



Composite Functions



REVISE THIS
TOPIC

1 $f(x) = 3x + 4$ $g(x) = x + 10$ $h(x) = x^2$



- 1 (a) Work out $fg(x)$.
Give your answer in the form $ax + b$ where a and b are integers

[2 marks]

$$\begin{aligned} fg(x) &= 3(x+10) + 4 \\ &= 3x + 30 + 4 \\ &= 3x + 34 \end{aligned}$$

$$fg(x) = 3x + 34$$

- 1 (b) Work out $gf(x)$.
Give your answer in the form $ax + b$ where a and b are integers

[2 marks]

$$\begin{aligned} gf(x) &= 3x + 4 + 10 \\ &= 3x + 14 \end{aligned}$$

$$gf(x) = 3x + 14$$

- 1 (c) Work out $gh(x)$

[1 mark]

$$gh(x) = x^2 + 10$$

$$gh(x) = x^2 + 10$$





2

$f(x) = x - 3$

$g(x) = x^2 + 1$

$h(x) = 10x$



2 (a)

Work out $fg(x)$

Fully simplify your answer.

[2 marks]

$$\begin{aligned} fg(x) &= x^2 + 1 - 3 \\ &= x^2 - 2 \end{aligned}$$

$$fg(x) = x^2 - 2$$

2 (b)

Work out $hg(x)$

Fully simplify your answer.

[2 marks]

$$\begin{aligned} hg(x) &= 10(x^2 + 1) \\ &= 10x^2 + 10 \end{aligned}$$

$$hg(x) = 10x^2 + 10$$

2 (c)

Work out $gh(x)$

Fully simplify your answer.

[2 marks]

$$\begin{aligned} gh(x) &= (10x)^2 + 1 \\ &= 100x^2 + 1 \end{aligned}$$

$$gh(x) = 100x^2 + 1$$





3

$f(x) = \frac{x}{4}$

$g(x) = 4x - 8$

$h(x) = \sqrt{x}$



3 (a)

Work out $fg(x)$

Fully simplify your answer.

[2 marks]

$$fg(x) = \frac{4x-8}{4}$$

$$= x - 2$$

$$fg(x) = x - 2$$

3 (b)

Work out $gf(x)$

Fully simplify your answer.

[2 marks]

$$gf(x) = 4\left(\frac{x}{4}\right) - 8$$

$$= x - 8$$

$$gf(x) = x - 8$$

3 (c)

Work out $hf(x)$.

Fully simplify your answer.

[2 marks]

$$hf(x) = \sqrt{\frac{x}{4}}$$

$$= \frac{\sqrt{x}}{\sqrt{4}}$$

$$hf(x) = \frac{\sqrt{x}}{2}$$





4 $f(x) = x - 5$

$g(x) = x^2 + 30$

- 4 (a) Work out $fg(x)$
Fully simplify your answer.

[2 marks]

$$fg(x) = x^2 + 30 - 5$$
$$= x^2 + 25$$

$$fg(x) = x^2 + 25$$

- 4 (b) Work out $fg(3)$

[2 marks]

$$fg(x) = x^2 + 25$$
$$fg(3) = 3^2 + 25$$
$$= 9 + 25$$

$$\text{Answer } 34$$

- 4 (c) Work out $gf(x)$
Give your answer in the form $ax^2 + bx + c$ where a , b and c are integers. [3 marks]

$$gf(x) = (x - 5)^2 + 30$$
$$= (x - 5)(x - 5) + 30$$
$$= x^2 - 5x - 5x + 25 + 30$$
$$= x^2 - 10x + 55$$

$$gf(x) = x^2 - 10x + 55$$





5

$f(x) = 2x + 1$

$g(x) = \sqrt{x + 3}$

5 (a) Work out $g(13)$

[1 mark]

$$g(13) = \sqrt{13 + 3}$$
$$= \sqrt{16}$$

Answer

4

5 (b) Work out $fg(13)$

[1 mark]

$$fg(13) = f(4)$$
$$= 2(4) + 1$$

Answer

9

5 (c) Work out $gf(16)$

[2 marks]

$$gf(x) = \sqrt{2x + 1 + 3}$$
$$= \sqrt{2x + 4}$$
$$gf(16) = \sqrt{2(16) + 4}$$
$$= \sqrt{36}$$

Answer

6





6

$f(x) = x + 2$

$g(x) = x^3$

$h(x) = \sqrt{x}$

6 (a) Work out $gf(3)$

[2 marks]

$$gf(x) = (x+2)^3$$
$$gf(3) = (3+2)^3$$
$$= 5^3$$

Answer 125

6 (b) Work out $gh(x)$ Give your answer in the form x^k where k is a fraction.

[2 marks]

$$gh(x) = (\sqrt{x})^3$$
$$= (x^{1/2})^3$$
$$= x^{3/2}$$

$gh(x) = x^{3/2}$

6 (c) Work out $gf(x)$ Give your answer in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

[3 marks]

$$gf(x) = (x+2)^3$$
$$= (x+2)(x+2)(x+2)$$
$$= (x^2 + 2x + 2x + 4)(x+2)$$
$$= (x^2 + 4x + 4)(x+2)$$
$$= x^3 + 2x^2 + 4x^2 + 8x + 4x + 8$$

$gf(x) = x^3 + 6x^2 + 12x + 8$





7

$f(x) = 2^x$

$g(x) = 1 - x$

$h(x) = 2 + x$

7 (a) Work out $gf(-3)$

[2 marks]

$$gf(x) = 1 - 2^x$$

$$gf(-3) = 1 - 2^{-3}$$

$$2^{-3} = \frac{1}{2^3}$$

$$1 - \frac{1}{8}$$

$$\frac{7}{8}$$

Answer

7 (b) $hg(x) - gh(x) = k$ where k is an integer.
Find the value of k .

[4 marks]

$$hg(x) = 2 + (1 - x)$$

$$= 2 + 1 - x$$

$$= 3 - x$$

$$gh(x) = 1 - (2 + x)$$

$$= 1 - 2 - x$$

$$= -1 - x$$

$$hg(x) - gh(x) = (3 - x) - (-1 - x)$$

$$= 3 - x + 1 + x$$

$$k = 4$$

7 (c) Show that $\frac{fh(x)}{fg(x)} = 2^{ax+b}$ where a and b are integers.

[3 marks]

$$fh(x) = 2^{2+x}$$

$$\frac{fh(x)}{fg(x)} = \frac{2^{2+x}}{2^{1-x}}$$

$$fg(x) = 2^{1-x}$$

$$= 2^{(2+x)-(1-x)}$$

$$= 2^{2+x-1+x}$$

$$= 2^{2x+1}$$

