Discrete Random Variables

SCAN ME

REVISE THIS TOPIC

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1 X and Y are independent discrete random variables with the following probability distributions.

x	1	2	3	4
$P(X = x)$	p	0.25	0.25	0.35

y	2	4	6	8
$P(Y = y)$	0.2	0.3	0.4	q

(a) Find the values of p and q **(2)**

(b) Find

(i) $P(X > 2)$ **(1)**

(ii) $P(Y < 6)$ **(1)**

(iii) $P(Y \leq 6)$ **(1)**

(iv) $P(X > Y)$ **(2)**

Z is a discrete random variable with values $z = 0, 1, 2, 3, 4$
The probability distribution for Z is a discrete uniform distribution.

(c) Find

(i) $P(Z = 3)$ **(1)**

(ii) $P(Z > 1)$ **(1)**

(Total for Question 1 is 9 marks)

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2 X is a random variable with the following probability distribution.

x	0	1	2	3	4	5	6
$P(X = x)$	p	q	r	0.14	0.16	0.19	0.21

(a) Find $P(4 \leq X < 6)$ **(1)**

$P(X > 1) = 0.88$
$P(X = 0) = 2 \times P(X = 1)$

(b) Find the values of p, q and r. **(3)**

The random variable $Y = X^2$

(c) Find
(i) $P(Y = 3)$ **(1)**
(ii) $P(Y > 4)$ **(1)**
(iii) $P(X < Y)$ **(1)**

(Total for Question 2 is 7 marks)

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3 X is a random variable with the following probability distribution.

x	5	6	7	8	9
$P(X = x)$	k	k	$3k$	$5k$	$5k$

(a) Find the value of k **(2)**

(b) Find

(i) $P(X \leq 6)$ **(1)**

(ii) $P(5 < X < 9)$ **(1)**

The random variable $Y = 4X$

n is an integer with $P(X + Y < n) > 0.5$

(c) Find the minimum possible value for n. **(3)**

(Total for Question 3 is 7 marks)

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4 X is a random variable with the following probability distribution.

x	1	2	3	4	5
$P(X = x)$	p	$p + q$	$p - q$	$4q$	$p + 0.08$

$P(X < 3) = 0.35$

(a) Find $P(X = 3)$ **(4)**

Y is a random variable with the following probability distribution.

y	1	2	3
$P(Y = y)$	a	$12b^2$	$4c$

The probability distribution for Y is a discrete uniform distribution.

(b) Given that a, b and c are all positive, find the value of $a + b + c$. **(4)**

(Total for Question 4 is 8 marks)

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5 X, Y and Z are independent discrete random variables with the following probability distributions.

x	1	2	3	4	5
$P(X = x)$	0.2	0.1	0.05	0.25	0.4

y	0	2	4	6	8
$P(Y = y)$	0.24	0.18	0.11	0.15	0.32

z	5	6	7	8
$P(Z = z)$	0.4	0.07	0.13	0.4

(a) Find

(i) $P(2 < X < 5)$ **(1)**

(ii) $P(Y > 2)$ **(1)**

(b) Find

(i) $P(X + Y > 11)$ **(2)**

(ii) $P(X + Z = 11)$ **(2)**

(iii) $P(Y > Z)$ **(3)**

(Total for Question 5 is 9 marks)

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6 A bag contains 5 blue counters, 1 red counter and 4 green counters.

Three counters are taken at random from the bag without replacement.

The discrete random variable X represents the number of red counters selected.
The discrete random variable Y represents the number of green counters selected.

(a) Find the complete probability distribution of X. **(3)**
(b) Find the complete probability distribution of Y. **(4)**

A different bag contains 4 blue counters and 1 red counter.

Tom randomly takes counters from the bag until he has taken out the red counter.
The discrete random variable M represents the number of counters Tom takes from the bag.

(c) Find the complete probability distribution for M. **(3)**

(Total for Question 6 is 10 marks)

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7 At the end of the year Hannah completes exams in three subjects.

The probability that Hannah passes each of the subjects is shown below.

Subject	Maths	Chemistry	History
Probability of passing	0.9	p	0.6

The outcome of each exam is independent of the others.
The probability that Hannah passes all three exams is 0.459

(a) Find the value of p. **(1)**

The discrete random variable X represents the number of exams that Hannah passes.

(b) Find the complete probability distribution of X. **(4)**

(Total for Question 7 is 5 marks)

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8 Kat plays a game at a fair that costs £5 per play.

The probability that Kat wins the game is 0.3

Kat has £20 and plays the game either until she has won the game twice or until she has no money left.

The discrete random variable X represents the number of times that Kat plays the game.

(a) Find the complete probability distribution of X. **(4)**

The discrete random variable Y represents the number of times that Kat wins the game.

(b) Find the complete probability distribution of Y. **(4)**

(Total for Question 8 is 8 marks)

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9 A biased dice can land on the numbers 1, 2, 3, 4, 5, or 6.

The random variable X represents the number that the dice lands on.

$P(X = r) = P(X = 7 - r)$ for $r = 1, 2, 3$

Given that $P(X = 3) = 0.05$ and $P(X = 2) = 2 \times P(X = 1)$

(a) Find the complete probability distribution of X. **(4)**

The dice is rolled 3 times.

The random variable Y represents the number of times that the dice lands on the number 2.

(b) Find the complete probability distribution of Y. **(4)**

(Total for Question 9 is 8 marks)

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10 X is a random variable with the following probability distribution.

x	1	2	3	4	5
$P(X = x)$	0.12	0.16	0.2	0.24	0.28

$Y = X^2$
$Z = 2X$

(a) Find
(i) $P(X < 3)$ **(1)**
(ii) $P(Y < 3)$ **(1)**
(iii) $P(2 < Z \leq 10)$ **(1)**

(b) Find
(i) $P(Y = Z)$ **(1)**
(ii) $P(Y - 4X = -3)$ **(3)**
(iii) $P(Y + 8 \leq 3Z)$ **(3)**

(Total for Question 10 is 10 marks)

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11 The random variable X has a probability function

$$P(X = x) = kx^2 \qquad x = 1, 2, 3$$

where k is a constant.

(a) Find the value of k **(2)**

(b) Find $P(X = 3)$ **(2)**

The random variable Y has a probability function

$$P(Y = y) = (c - 3)y \qquad y = 1, 2, 3, 4$$

where c is a constant.

(c) Find the value of c **(2)**

(d) Find $P(Y = 2)$ **(2)**

(Total for Question 11 is 8 marks)

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12 The random variable X has a probability function

$$P(X = x) = (k + 1)x \qquad x = 1, 2, 3$$

where k is a constant.

(a) By working out the value of k, find the complete probability distribution of X. **(4)**

The random variable Y has a probability function

$$P(Y = y) = \frac{c - y^2}{100} \qquad y = 1, 2, 3, 4$$

where c is a constant.

(b) By working out the value of c, find the complete probability distribution of Y. **(4)**

(Total for Question 12 is 8 marks)

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13 The random variable X has a probability function

$$P(X = x) = \frac{1}{k - x} \qquad x = 1, 3$$

where k is a constant.

Find the value of k giving your answer in the form $a + \sqrt{b}$, where a and b are integers. **(5)**

(Total for Question 13 is 5 marks)

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14 The random variable X has a probability function

$$P(X = x) = kx - c \qquad x = 5, 6, 7, 8$$

where k and c are constants.

Given that $P(X = 5) = 0.01$ find the values of k and c. **(4)**

(Total for Question 14 is 4 marks)

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15 The random variable X has a probability function

$$P(X = x) = \begin{cases} kx & x = 1, 2 \\ (k - 0.2)x & x = 3, 4 \end{cases}$$

where k is a constant.

(a) By working out the value of k, find the complete probability distribution of X. **(5)**

The random variable Y has a probability function

$$P(Y = y) = \begin{cases} c(y + 0.5) & y = 0, 1 \\ cy^2 & y = 2, 3, 4 \end{cases}$$

where c is a constant.

(b) By working out the value of c, find the complete probability distribution of Y. **(5)**

(Total for Question 15 is 10 marks)

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