

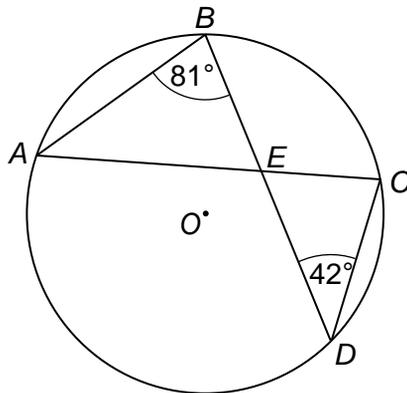


# Circle Theorems



← REVISE THIS TOPIC

1 A, B, C and D are points on a circle, centre O.



1 (a) Write down the size of angle CAB. [1 mark]

Answer 42 degrees

1 (b) Write down the size of angle ACD. [1 mark]

Answer 81 degrees

1 (c) Write down the size of angle AEB. [1 mark]

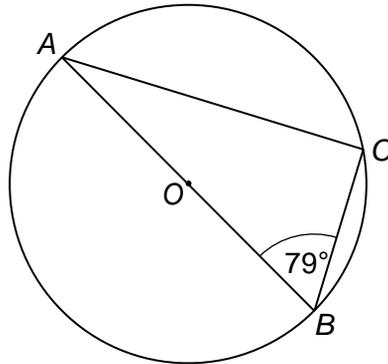
Answer 57 degrees

1 (d) Write down the size of angle BEC. [1 mark]

Answer 123 degrees



2 A, B, and C are points on a circle, centre O.



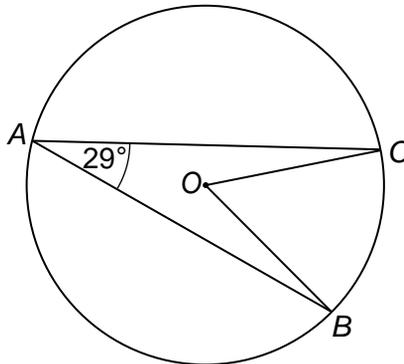
Work out the size of angle  $CAB$ .  
Give a reason for your answer.

[2 marks]

Answer 11 degrees

Reason The angle in a semicircle is  $90^\circ$   
Angles in a triangle add to  $180^\circ$

3 A, B, and C are points on a circle, centre O.



Work out the size of angle  $COB$ .  
Give a reason for your answer.

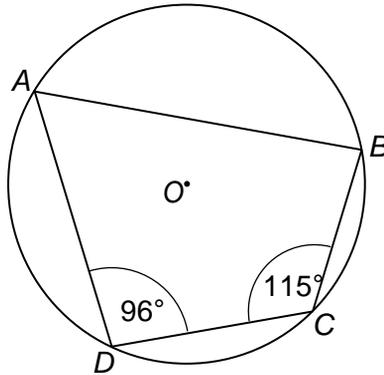
[2 marks]

Answer 58 degrees

Reason The angle at the centre is twice  
the angle at the circumference.



- 4  $A, B, C$  and  $D$  are points on a circle, centre  $O$ .



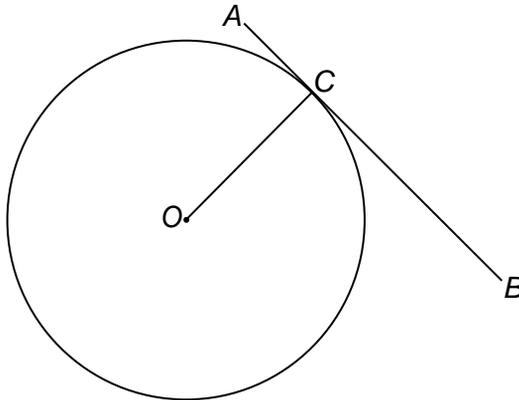
Work out the size of angle  $ABC$ .  
Give a reason for your answer.

[2 marks]

Answer 94 degrees

Reason Opposite angles in a cyclic quadrilateral add to  $180^\circ$

- 5  $A, B,$  and  $C$  are points on a circle, centre  $O$ .  
 $AB$  is a tangent.



Work out the size of angle  $OCB$ .  
Give a reason for your answer.

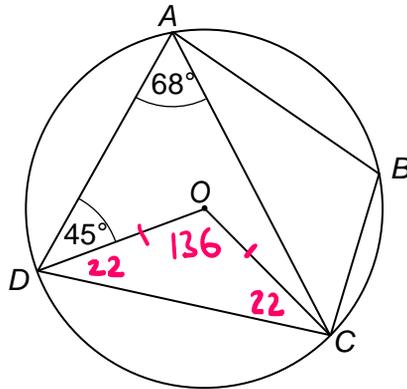
[2 marks]

Answer 90 degrees

Reason A tangent meets a radius at  $90^\circ$



6 A, B, C and D are points on a circle, centre O.



Work out the size of angle  $ABC$ .  
Give reasons for your answer.

[4 marks]

$$\text{Angle } DOC = 136^\circ$$

The angle at the centre is twice the angle at the circumference.

$$\text{Angle } ODC = \text{Angle } OCD = 22^\circ$$

Base angles in an isosceles triangle are equal

$$\begin{aligned}\text{Angle } ADC &= 22 + 45 \\ &= 67\end{aligned}$$

$$\begin{aligned}\text{Angle } ABC &= 180 - 67 \\ &= 113\end{aligned}$$

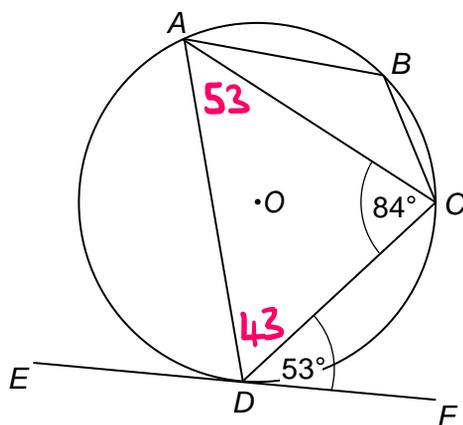
Opposite angles in a cyclic quadrilateral add to  $180^\circ$

Answer 113 degrees





7 A, B, C and D are points on a circle, centre O.  
EF is a tangent.



Work out the size of angle ABC.  
Give reasons for your answer.

[4 marks]

Angle DAC = 53°

Alternate segment theorem

Angle ADC = 43°

Angles in a triangle add to 180°

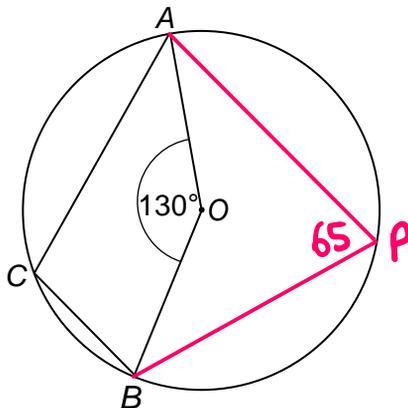
Angle ABC = 180 - 43  
= 137°

Opposite angles in a cyclic quadrilateral  
add to 180°

Answer 137 degrees



8 A, B, and C are points on a circle, centre O.



Work out the size of angle  $ACB$ .  
Give reasons for your answer.

[3 marks]

$$\text{Angle } APB = 65^\circ$$

The angle at the centre is twice the angle at the circumference.

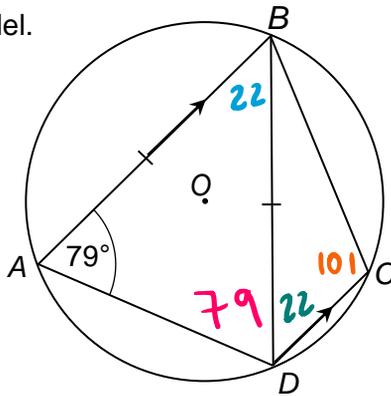
$$\begin{aligned}\text{Angle } ABC &= 180 - 65 \\ &= 115\end{aligned}$$

Opposite angles in a cyclic quadrilateral add to  $180^\circ$

Answer 115 degrees



- 9 A, B, C and D are points on a circle, centre O.  
 $BA = BD$   
AB and DC are parallel.



Work out the size of angle  $DBC$ .  
Give reasons for your answer.

[5 marks]

$$\text{Angle } BAD = \text{Angle } BDA = 79^\circ$$

Base angles in an isosceles triangle are equal

$$\text{Angle } ABD = 22^\circ$$

Angles in a triangle add to  $180^\circ$

$$\text{Angle } BDC = \text{Angle } ABD$$

Alternate angles are equal

$$\begin{aligned} \text{Angle } ABC &= 180 - 43 \\ &= 137^\circ \end{aligned}$$

Opposite angles in a cyclic quadrilateral add to  $180^\circ$

$$\text{Angle } DBC = 57^\circ$$

Angles in a triangle add to  $180^\circ$

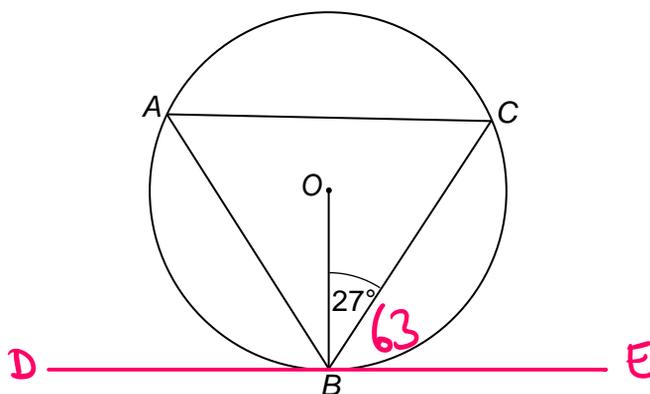
Answer

57

degrees



10 A, B, and C are points on a circle, centre O.



Work out the size of angle  $BAC$ .  
Give reasons for your answer.

[4 marks]

$$\begin{aligned} \text{Angle } CBE &= 90 - 27 \\ &= 63^\circ \end{aligned}$$

A tangent meets a radius at  $90^\circ$

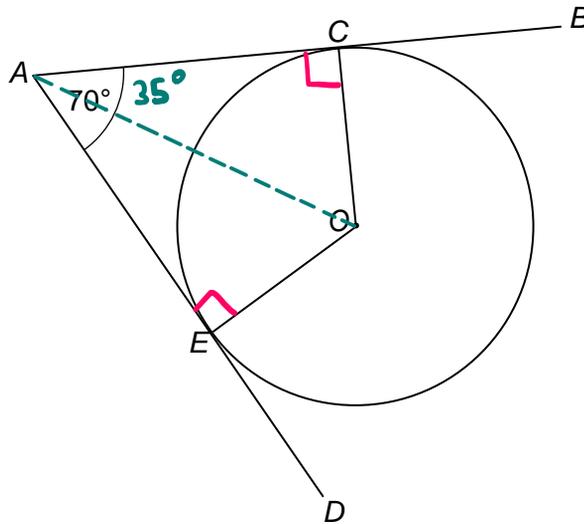
$$\text{Angle } BAC = 63^\circ$$

Alternate segment theorem

Answer 63 degrees



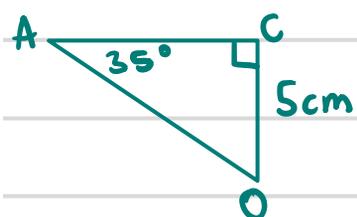
- 11 C and E are points on a circle, centre O.  
AB and AD are tangents.



- 11 (a) Work out the size of angle COE. [2 marks]

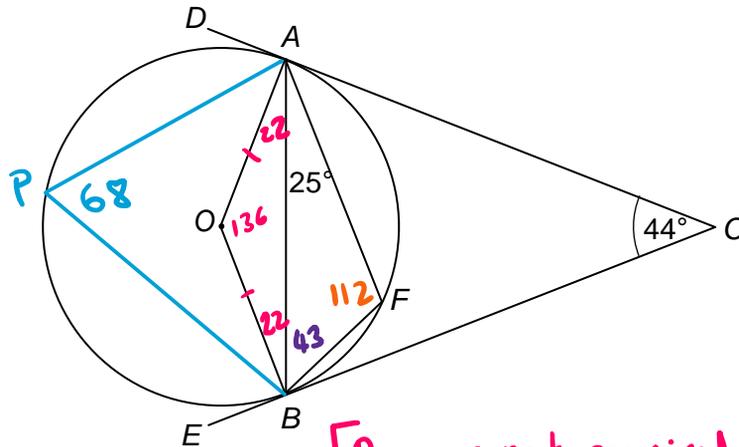
Angle AEO = Angle ACO =  $90^\circ$   
A tangent meets a radius at  $90^\circ$   
Angle COE =  $110^\circ$  Angles in a quadrilateral add to  $360^\circ$   
Answer 110 degrees

- 11 (b) OC = 5 cm  
Work out the length of CA to 1 decimal place. [2 marks]

Angle CAO =  $35^\circ$   $\tan(35) = \frac{5}{CA}$   
  
 $CA = \frac{5}{\tan(35)}$   
 $CA = 7.1407\dots$   
Answer 7.1 cm



- 12 A and B are points on a circle, centre O.  
DC and EC are tangents.



Work out the size of angle  $FBC$ .

[4 marks]

Angle  $AOB = 136^\circ$  A tangent meets a radius at  $90^\circ$   
and angles in a quadrilateral add to  $360^\circ$

Angle  $ABO = \text{Angle } BAO = 22^\circ$

Base angles in an isosceles triangle are equal

Angle  $APB = 68^\circ$

The angle at the centre is twice the  
angle at the circumference.

Angle  $AFB = 112^\circ$  Opposite angles in a  
cyclic quadrilateral add to  $180^\circ$

Angle  $ABF = 43^\circ$  Angles in a triangle add to  $180^\circ$

Angle  $FBC = 25^\circ$  A tangent meets a radius at  $90^\circ$

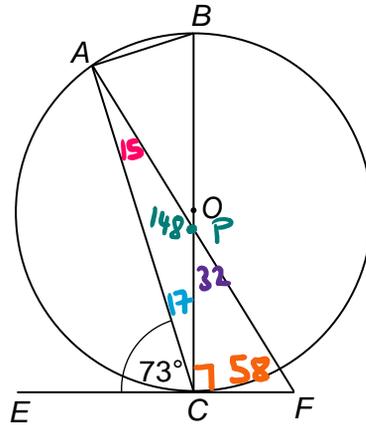
Answer

25

degrees



- 13 A, B, and C are points on a circle, centre O.  
EF is a tangent.  
Angle FAB = 5 × Angle CAF.



[Reasons not required in this Q]

Work out the size of angle AFC.

[4 marks]

Angle

$ACB = 90^\circ$  The angle in a semicircle is  $90^\circ$

$$90 \div 6 = 15$$

$$\text{Angle } CAF = 15^\circ$$

$$\text{Angle } ECB = 90^\circ \quad \text{Angle } ACB = 17^\circ$$

A tangent meets a radius at  $90^\circ$

Angle APC =  $148^\circ$  Angles in a triangle add to  $180^\circ$

Angle CPF =  $32^\circ$  Angles on a straight line add to  $180^\circ$

Angle AFC =  $58^\circ$  Angles in a triangle add to  $180^\circ$

Answer

58

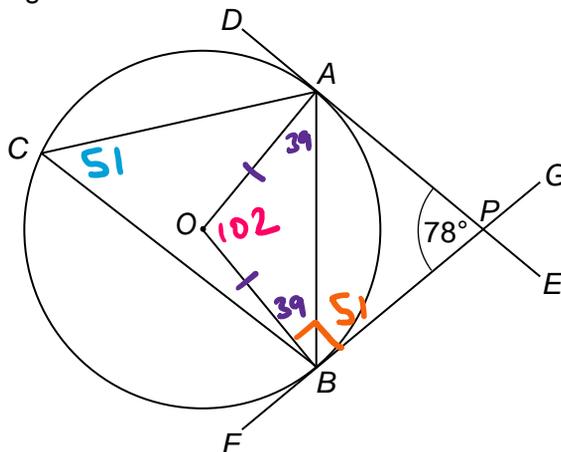
degrees

8

Turn over ►



- 14 A, B, and C are points on a circle, centre O.  
DE and FG are tangents.



[Reasons not required in this Q]

- 14 (a) Work out the size of angle ACB.

[2 marks]

Angle  $AOB = 102^\circ$  A tangent meets a radius at  $90^\circ$   
and angles in a quadrilateral add to  $360^\circ$

Angle  $ACB = 51^\circ$  Angle at the circumference is  
half the angle at the centre

Answer 51 degrees

- 14 (b) Work out the size of angle ABP.

[2 marks]

Angle  $ABO = \text{Angle } BAO = 39^\circ$

Base angles in an isosceles triangle are equal

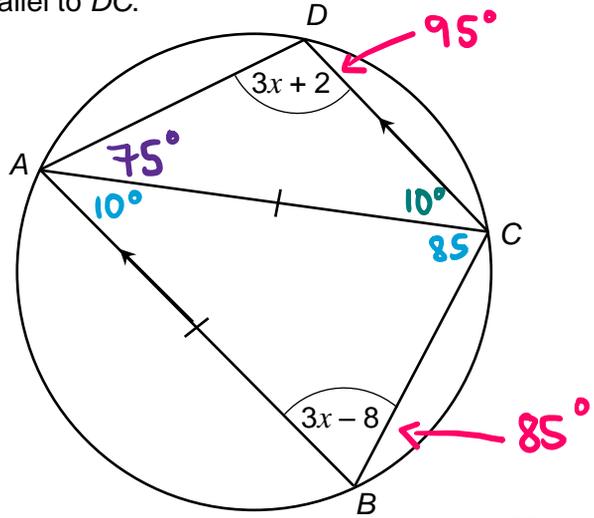
Angle  $ABP = 90 - 39$  A tangent meets  
 $= 51$  a radius at  $90^\circ$

Answer 51 degrees



15

$A, B, C$  and  $D$  are points on a circle, centre  $O$ .  
 $ABCD$  is a trapezium with  $AB$  parallel to  $DC$ .  
 $AB = AC$



Work out the size of angle  $DAC$ .

[5 marks]

[Reasons not required in this Q]

$$3x + 2 + 3x - 8 = 180^\circ$$

Opposite angles in a cyclic quadrilateral  
 add to  $180^\circ$

$$6x - 6 = 180$$

$$3(31) + 2 = 95$$

$$6x = 186$$

$$3(31) - 8 = 85$$

$$x = 31^\circ$$

Angle  $ACB =$  Angle  $ABC = 85$

Angle  $CAB = 10^\circ$  Angles in a triangle add to  $180^\circ$

Angle  $DCA =$  Angle  $CAB$  alternate angles are equal

Angle  $DAC = 75^\circ$  Angles in a triangle add to  $180^\circ$

Answer

75

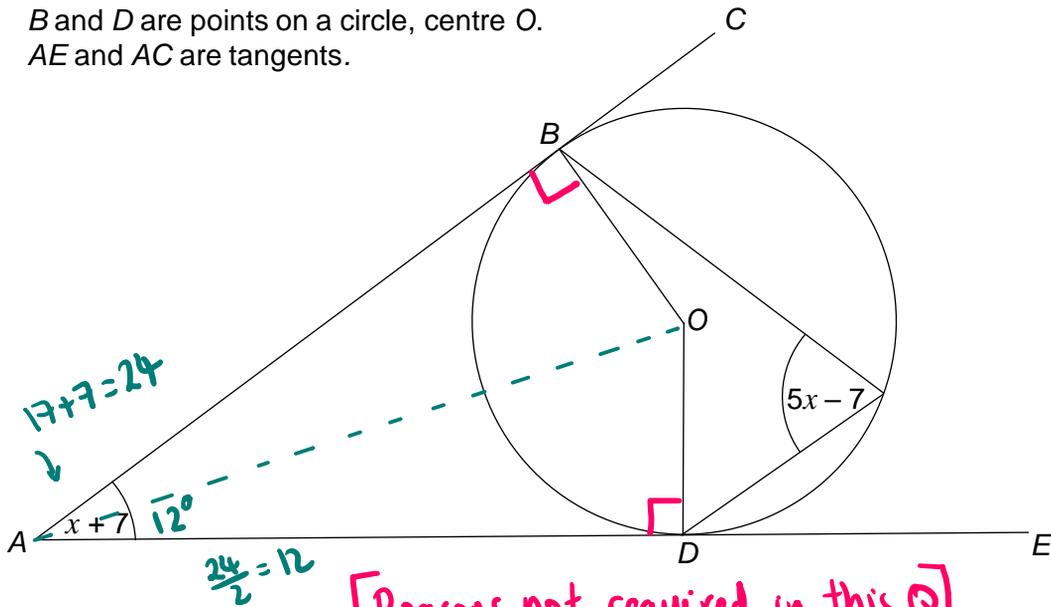
degrees

9



Turn over ▶

16 B and D are points on a circle, centre O.  
AE and AC are tangents.



16 (a) Work out the value of  $x$  [Reasons not required in this Q] [3 marks]

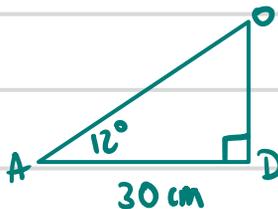
$$\begin{aligned} \text{Angle } BOD &= 2(5x-7) & 10x-14 &= 180-x-7 \\ \text{(twice angle at circumference)} & & 10x-14 &= 173-x \\ \text{Angle } BOD &= 180-(x+7) & 11x &= 187 \\ \text{(angles in quadrilateral add to } 360^\circ) & & x &= 17 \end{aligned}$$

Answer 17 degrees

16 (b)  $AD = 30$  cm  
Work out the length of  $OD$  to 3 significant figures. [3 marks]

$$\begin{aligned} \angle BAD &= 24^\circ \\ \angle OAD &= 12^\circ \end{aligned}$$

$$\tan(12) = \frac{OD}{30}$$



$$OD = 30 \tan(12)$$

Answer 3.38 cm

