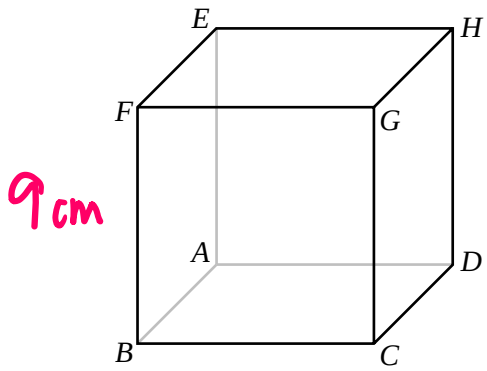
3D Trig/Pythagoras

SCAN ME

REVISE THIS TOPIC

1 *ABCDEFGH* is a cube.



BF = 9 cm

(a) Work out the length of *AC* giving your answer to 1 decimal place.

$AC^2 = AB^2 + BC^2$

$AC^2 = 9^2 + 9^2$

$AC^2 = 162$

$AC = \sqrt{162}$

$AC = 12.7279...$

12.7 cm

(2)

(b) Work out the length of *CE* giving your answer to 1 decimal place.

$CE^2 = AC^2 + CE^2$

$CE^2 = (\sqrt{162})^2 + 9^2$

$CE^2 = 243$

$CE = \sqrt{243}$

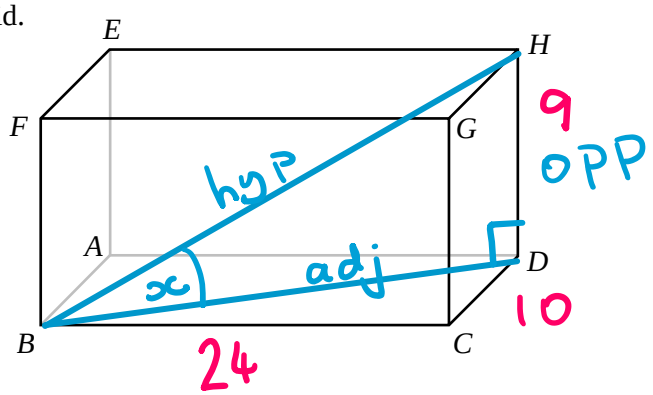
$CE = 15.5884...$

15.6 cm

(2)

(Total for Question 1 is 4 marks)

2 *ABCDEFGH* is a cuboid.



BC = 24 cm
CD = 10 cm
DH = 9 cm

(a) Work out the length of *BD*.

$BD^2 = BC^2 + CD^2$
$BD^2 = 24^2 + 10^2$
$BD^2 = 676$

$BD = \sqrt{676}$

26 cm

(2)

(b) Work out the length of *BH* giving your answer to 1 decimal place.

$BH^2 = BD^2 + DH^2$
$BH^2 = 26^2 + 9^2$
$BH^2 = 757$

$BH = \sqrt{757}$
$BH = 27.5136\ldots$

27.5 cm

(2)

(c) Work out the size of angle *DBH* giving your answer to 1 decimal place.

$\tan(x) = \frac{9}{27.5\ldots}$

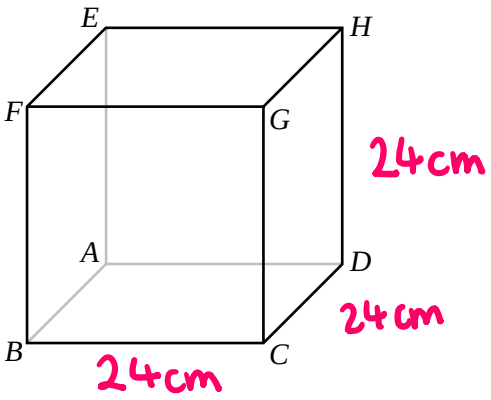
$x = \tan^{-1}\left(\frac{9}{27.5\ldots}\right)$
$x = 18.1134\ldots$

18.1 °

(2)

(Total for Question 2 is 6 marks)

3 *ABCDEFGH* is a cube.



The surface area of the cube is 3456 cm^2

Work out the length of *EC* giving your answer to 1 decimal place.

$3456 \div 6 = 576 cm^2$ (area of one face)

$\sqrt{576} = 24cm$ (length of one edge)

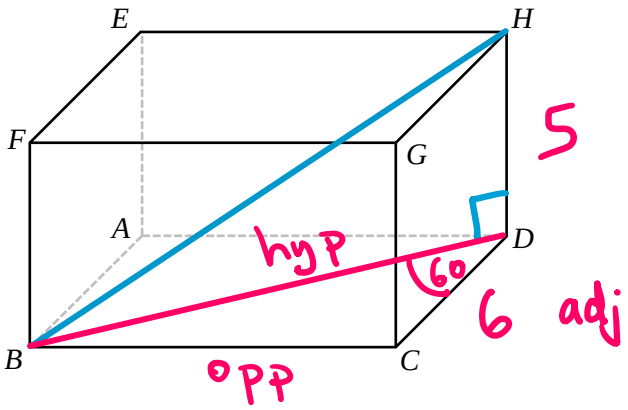
$AC^2 = AB^2 + BC^2$
$AC^2 = 24^2 + 24^2$
$AC^2 = 1152$
$AC = \sqrt{1152}$
$AC = 33.94...$

$EC^2 = AC^2 + AE^2$
$EC^2 = (\sqrt{1152})^2 + 24^2$
$EC^2 = 1728$
$EC = \sqrt{1728}$
$EC = 41.5692...$

41.6 ..... cm

(Total for Question 3 is 5 marks)

4 *ABCDEFGH* is a cuboid.



CD = 6 cm
DH = 5 cm
Angle *BDC* = 60°

Work out the perimeter of triangle *BDH*.

$\cos(60) = \frac{6}{BD}$

$BD = \frac{6}{\cos(60)}$

$BD = \frac{6}{0.5}$

$BD = 12\text{ cm}$

$BH^2 = BD^2 + DH^2$

$BH^2 = 12^2 + 5^2$

$BH^2 = 169$

$BH = \sqrt{169}$

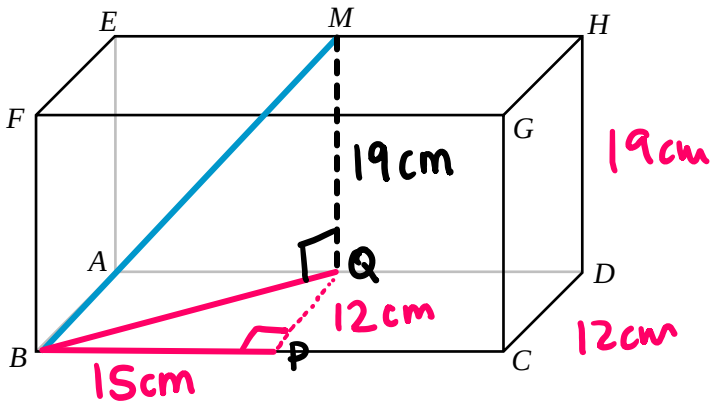
$BH = 13\text{ cm}$

Perimeter = 5 + 12 + 13

30cm ..... cm

(Total for Question 4 is 4 marks)

5 *ABCDEFGH* is a cuboid.



M is the midpoint of line *EH*.
BC = 30 cm
CD = 12 cm
DH = 19 cm

Work out the length of *BM* giving your answer to 1 decimal place

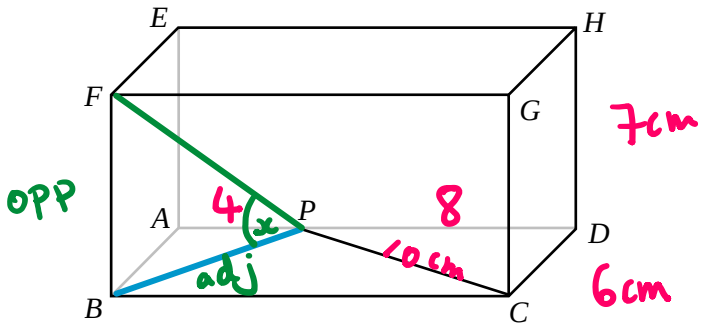
$BQ^2 = BP^2 + PQ^2$
$BQ^2 = 15^2 + 12^2$
$BQ^2 = 369$
$BQ = \sqrt{369}$
$BQ = 19.209...$

$BM^2 = BQ^2 + QM^2$
$BM^2 = (\sqrt{369})^2 + 19^2$
$BM^2 = 730$
$BM = \sqrt{730}$
$BM = 27.0185...$

27.0 ..... cm

(Total for Question 5 is 4 marks)

6 *ABCDEFGH* is a cuboid.



CD = 6 cm
DH = 7 cm
PC = 10 cm

P is the point on the line *AD* so that *AP* : *PD* = 1 : 2

(a) Work out the length of *BC* giving your answer to 1 decimal place.

$PD^2 = PC^2 - CD^2$
$PD = \sqrt{10^2 - 6^2}$
$PD = 8$

$AP = 8 \div 2 = 4$
$BC = 4 + 8$

12 cm

(3)

(b) Work out the length of *BP* giving your answer to 1 decimal plac.

$BP^2 = BA^2 + AP^2$
$BP^2 = 6^2 + 4^2$
$BP^2 = 52$

$BP = \sqrt{52}$
$BP = 7.211\ldots$

7.2 cm

(2)

(c) Work out the size of angle *BPF* giving your answer to 1 decimal place.

$\tan(x) = \frac{7}{7.21\ldots}$

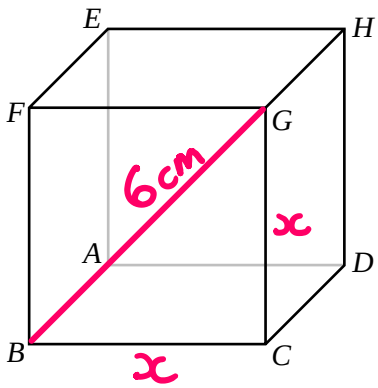
$x = \tan^{-1}\left(\frac{7}{7.21\ldots}\right)$
$x = 44.148\ldots$

44.1 °

(2)

(Total for Question 6 is 7 marks)

7 *ABCDEFGH* is a cube.



BG = 6 cm

Work out the volume of the cube giving your answer to 1 decimal place.

$BG^2 = BC^2 + CG^2$

$6^2 = x^2 + x^2$

$36 = 2x^2$ ÷2

$18 = x^2$ ÷2

$x = \sqrt{18}$

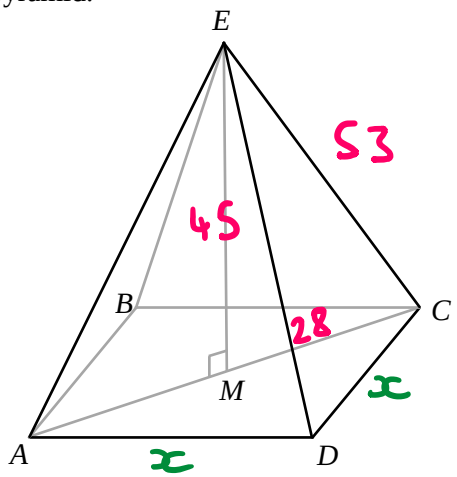
Volume $= \sqrt{18} \times \sqrt{18} \times \sqrt{18}$

$= 76.367...$

76.4 ..... cm^3

(Total for Question 7 is 4 marks)

8 *ABCDE* is a square-based pyramid.



M is the midpoint of the line *AC* and *AC* is perpendicular to *ME*.

EC = 53 cm
EM = 45 cm

Work out the volume of the pyramid.

$MC^2 = EC^2 - ME^2$
$MC^2 = 53^2 - 45^2$
$MC^2 = 784$
$MC = \sqrt{784}$
$MC = 28\text{cm}$

$AC = 2 \times MC$
$= 56\text{cm}$

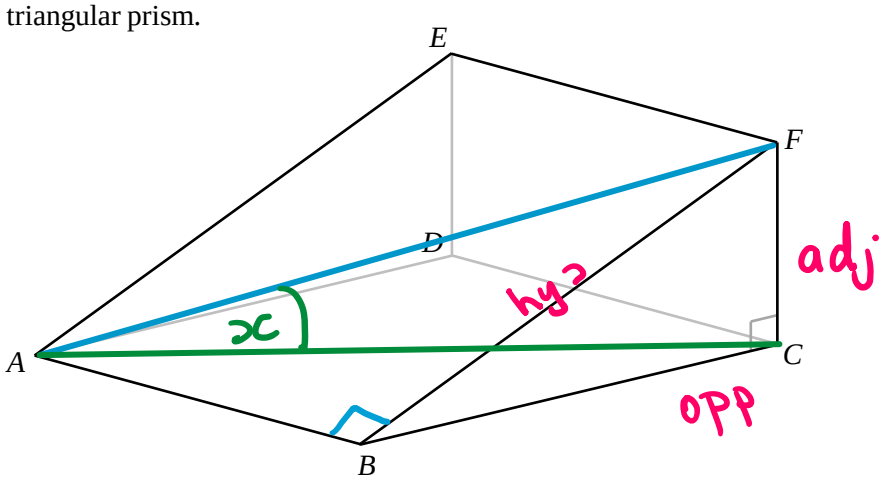
$AC^2 = x^2 + x^2$
$56^2 = x^2 + x^2$
$3136 = 2x^2$ ÷2
$1568 = x^2$ ÷2
$x = \sqrt{1568}$

$\text{Volume} = \frac{1}{3} \times \sqrt{1568} \times \sqrt{1568} \times 45$

23520 cm^3

(Total for Question 8 is 6 marks)

9 *ABCDEF* is a triangular prism.



AB = 18 cm *BC* = 22 cm Angle *BFC* = 70°

(a) Work out the length of *AF* giving your answer to 1 decimal place.

$\sin(70) = \frac{22}{BF}$

$BF = \frac{22}{\sin(70)}$

$BF = 23.411\ldots$

$AF^2 = AB^2 + BF^2$

$AF^2 = 18^2 + 23.4\ldots^2$

$AF^2 = 872.11757\ldots$

$AF = \sqrt{872.11757\ldots}$

$AF = 29.5316\ldots$

29.5 ..... cm

(4)

(b) Work out the size of the angle between *AF* and the plane *ABCD*.
Give your answer to 1 decimal place.

$AC^2 = AB^2 + BC^2$

$AC^2 = 18^2 + 22^2$

$AC^2 = 808$

$AC = \sqrt{808}$

$\cos(x) = \frac{\sqrt{808}}{29.53\ldots}$

$\cos(x) = 0.9625\ldots$

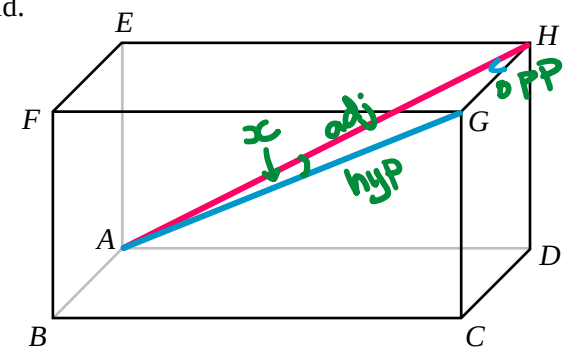
$x = \cos^{-1}(0.9625\ldots)$

15.7 .....°

(4)

(Total for Question 9 is 8 marks)

10 *ABCDEFGH* is a cuboid.



CD = 3.5 cm
DH = 4.5 cm
Angle *HAD* = 38°

(a) Work out the length of *AG* giving your answer to 1 decimal place.

$\sin(38) = \frac{4.5}{AH}$

$AH = \frac{4.5}{\sin(38)}$

$= 7.309\ldots$

$AG^2 = AH^2 + HG^2$

$AG^2 = 7.309\ldots^2 + 3.5^2$

$AG^2 = 65.674\ldots$

$AG = \sqrt{65.674\ldots}$

$AG = 8.1039\ldots$

8.1 ..... cm

(4)

(a) Work out the size of the angle between *AG* and the plane *ABCD*.
Give your answer to 1 decimal place.

$\sin(x) = \frac{3.5}{8.10\ldots}$

$\sin(x) = 0.43188\ldots$

$x = \sin^{-1}(0.43188\ldots)$

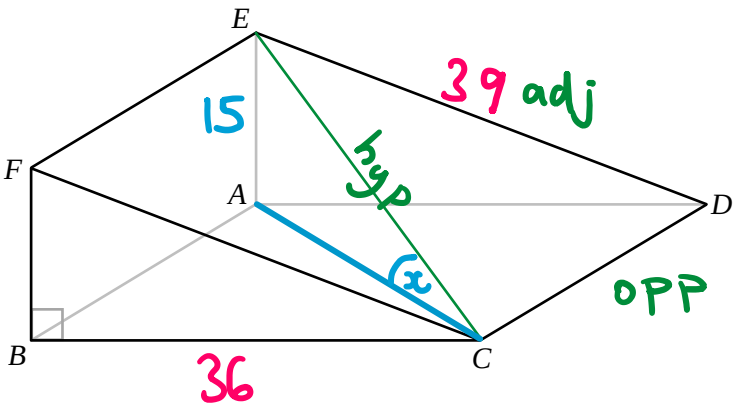
$x = 25.5873\ldots$

25.6 °

(2)

(Total for Question 10 is 6 marks)

11 *ABCDEF* is a triangular prism.



BF = 15 cm
Angle *CED* = 33°
BF : *BC* = 5 : 12

Work out the size of the angle between *CE* and the plane *ABCD*.
Give your answer to 1 decimal place.

$BC = \frac{15}{5} \times 12$

$= 36\text{cm}$

$FC^2 = 15^2 + 36^2$

$FC^2 = 1521$

$FC = \sqrt{1521}$

$FC = 39$

$FC = ED = 39$

$\cos(33) = \frac{39}{EC}$

$EC = \frac{39}{\cos(33)}$

$EC = 46.502\ldots$

$\sin(x) = \frac{15}{46.502\ldots}$

$\sin(x) = 0.3225\ldots$

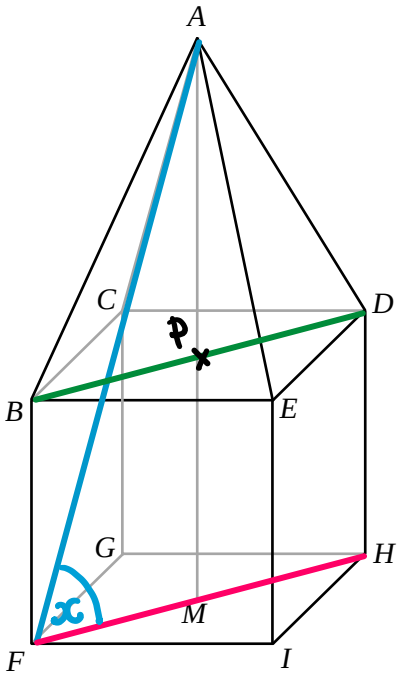
$x = \sin^{-1}(0.322\ldots)$

$x = 18.81815\ldots$

18.8 °

(Total for Question 11 is 6 marks)

12 *ABCDE* is a square-based pyramid placed on top of cube *BCDEFGHI*



M is the midpoint of the line *FH* with *FH* perpendicular to *MA*.
F = 12 cm
AD = 19 cm

Work out the size of the angle between *AF* and the plane *FGHI*.
Give your answer to 1 decimal place.

$FH^2 = 12^2 + 12^2$
$FH^2 = 288$
$FH = \sqrt{288}$
$MH = \sqrt{288} \div 2$
$MH = 8.485\ldots$

$\tan(x) = \dfrac{32.808\ldots}{\sqrt{288} \div 2}$

$\tan(x) = 3.86653\ldots$

$x = \tan^{-1}(3.866\ldots)$

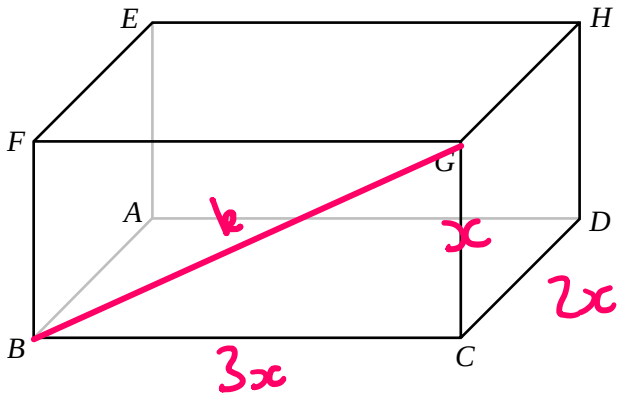
$PA^2 = 8.485\ldots^2 + 19^2$
$PA^2 = 433$
$PA = \sqrt{433}$
$PA = 20.808652\ldots$
$MA = 20.8\ldots + 12$
$MA = 32.808\ldots$

$x = 75.499366$

75.5 °

(Total for Question 12 is 6 marks)

13 *ABCDEFGH* is a cuboid.



$CG : CD : CB = 1 : 2 : 3$
$BG = k$ cm

Show that the volume of the cuboid can be written in the form $\frac{3\sqrt{a}}{b}k^3$ where a and b are integers.

$x^2 + (3x)^2 = k^2$

$x^2 + 9x^2 = k^2$

$10x^2 = k^2$

$x^2 = \frac{k^2}{10}$

$x = \frac{k}{\sqrt{10}}$

$x = \frac{k\sqrt{10}}{10}$

Volume $= x \times 2x \times 3x$

$= 6x^3$

$= 6 \times \left(\frac{k\sqrt{10}}{10}\right)^3$

$= 6 \times \frac{k^3 \times 10\sqrt{10}}{1000}$

$= \frac{60\sqrt{10}\,k^3}{1000}$

$= \frac{3\sqrt{10}}{50}k^3$

(Total for Question 13 is 6 marks)