

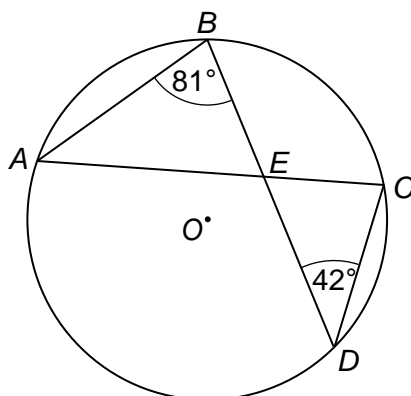


Circle Theorems



REVISE THIS
TOPIC

- 1 A, B, C and D are points on a circle, centre O .



- 1 (a) Write down the size of angle CAB . [1 mark]

Answer 42 degrees

- 1 (b) Write down the size of angle ACD . [1 mark]

Answer 81 degrees

- 1 (c) Write down the size of angle AEB . [1 mark]

Answer 57 degrees

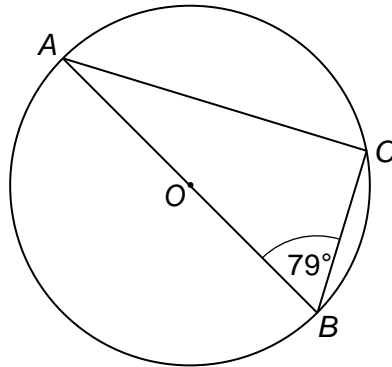
- 1 (d) Write down the size of angle BEC . [1 mark]

Answer 123 degrees





- 2 A, B, and C are points on a circle, centre O.



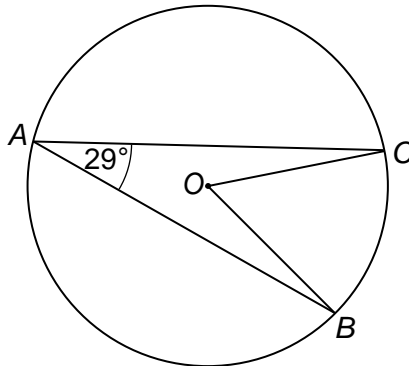
Work out the size of angle CAB .
Give a reason for your answer.

[2 marks]

Answer 11 degrees

Reason The angle in a semicircle is 90°
Angles in a triangle add to 180°

- 3 A, B, and C are points on a circle, centre O.



Work out the size of angle COB .
Give a reason for your answer.

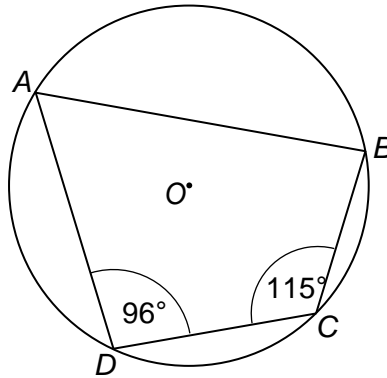
[2 marks]

Answer 58 degrees

Reason The angle at the centre is twice
the angle at the circumference.



- 4 A, B, C and D are points on a circle, centre O .



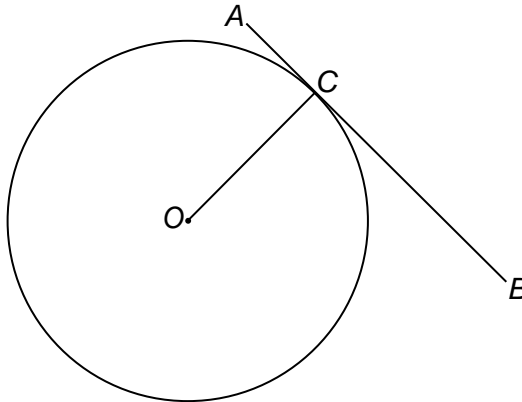
Work out the size of angle ABC .
Give a reason for your answer.

[2 marks]

Answer 94 degrees

Reason Opposite angles in a cyclic quadrilateral add to 180°

- 5 A, B , and C are points on a circle, centre O .
 AB is a tangent.



Work out the size of angle OCB .
Give a reason for your answer.

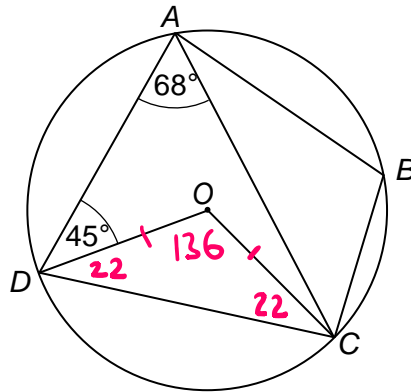
[2 marks]

Answer 90 degrees

Reason A tangent meets a radius at 90°



6 A, B, C and D are points on a circle, centre O.



Work out the size of angle ABC.
Give reasons for your answer.

[4 marks]

$$\text{Angle } DOC = 136^\circ$$

The angle at the centre is twice the angle at the circumference.

$$\text{Angle } ODC = \text{Angle } OCD = 22^\circ$$

Base angles in an isosceles triangle are equal

$$\begin{aligned}\text{Angle } ADC &= 22 + 45 \\ &= 67\end{aligned}$$

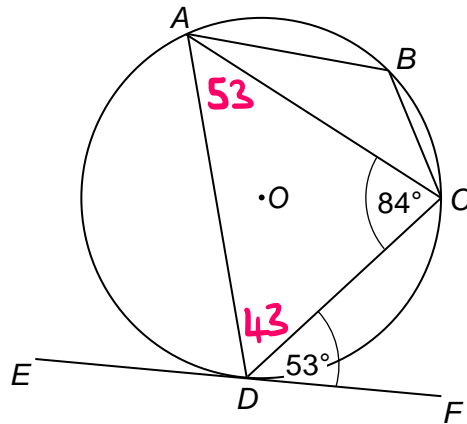
$$\begin{aligned}\text{Angle } ABC &= 180 - 67 \\ &= 113\end{aligned}$$

Opposite angles in a cyclic quadrilateral add to 180°

Answer 113 degrees



- 7 A, B, C and D are points on a circle, centre O.
EF is a tangent.



Work out the size of angle ABC.
Give reasons for your answer.

[4 marks]

$$\text{Angle DAC} = 53^\circ$$

Alternate segment theorem

$$\text{Angle ADC} = 43^\circ$$

Angles in a triangle add to 180°

$$\begin{aligned}\text{Angle ABC} &= 180 - 43 \\ &= 137^\circ\end{aligned}$$

Opposite angles in a cyclic quadrilateral
add to 180°

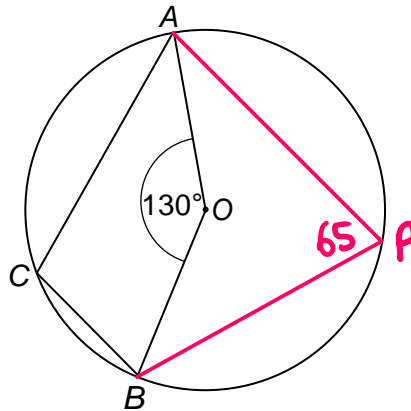
Answer

137

degrees



8 A, B, and C are points on a circle, centre O.



Work out the size of angle ACB.
Give reasons for your answer.

[3 marks]

$$\text{Angle APB} = 65^\circ$$

The angle at the centre is twice the angle at the circumference.

$$\begin{aligned}\text{Angle ABC} &= 180 - 65 \\ &= 115\end{aligned}$$

Opposite angles in a cyclic quadrilateral add to 180°

Answer 115 degrees

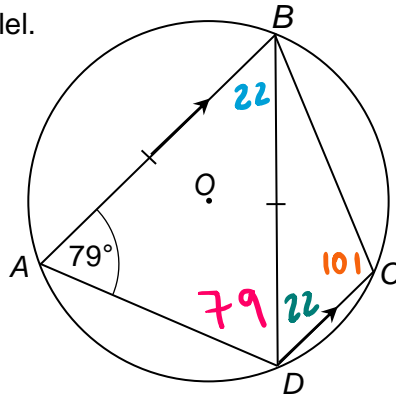


9

A, B, C and D are points on a circle, centre O.

$BA = BD$

AB and DC are parallel.



Work out the size of angle DBC .

Give reasons for your answer.

[5 marks]

Angle $BAD = \text{Angle } BDA = 79^\circ$

Base angles in an isosceles triangle are equal

Angle $ABD = 22^\circ$

Angles in a triangle add to 180°

Angle $BDC = \text{Angle } ABD$

Alternate angles are equal

Angle $ABC = 180 - 43$
 $= 137^\circ$

Opposite angles in a cyclic quadrilateral add to 180°

Angle $DBC = 57^\circ$

Angles in a triangle add to 180°

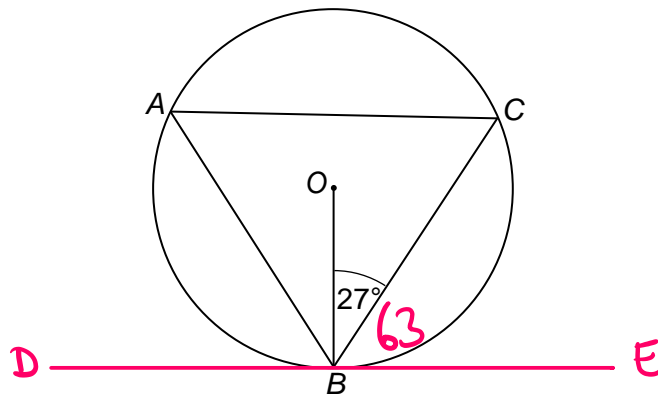
57

Answer

degrees



10 A, B, and C are points on a circle, centre O.



Work out the size of angle BAC.
Give reasons for your answer.

[4 marks]

$$\begin{aligned} \text{Angle } CBE &= 90 - 27 \\ &= 63^\circ \end{aligned}$$

A tangent meets a radius at 90°

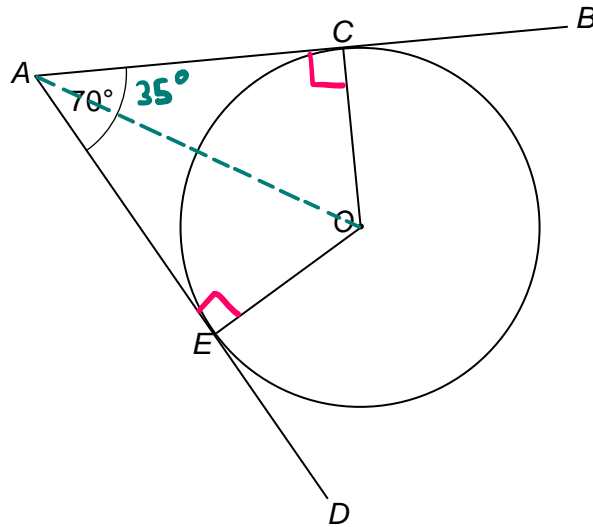
$$\text{Angle } BAC = 63^\circ$$

Alternate segment theorem

Answer 63 degrees



- 11 C and E are points on a circle, centre O.
AB and AD are tangents.



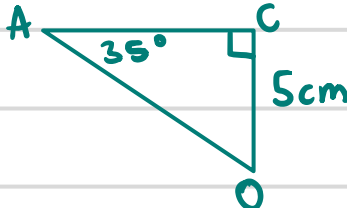
- 11 (a) Work out the size of angle COE.

[2 marks]

Angle AEO = Angle ACO = 90°
A tangent meets a radius at 90°
Angle COE = 110° Angles in a quadrilateral add to 360°
Answer 110 degrees

- 11 (b) OC = 5 cm
Work out the length of CA to 1 decimal place.

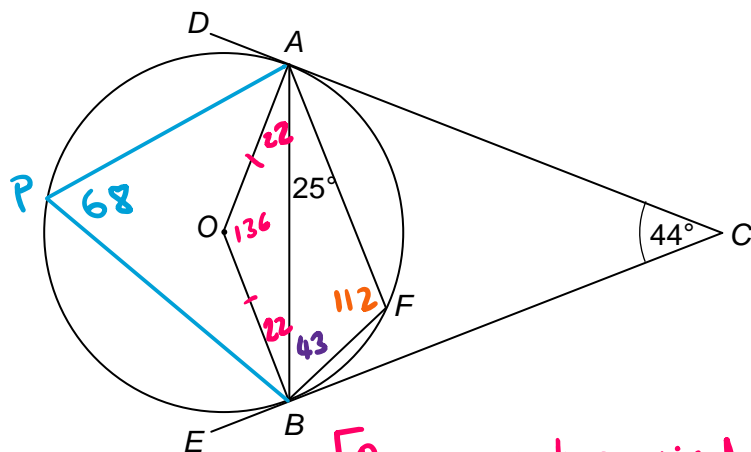
[2 marks]

Angle CAO = 35° $\tan(35) = \frac{5}{CA}$

 $CA = \frac{5}{\tan(35)}$
 $CA = 7.1407...$
Answer 7.1 cm



Turn over ►

- 12 A and B are points on a circle, centre O.
DC and EC are tangents.



[Reasons not required in this Q]

Work out the size of angle FBC.

[4 marks]

Angle AOB = 136° A tangent meets a radius at 90°
and angles in a quadrilateral add to 360°

Angle ABO = Angle BAO = 22°

Base angles in an isosceles triangle are equal

Angle APB = 68°

The angle at the centre is twice the angle at the circumference.

Angle AFB = 112° Opposite angles in a cyclic quadrilateral add to 180°

Angle ABF = 43° Angles in a triangle add to 180°

Angle FBC = 25° A tangent meets a radius at 90°

Answer

25

degrees

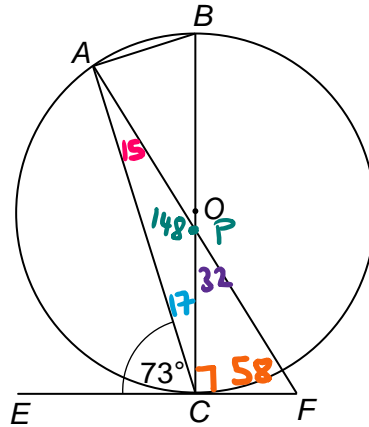


13

A, B, and C are points on a circle, centre O.

EF is a tangent.

Angle $FAB = 5 \times$ Angle CAF .



[Reasons not required in this Q]

Work out the size of angle AFC .

[4 marks]

Angle

$ACB = 90^\circ$ The angle in a semicircle is 90°

$90 \div 6 = 15$

Angle $CAF = 15^\circ$

Angle $ECB = 90^\circ$ Angle $ACB = 17^\circ$

A tangent meets a radius at 90°

Angle $APC = 148^\circ$ Angles in a triangle add to 180°

Angle $CPF = 32^\circ$ Angles on a straight line add to 180°

Angle $AFC = 58^\circ$ Angles in a triangle add to 180°

Answer

58

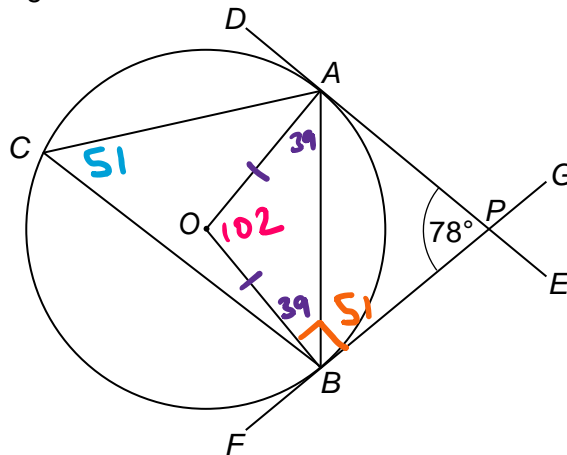
degrees

8

Turn over ►



- 14 A, B, and C are points on a circle, centre O.
DE and FG are tangents.



[Reasons not required in this Q]

- 14 (a) Work out the size of angle ACB.

[2 marks]

Angle AOB = 102° A tangent meets a radius at 90°
and angles in a quadrilateral add to 360°

Angle ACB = 51° Angle at the circumference is
half the angle at the centre

Answer 51 degrees

- 14 (b) Work out the size of angle ABP.

[2 marks]

Angle ABO = Angle BAO = 39°

Base angles in an isosceles triangle are equal

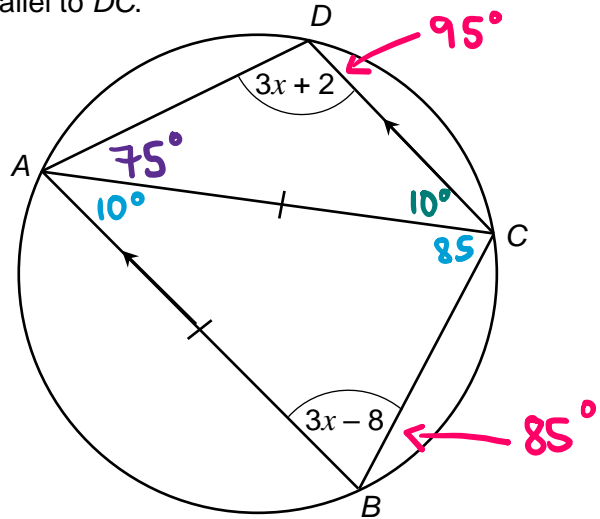
Angle ABP = $90 - 39$ A tangent meets
= 51 a radius at 90°

Answer 51 degrees



15

A, B, C and D are points on a circle, centre O.
ABCD is a trapezium with AB parallel to DC.
AB = AC



Work out the size of angle DAC.

[5 marks]

[Reasons not required in this Q]

$$3x + 2 + 3x - 8 = 180^\circ$$

Opposite angles in a cyclic quadrilateral
add to 180°

$$6x - 6 = 180$$

$$3(31) + 2 = 95$$

$$6x = 186$$

$$3(31) - 8 = 85$$

$$x = 31^\circ$$

$$\text{Angle ACB} = \text{Angle ABC} = 85$$

$$\text{Angle CAB} = 10^\circ \text{ Angles in a triangle add to } 180^\circ$$

$$\text{Angle DCA} = \text{Angle CAB} \text{ alternate angles are equal}$$

$$\text{Angle DAC} = 75^\circ \text{ Angles in a triangle add to } 180^\circ$$

Answer

75

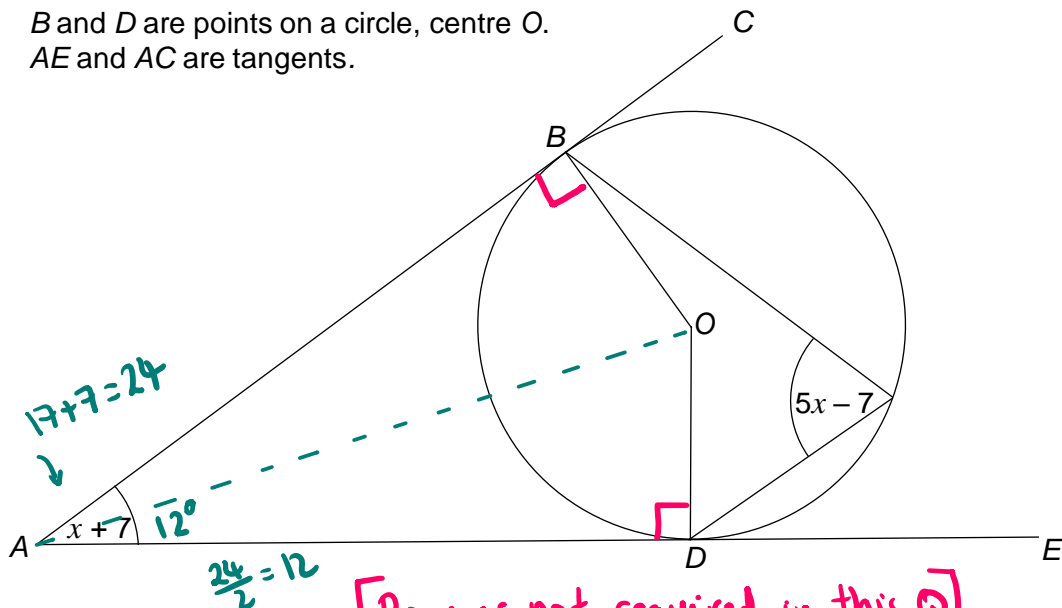
degrees

9

Turn over ►



- 16 B and D are points on a circle, centre O.
AE and AC are tangents.



- 16 (a) Work out the value of x [Reasons not required in this Q] [3 marks]

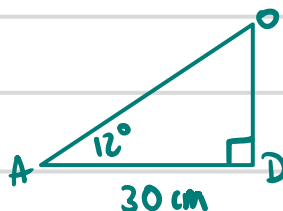
$$\begin{aligned} \text{Angle } BOD &= 2(5x - 7) & 10x - 14 &= 180 - x - 7 \\ (\text{twice angle at circumference}) & & 10x - 14 &= 173 - x \\ \text{Angle } BOD &= 180 - (x + 7) & 11x &= 187 \\ (\text{angles in quadrilateral add to } 360^\circ) & & x &= 17 \end{aligned}$$

Answer 17 degrees

- 16 (b) $AD = 30$ cm
Work out the length of OD to 3 significant figures. [3 marks]

$$\begin{aligned} \angle BAD &= 24^\circ \\ \angle ADE &= 12^\circ \end{aligned}$$

$$\tan(12) = \frac{OD}{30}$$



$$OD = 30 \tan(12)$$

Answer 3.38 cm

