

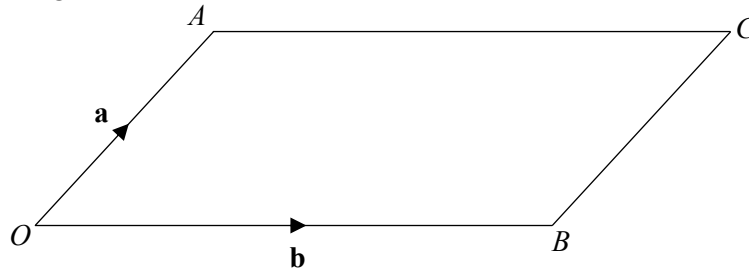


Vectors



REVISE THIS TOPIC

1 $OACB$ is a parallelogram.



$$\vec{OA} = \mathbf{a} \quad \vec{OB} = \mathbf{b}$$

Work out the following vectors in terms of \mathbf{a} and \mathbf{b} .

(a) \vec{AO}

$$\underline{\hspace{10em} - \mathbf{a} \hspace{10em}} \quad (1)$$

(b) \vec{BC}

$$\underline{\hspace{10em} \mathbf{a} \hspace{10em}} \quad (1)$$

(c) \vec{AB}

$$\underline{\hspace{10em} \mathbf{b} - \mathbf{a} \hspace{10em}} \quad (1)$$

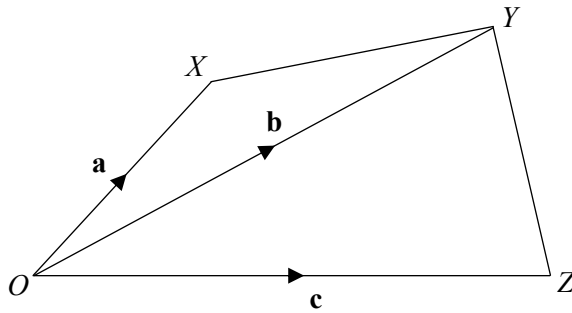
(d) \vec{CO}

$$\underline{\hspace{10em} - \mathbf{a} - \mathbf{b} \hspace{10em}} \quad (1)$$

(Total for Question 1 is 4 marks)



2 $OXYZ$ is a quadrilateral.



$\vec{OX} = \mathbf{a}$ $\vec{OY} = \mathbf{b}$ $\vec{OZ} = \mathbf{c}$

Work out the following vectors in terms of \mathbf{a} , \mathbf{b} and \mathbf{c} .

(a) \vec{ZO}

$-c$

(1)

(b) \vec{XY}

$b - a$

(1)

(c) \vec{ZY}

$b - c$

(1)

(d) \vec{XZ}

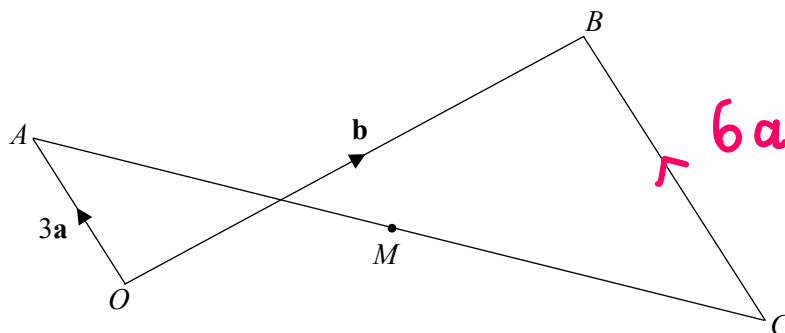
$c - a$

(1)

(Total for Question 2 is 4 marks)



3



$$\vec{OA} = 3\mathbf{a} \quad \vec{OB} = \mathbf{b} \quad \vec{CB} = 2\vec{OA}$$

Work out the following vectors in terms of \mathbf{a} and \mathbf{b} .

(a) \vec{AB}

$$\underline{b - 3a}$$

(1)

(b) \vec{CA}

$$\begin{aligned} \vec{CA} &= \vec{CB} + \vec{BO} + \vec{OA} \\ &= 6\mathbf{a} - \mathbf{b} + 3\mathbf{a} \end{aligned}$$

$$\underline{9\mathbf{a} - \mathbf{b}}$$

(2)

M is the midpoint of AC .

(c) Write \vec{CM} in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{CM} &= \frac{1}{2} \vec{CA} \\ &= \frac{1}{2} (9\mathbf{a} - \mathbf{b}) \end{aligned}$$

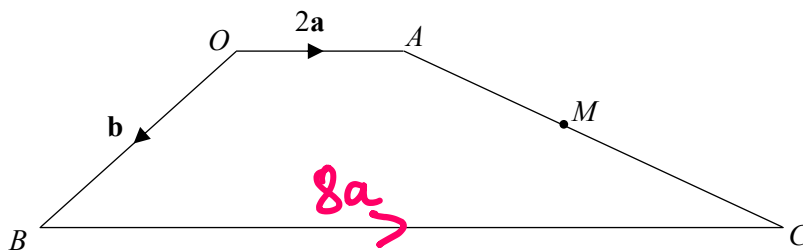
$$\underline{\frac{9}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}}$$

(2)

(Total for Question 3 is 5 marks)



4 $OACB$ is a trapezium



$$\vec{OA} = 2\mathbf{a} \quad \vec{OB} = \mathbf{b} \quad \vec{BC} = 4\vec{OA}$$

(a) Write \vec{AC} in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{AC} &= \vec{AO} + \vec{OB} + \vec{BC} \\ &= -2\mathbf{a} + \mathbf{b} + 8\mathbf{a} \end{aligned}$$

$$\underline{6\mathbf{a} + \mathbf{b}}$$

(2)

M is the midpoint of AC .

(b) Write \vec{BM} in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{BM} &= \vec{BO} + \vec{OA} + \frac{1}{2}\vec{AC} \\ &= -\mathbf{b} + 2\mathbf{a} + \frac{1}{2}(6\mathbf{a} + \mathbf{b}) \\ &= -\mathbf{b} + 2\mathbf{a} + 3\mathbf{a} + \frac{1}{2}\mathbf{b} \end{aligned}$$

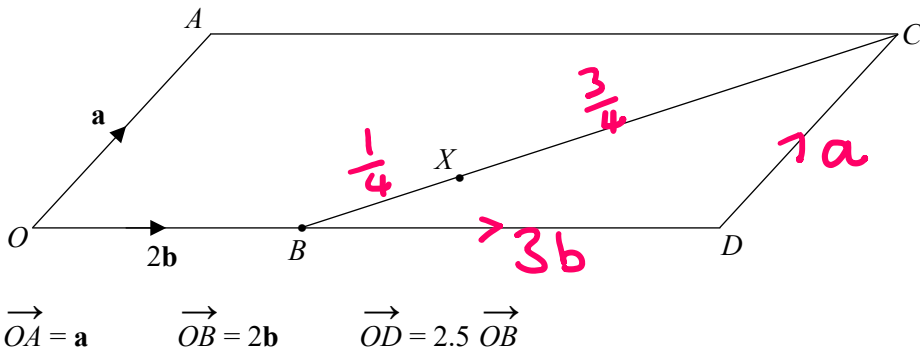
$$\underline{5\mathbf{a} - \frac{1}{2}\mathbf{b}}$$

(3)

(Total for Question 4 is 5 marks)



5 $OACD$ is a parallelogram.



(a) Write \vec{AD} in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{AD} &= \vec{AO} + \vec{OD} \\ &= -\mathbf{a} + 5\mathbf{b} \end{aligned}$$

$$\underline{5\mathbf{b} - \mathbf{a}}$$

(2)

(b) Write \vec{BC} in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{BC} &= \vec{BD} + \vec{DC} \\ &= 3\mathbf{b} + \mathbf{a} \end{aligned}$$

$$\underline{3\mathbf{b} + \mathbf{a}}$$

(2)

$BX : XC = 1 : 3$

(c) Write \vec{OX} in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{OX} &= \vec{OB} + \vec{BX} \\ &= 2\mathbf{b} + \frac{1}{4}\vec{BC} \\ &= 2\mathbf{b} + \frac{1}{4}(3\mathbf{b} + \mathbf{a}) \\ &= 2\mathbf{b} + \frac{3}{4}\mathbf{b} + \frac{1}{4}\mathbf{a} \end{aligned}$$

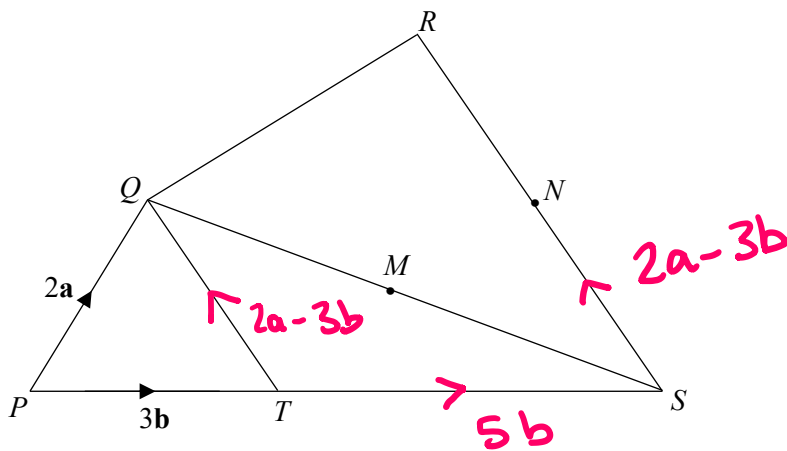
$$\underline{\frac{11}{4}\mathbf{b} + \frac{1}{4}\mathbf{a}}$$

(2)

(Total for Question 5 is 6 marks)



6 PQRS is a quadrilateral



$$\vec{PQ} = 2\mathbf{a} \quad \vec{PT} = 3\mathbf{b} \quad \vec{RS} = 2\vec{QT}$$

PTS is a straight line with $PT : TS = 3 : 5$

M is the midpoint of QS.
N is the midpoint of RS.

↑ $TS = 5b$

Write \vec{MN} in term of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{TQ} &= 2\mathbf{a} - 3\mathbf{b} & \vec{RS} &= 2(2\mathbf{a} - 3\mathbf{b}) \\ \vec{SN} &= 2\mathbf{a} - 3\mathbf{b} & &= 4\mathbf{a} - 6\mathbf{b} \end{aligned}$$

$$\begin{aligned} \vec{QS} &= \vec{QT} + \vec{TS} \\ &= 3\mathbf{b} - 2\mathbf{a} + 5\mathbf{b} \\ &= 8\mathbf{b} - 2\mathbf{a} \end{aligned}$$

$$\begin{aligned} \vec{MS} &= \frac{1}{2}(8\mathbf{b} - 2\mathbf{a}) \\ &= 4\mathbf{b} - \mathbf{a} \end{aligned}$$

$$\begin{aligned} \vec{MN} &= \vec{MS} + \vec{SN} \\ &= 4\mathbf{b} - \mathbf{a} + 2\mathbf{a} - 3\mathbf{b} \end{aligned}$$

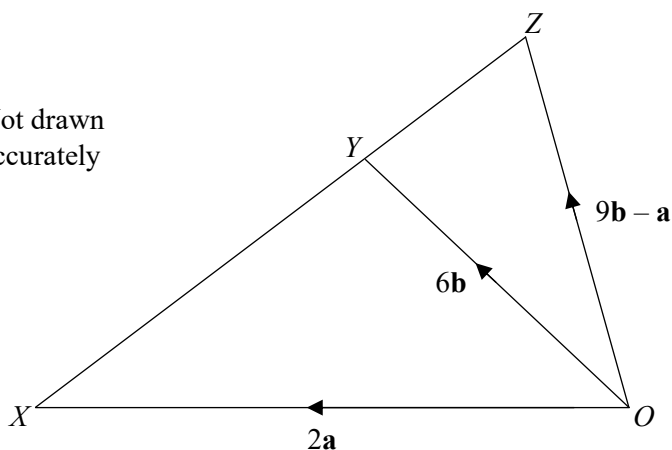
$\underline{\hspace{1cm}} \mathbf{a} + \mathbf{b}$

(Total for Question 6 is 4 marks)



7

Not drawn accurately



Prove, using vectors, that XYZ is a straight line.

$$\begin{aligned}
 \vec{XY} &= 6b - 2a \\
 \vec{XZ} &= 9b - a - 2a \\
 &= 9b - 3a \\
 &= \frac{3}{2}(6b - 2a)
 \end{aligned}$$

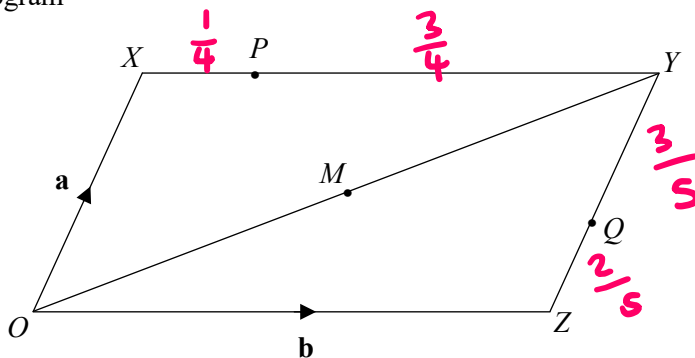
$$\vec{XZ} = \frac{3}{2}\vec{XY}$$

therefore XYZ is a straight line



(Total for Question 7 is 3 marks)

8 OXYZ is a parallelogram



$$\vec{OX} = \mathbf{a} \quad \vec{OZ} = \mathbf{b}$$

$$XP : PY = 1 : 3$$

$$ZQ : QY = 2 : 3$$

M is the midpoint of OY

(a) Write \vec{PQ} in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{PQ} &= \vec{PY} + \vec{YQ} \\ &= \frac{3}{4}\mathbf{b} - \frac{3}{5}\mathbf{a} \end{aligned}$$

$$\frac{3}{4}\mathbf{b} - \frac{3}{5}\mathbf{a}$$

(2)

(b) Write \vec{MQ} in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned} \vec{MQ} &= \vec{MY} + \vec{YQ} \\ &= \frac{1}{2}(\mathbf{a} + \mathbf{b}) + \vec{YQ} \\ &= \frac{1}{2}(\mathbf{a} + \mathbf{b}) - \frac{3}{5}\mathbf{a} \\ &= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{3}{5}\mathbf{a} \end{aligned}$$

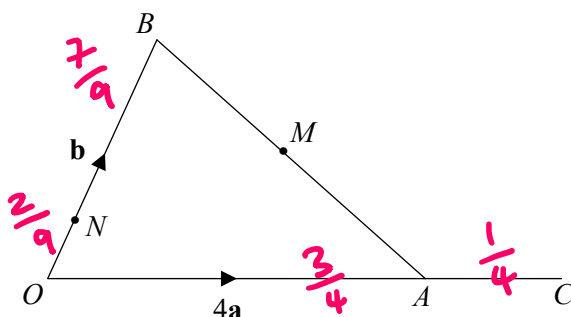
$$\frac{1}{2}\mathbf{b} - \frac{1}{10}\mathbf{a}$$

(3)

(Total for Question 8 is 5 marks)



9



$$\vec{OA} = 4\mathbf{a} \quad \vec{OB} = \mathbf{b}$$

$$OA : OC = 3 : 4$$

$$ON : OB = 2 : 9$$

M is the midpoint of AB

(a) Write \vec{MC} in terms of \mathbf{a} and \mathbf{b} .

$$\begin{aligned}
 \vec{MC} &= \vec{MA} + \vec{AC} \\
 &= \frac{1}{2} \vec{BA} + \vec{AC} \\
 &= \frac{1}{2}(4\mathbf{a} - \mathbf{b}) + \frac{4}{3}\mathbf{a} \\
 &= 2\mathbf{a} - \frac{1}{2}\mathbf{b} + \frac{4}{3}\mathbf{a}
 \end{aligned}$$

$$\frac{10}{3}\mathbf{a} - \frac{1}{2}\mathbf{b} \quad (3)$$

(b) Write \vec{NM} in terms of \mathbf{a} and \mathbf{b} .

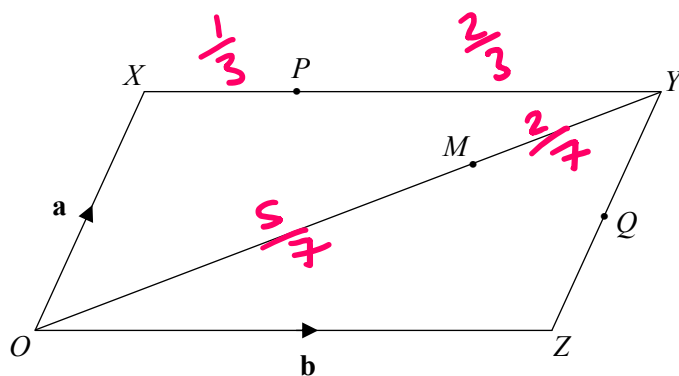
$$\begin{aligned}
 \vec{NM} &= \vec{NB} + \vec{BM} \\
 &= \frac{7}{9}\mathbf{b} + 2\mathbf{a} - \frac{1}{2}\mathbf{b}
 \end{aligned}$$

$$\frac{5}{18}\mathbf{b} + 2\mathbf{a} \quad (2)$$

(Total for Question 9 is 5 marks)



10 $OXYZ$ is a parallelogram



$$\vec{OX} = \mathbf{a} \quad \vec{OZ} = \mathbf{b}$$

$$ZQ = QY$$

$$XP : PY = 1 : 2$$

$$OM : MY = 5 : 2$$

Prove, using vectors, that PMQ is a straight line.

$$\begin{aligned}
 \vec{PM} &= \vec{PY} + \vec{YM} \\
 &= \vec{PY} + \frac{2}{7}(\vec{YO}) \\
 &= \frac{2}{3}\mathbf{b} + \frac{2}{7}(-\mathbf{b}-\mathbf{a}) \\
 &= \frac{2}{3}\mathbf{b} - \frac{2}{7}\mathbf{b} - \frac{2}{7}\mathbf{a} \\
 &= \frac{8}{21}\mathbf{b} - \frac{2}{7}\mathbf{a}
 \end{aligned}$$

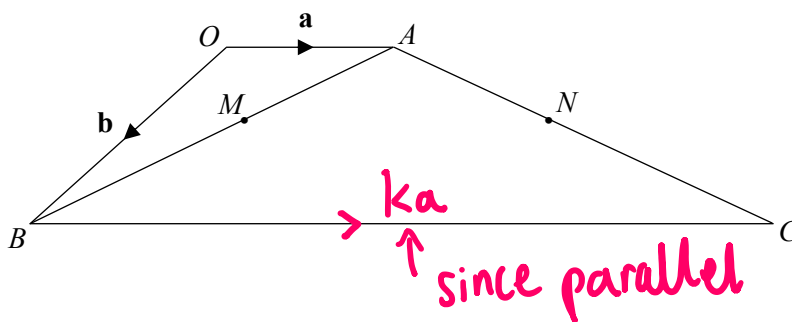
$$\begin{aligned}
 \vec{PQ} &= \vec{PY} + \vec{YQ} \\
 &= \frac{2}{3}\mathbf{b} - \frac{1}{2}\mathbf{a}
 \end{aligned}$$

$$\vec{PQ} = \frac{7}{4}\vec{PM}$$

therefore PMQ is a straight line



11 $OACB$ is a trapezium



$$\vec{OA} = \mathbf{a} \quad \vec{OB} = \mathbf{b}$$

M and N are the midpoints of AB and AC .

Prove, using vectors, that MN is parallel to OA .

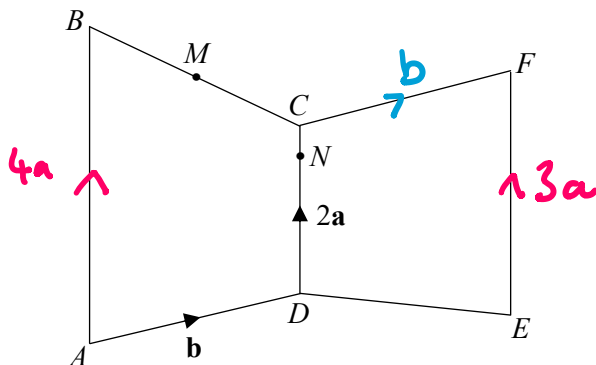
$$\begin{aligned}
 \vec{MN} &= \vec{MA} + \vec{AN} \\
 &= \frac{1}{2}\vec{BA} + \frac{1}{2}\vec{AC} \\
 &= \frac{1}{2}(\mathbf{a} - \mathbf{b}) + \frac{1}{2}(-\mathbf{a} + \mathbf{b} + k\mathbf{a}) \\
 &= \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} + \frac{k}{2}\mathbf{a} \\
 &= \frac{k}{2}\mathbf{a}
 \end{aligned}$$

$$\vec{MN} = \frac{k}{2}\vec{OA}$$

therefore they are parallel



12 $ABCD$ and $CDEF$ are trapeziums



$$\vec{CD} = 2\mathbf{a} \quad \vec{AD} = \vec{CF} = \mathbf{b}$$

$AB : DC : EF = 4 : 2 : 3$
 M is the midpoint of BC .
 N is on the line CD .

MNE is a straight line.

$DN : NC = k : 1$, where k is an integer.

Work out the value of k .

$$\begin{aligned}
 \vec{MN} &= \vec{MC} + \vec{CN} \\
 &= \frac{1}{2}(\vec{BC}) + \vec{CN} \\
 &= \frac{1}{2}(-4\mathbf{a} + \mathbf{b} + 2\mathbf{a}) - \alpha(2\mathbf{a}) \\
 &= \frac{1}{2}\mathbf{b} - \mathbf{a} - 2\alpha\mathbf{a} \\
 &= \frac{1}{2}\mathbf{b} - (2\alpha + 1)\mathbf{a}
 \end{aligned}$$

$$\begin{aligned}
 \vec{NE} &= \vec{NC} + \vec{CF} + \vec{FE} \\
 &= \alpha(2\mathbf{a}) + \mathbf{b} - 3\mathbf{a} \\
 &= \mathbf{b} - (3 - 2\alpha)\mathbf{a}
 \end{aligned}$$

$$\begin{aligned}
 \vec{NE} &= 2\vec{MN} \\
 3 - 2\alpha &= 2(2\alpha + 1) \\
 3 - 2\alpha &= 4\alpha + 2 \\
 \alpha &= \frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 \text{so } NC &= \frac{1}{6}DC \\
 DC : NC &= 6 : 1 \\
 DN : NC &= 5 : 1
 \end{aligned}$$

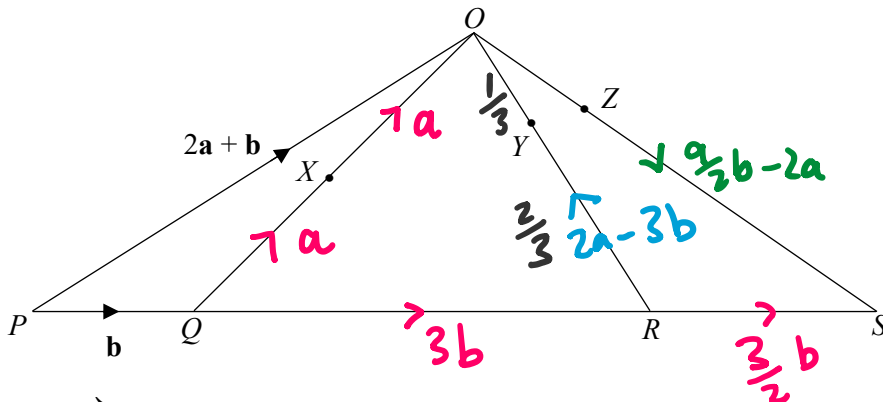


$$k = \dots\dots\dots 5$$

(Total for Question 12 is 5 marks)



13 POS is a triangle.



$\vec{PQ} = b$ $\vec{PO} = 2a + b$

X is the midpoint of OQ

OY : YR = 1 : 2

PQ : QR : RS = 2 : 6 : 3

$$\vec{RO} = \vec{RQ} + \vec{QO}$$

$$= -3b + 2a$$

$$= 2a - 3b$$

$$\vec{OS} = \vec{OZ} + \vec{ZS}$$

$$= 3b - 2a + \frac{3}{2}b$$

$$= \frac{9}{2}b - 2a$$

XYZ is a straight line.

OZ : OS = 1 : k

Work out the value of k.

$$\vec{XY} = \vec{XO} + \vec{OY}$$

$$= a - \frac{1}{3}(2a - 3b)$$

$$= a - \frac{2}{3}a + b$$

$$= b + \frac{1}{3}a$$

$$\vec{XZ} = \vec{XO} + \vec{OZ}$$

$$= a + \frac{1}{k}(\frac{9}{2}b - 2a)$$

$$= a + \frac{9}{2k}b - \frac{2}{k}a$$

$$= (1 - \frac{2}{k})a + \frac{9}{2k}b$$

$$\vec{XZ} = n(\vec{XY})$$

$$= n(b + \frac{1}{3}a)$$

$$= nb + \frac{n}{3}a$$

← equate →

$$3 - \frac{6}{k} = \frac{9}{2k}$$

$$3k - 6 = \frac{9}{2}$$

$$6k - 12 = 9$$

$$k = \frac{21}{6} = \frac{7}{2}$$

$$1 - \frac{2}{k} = \frac{n}{3}$$

$$3 - \frac{6}{k} = n$$

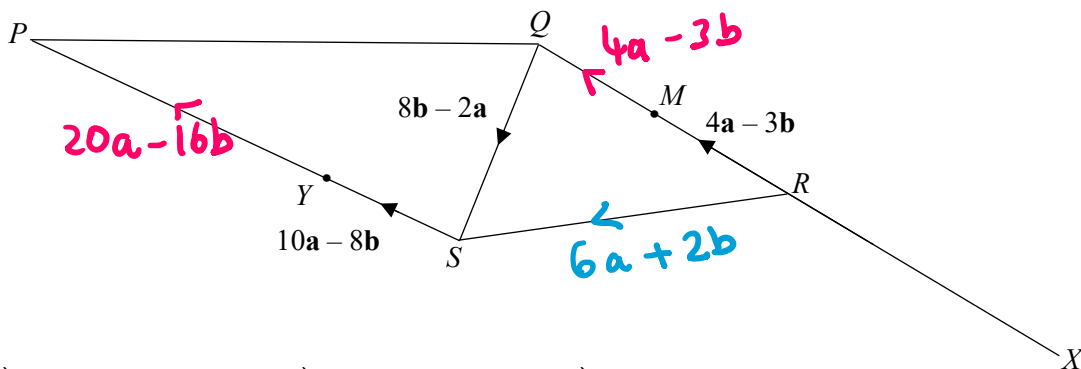
and $\frac{9}{2k} = n$

k = 3.5

(Total for Question 13 is 6 marks)



14 PQRS is a quadrilateral.



$$\vec{SY} = 10a - 8b \quad \vec{QS} = 8b - 2a \quad \vec{RM} = 4a - 3b$$

$$RM = MQ$$

$$SY : YP = 1 : 2$$

QRX is a straight line.

XS is parallel to RP.

Work out $XS : RP$

Give your answer in the form $n : 1$

$$\begin{aligned} \vec{RP} &= \vec{RQ} + \vec{QS} + \vec{SP} \\ &= 8a - 6b + 8b - 2a + 30a - 24b \\ &= 36a - 22b \end{aligned}$$

$$\begin{aligned} \vec{XS} &= \vec{XR} + \vec{RS} \\ &= k(4a - 3b) + 6a + 2b \\ &= (4k + 6)a - (3k - 2)b \end{aligned}$$

$$\begin{aligned} \vec{XS} &= n \vec{RP} \\ &= 36n a - 22n b \end{aligned}$$

$$4k + 6 = 36n \quad (\times 3)$$

$$3k - 2 = 22n \quad (\times 4)$$

$$12k + 18 = 108n$$

$$12k - 8 = 88n$$

$$\hline 26 = 20n$$

$$n = \frac{26}{20}$$

$$n = \frac{13}{10}$$

$$1.3 : 1$$

(Total for Question 14 is 6 marks)

