Variable Acceleration

SCAN ME

REVISE THIS TOPIC

CHECK YOUR ANSWERS

SCAN ME

1 A fixed point O lies on a straight line.

A particle, P, moves along the straight line.

At time t seconds, $0 \leq t \leq 10$, the distance, s metres, of P from O is given by

$$s = 10t - t^2$$

When $t = 1$, P is at point X.

When $t = T$, P is at point Y, where $OY = 21$ m

Find,

(a) the distance OX. **(1)**

(b) two possible values for T. **(2)**

(c) the maximum distance of point P from the point O in the interval $0 \leq t \leq 10$ **(2)**

(d) the total distance travelled by the point P in the interval $0 \leq t \leq 10$ **(2)**

(Total for Question 1 is 7 marks)

2 A fixed point O lies on a straight line.

A particle, P, moves along the straight line.

At time t seconds, $t \geq 0$, the distance, s metres, of P from O is given by

$$s = 36t - t^3$$

When $t = 1$, P is at point X.
When $t = 2$, P is at point Y.
When $t = T$, $T > 0$, P is at point O.

Find,

(a) XY **(2)**

(b) T **(2)**

(c) the maximum distance of point P from the point O in the interval $0 \leq t \leq T$ **(3)**

(d) the total distance travelled by P in the interval $0 \leq t \leq T$ **(2)**

(Total for Question 2 is 8 marks)

3 A particle, P, moves along the x-axis. At time t seconds, $t \geq 0$, the displacement, x metres, of P from the origin O, is given by $x = 4t^3 - 28t^2 + 49t$

(a) Find the velocity of P at time t seconds. **(2)**

(b) Find the speed of P when $t = 2$ **(2)**

(c) Find the values of t when P is instantaneously at rest. **(3)**

When $t = 0$, P is at point O.

(d) Find the other value of t when P is at point O. **(2)**

(e) Show that P will never move along the negative x-axis. **(2)**

(Total for Question 3 is 11 marks)

4 A fixed point O lies on a straight line.

A particle, P, moves along the straight line.

At time t seconds, $t \geq 0$, the distance, s metres, of P from O is given by

$$s = \frac{1}{3}t^3 - 4t^2 + 7t$$

Find,

(a) the maximum speed of P in the interval $0 \leq t \leq 8$ **(4)**

(b) the magnitude of the acceleration of P when $t = 3$ **(2)**

(c) The total distance travelled by P in the interval $0 \leq t \leq 8$ **(4)**

(Total for Question 4 is 10 marks)

5 A particle, P, moves along the x-axis. At time t seconds, $t \geq 0$, the displacement, x metres, of P from the origin O, is given by $x = \frac{1}{3}t^3 - 3t^2 + 5t$

At $t = 6$, P is at the point M.

(a) Find the distance OM. **(2)**

(b) Find the values of t for which P is at instantaneous rest. **(3)**

(c) Find the total distance travelled by P in the interval $0 \leq t \leq 6$ **(4)**

(d) Find the value of t at which the acceleration of P is zero. **(2)**

(Total for Question 5 is 11 marks)

6 A particle, P, moves along a straight line such that at time t seconds, $t \geq 0$, the velocity of P, v ms^{-1}, is modelled as

$$v = t^3 + 3t^2$$

(a) Find the velocity of P when $t = 5$ **(1)**

(b) Find the acceleration of P at time t seconds. **(2)**

(c) Show that P never changes direction. **(2)**

(d) Find the total distance travelled by P in the interval $0 \leq t \leq 4$ **(3)**

(Total for Question 6 is 8 marks)

7 A fixed point O lies on a straight line.

A particle, P, moves along the straight line such that at time t seconds, $t \geq 0$, after passing through point O, the velocity of P, v ms^{-1} is modelled as

$$v = t^2 - 6t + 8$$

(a) Find the range of values of t for which the velocity of P is negative. **(2)**

(b) Find the greatest speed of P in the interval $0 \leq t \leq 5$ **(3)**

(c) Find the magnitude of the acceleration of P the first time that P at instantaneous rest. **(4)**

When $t = 3$, P is at point X.

(d) Find the distance OX. **(3)**

(Total for Question 7 is 12 marks)

8 A particle, P, moves along a straight line such that at time t seconds, $t \geq 0$, the velocity of P, v ms^{-1}, is modelled as

$$v = 4t^3 - 24t^2 + 20t$$

(a) Find the values of t for which P is at instantaneous rest. **(3)**

(b) Find the magnitude of the acceleration of P when $t = 1$. **(2)**

M and N are fixed points on the straight line.

When $t = 0$, the displacement of P from point M is 25 metres.
When $t = 3$, P is at point N.

(c) Find the distance MN. **(4)**

(Total for Question 8 is 9 marks)

9 A particle, P, moves along a straight line such that at time t seconds, $t \geq 0$, the velocity of P, v ms^{-1}, is modelled as

$$v = 36t - 3t^2 - 33$$

(a) Find the greatest speed of P in the interval $0 \leq t \leq 12$ **(4)**

(b) Find the range of values of t for which the acceleration of P is positive. **(3)**

(c) Find the total distance travelled by P in the interval $0 \leq t \leq 12$ **(5)**

(Total for Question 9 is 12 marks)

10 A fixed point O lies on a straight line.

A particle, P, moves along the straight line such that at time t seconds, $t \geq 0$, after passing through point O, the velocity of P, v ms^{-1} is modelled as

$$v = 8 - t^3$$

(a) Show that the acceleration of P is never positive. **(2)**

(b) Find the distance travelled by P when it is at instantaneous rest. **(4)**

(c) Determine if P is moving towards or away from O when $t = 4$ **(3)**

(Total for Question 10 is 9 marks)

11 A particle, P, moves along the x-axis such that at time t seconds, $t > 0$, the velocity of P, v ms^{-1} is modelled as

$$v = 4\sqrt{t} - 5t$$

(a) Find the magnitude of the acceleration when P is at instantaneous rest. **(4)**

When $t = 1$, the displacement of P from the origin is 6 metres.

(b) Find the displacement of P from the origin at the instant when P stops accelerating. **(6)**

(Total for Question 11 is 10 marks)

12 A fixed point O lies on a straight line.

A particle, P, moves along the straight line such that at time t seconds, $t \geq 0$, after passing through point O, the velocity of P, v ms^{-1} is modelled as

$$v = t^2 - kt + 18$$

where k is a constant.

(a) Given that P at instantaneous rest when $t = 2$, find the value of k. **(2)**

(b) Find the magnitude of the acceleration of P when $t = 3$ **(2)**

(c) Find the maximum distance of P from O in the interval $0 \leq t \leq 10$ **(4)**

(Total for Question 12 is 8 marks)

13 Particles P and Q, move along the x-axis. At time t seconds, $t \geq 0$, the displacement, x metres, of P from the origin O, is given by $x = t^3 - 20t^2 + 77t$

At time t seconds, $t \geq 0$, the acceleration of Q, a ms^{-2}, is modelled as $a = 2t - 9$

When $t = 0$, Q is moving with a velocity of 12 ms^{-1}

(a) Find a range of values of t for which the velocity of Q is greater than the velocity of P. **(6)**

When $t = 0$, the displacement of Q from the origin O is 6 metres.

(b) Find the distance between P and Q when $t = 4$ **(4)**

(Total for Question 13 is 10 marks)

Solutions

14 A fixed point O lies on a straight line.

A particle, P, moves along the straight line such that at time t seconds, $t \geq 0$, the acceleration of P, a ms^{-2} is modelled as

$$a = 13 - 6t$$

When $t = 0$, the velocity of P is 10 ms^{-1} and the displacement of P from point O is 2 metres.

(a) Find the value of t for which P is at instantaneous rest. **(4)**

(b) Find the displacement of P from O when $t = 3$ **(4)**

(c) Find the total distance travelled by P in the interval $0 \leq t \leq 6$ **(4)**

(Total for Question 14 is 12 marks)

15 An electric scooter travels along a straight horizontal road.

At time t seconds, $0 \leq t \leq k$, where k is an integer, the acceleration of the scooter, a ms^{-2} is modelled as

$$a = 2 - 0.4t$$

When $t = 0$, the velocity of the scooter is 2.2 ms^{-1}

The scooter comes to rest when $t = k$

(a) Find the value of k **(4)**

(b) Find the magnitude of the acceleration of the scooter when $t = k$ **(2)**

(c) Find the distance travelled by the scooter in the interval $0 \leq t \leq k$ **(4)**

(Total for Question 15 is 10 marks)

16 At time $t = 0$, a drone takes off from rest at ground level and moves vertically such that at time t seconds, $0 \leq t \leq 4$, the velocity of the drone, v ms^{-1} is modelled as

$$v = t^3 - 7t^2 + 12t$$

The drone is at instantaneous rest when $t = 0$

(a) Find the other values of t for which the drone is at instantaneous rest. **(2)**

(b) Find the greatest height above ground level reached by the drone in the interval $0 \leq t \leq 4$ **(4)**

The acceleration of the drone, a ms^{-2}, is such that for $0 \leq t \leq 4$,

$$a \geq k, \text{ where } k \text{ is a constant}$$

(c) Find the greatest possible value for k. **(4)**

(Total for Question 16 is 10 marks)