} + 3\sqrt{10} - 10} \\
 &= \frac{12\sqrt{5} - 4 \times \sqrt{25} \times \sqrt{2}}{-1} \\
 &= 20\sqrt{2} - 12\sqrt{5}
 \end{aligned}$$

(Total for Question 23 is 4 marks)

24 Show that $\frac{8 - \sqrt{12}}{5 - \sqrt{3}}$ can be written in the form $\frac{a - \sqrt{3}}{b}$ where a and b are integers.

$$\begin{aligned}
 \sqrt{12} &= \sqrt{4} \times \sqrt{3} \\
 &= 2\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \frac{8 - 2\sqrt{3}}{5 - \sqrt{3}} \times \frac{5 + \sqrt{3}}{5 + \sqrt{3}} &= \frac{(8 - 2\sqrt{3})(5 + \sqrt{3})}{(5 - \sqrt{3})(5 + \sqrt{3})} \\
 &= \frac{40 + 8\sqrt{3} - 10\sqrt{3} - 6}{25 + 5\sqrt{3} - 5\sqrt{3} - 3} \\
 &= \frac{34 - 2\sqrt{3}}{22} \\
 &= \frac{17 - \sqrt{3}}{11}
 \end{aligned}$$

(Total for Question 24 is 4 marks)

