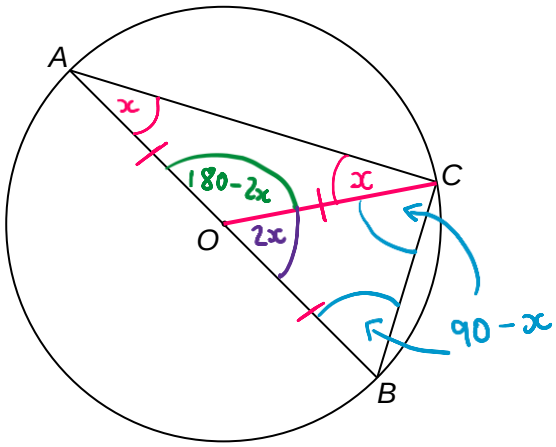
Circle Theorem Proofs

SCAN ME

REVISE THIS TOPIC

1



A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOB is a diameter of the circle.

Prove that angle *ACB* = 90° **[4 marks]**

OC = OA = OB (all radii)

Let angle OAC = x

Angle ACO = angle OAC = x

(Base angles in an isosceles triangle are equal)

Angle AOC = 180 - x - x

= 180 - 2x (angles in a triangle add to 180°)

Angle BOC = 180 - (180 - 2x)

= 2x (angles on a straight line add to 180°)

Angle OBC = Angle OCB = ½(180 - 2x)

= 90 - x

(Base angles in an isosceles triangle are equal)

Angle ACB = Angle ACO + Angle OCB

= x + 90 - x

= 90°

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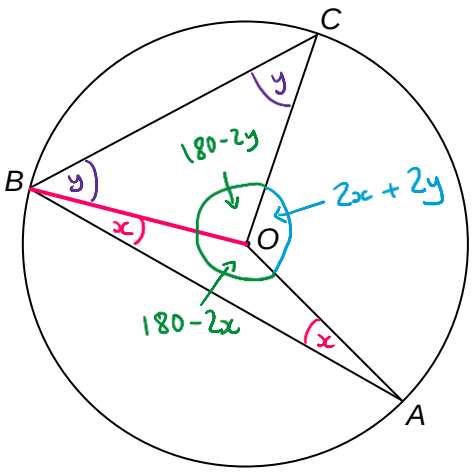
2



A, *B* and *C* are points on the circumference of a circle, centre *O*.

Prove that angle *AOC* = 2 × angle *ABC* **[4 marks]**

OC = OA = OB (all radii)

Let angle OAB = x Let angle OCB = y

Angle OBA = x Angle OBC = y

Base angles in an isosceles triangle are equal

Angle COB = 180 − 2y Angle AOB = 180 − 2x

Angles in a triangle add to 180°

Angle ABC = x + y

Angle AOC = 360 − (180 − 2y) − (180 − 2x)

= 360 − 180 + 2y − 180 + 2x

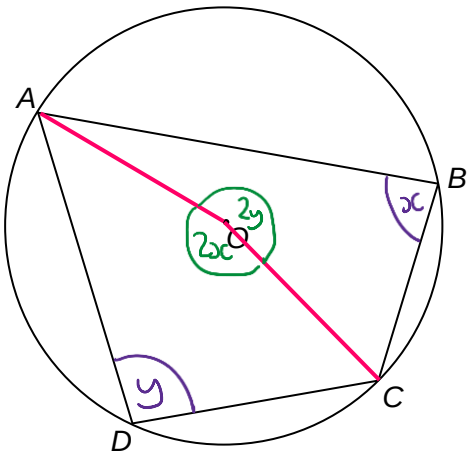
= 2x + 2y

= 2(x + y)

= 2 × Angle ABC

angles around a point add to 360°

3



A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.

Prove that angle *ABC* + angle *CDA* = 180° **[4 marks]**

Let angle ABC = x and Angle CDA = y

Minor angle AOC = 2x and major angle AOC = 2y

as angle at the centre is twice the angle at the circumference

$2x + 2y = 360°$ (angles around a point add to 360°)

÷2 ↙ ↘ ÷2

$x + y = 180°$

↓ ↘

Angle ABC + Angle CDA = 180°

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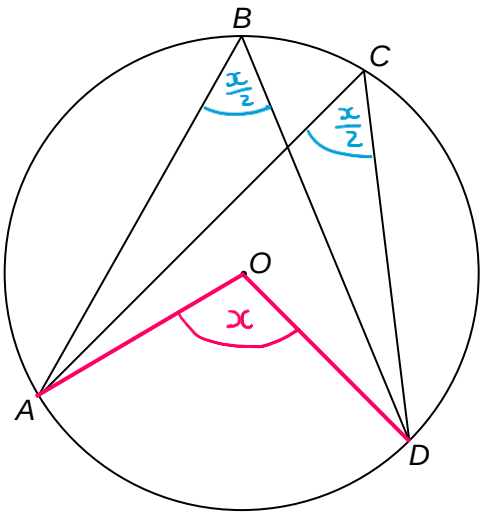
4



A, *B*, *C* and *D* are points on the circumference of a circle, centre *O*.

Prove that angle *ABD* = angle *ACD*

[2 marks]

Let angle AOD = x

Angle ABD = $\frac{x}{2}$ } angle at the centre is twice the angle at the circumference

Angle ACD = $\frac{x}{2}$ }

Angle ABD = Angle ACD = $\frac{x}{2}$

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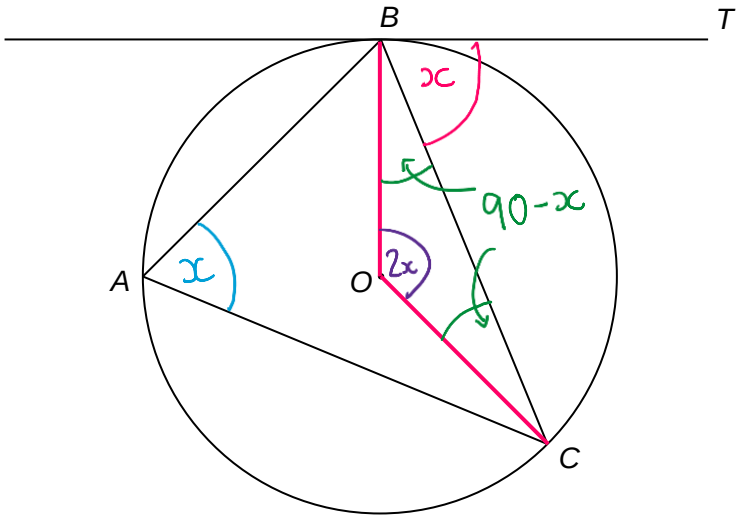

A, *B* and *C* are points on the circumference of a circle, centre *O*.
BT is the tangent to the circle at *B*.

Prove that angle *CAB* = angle *CBT* **[4 marks]**

Let angle CBT = x

Angle OBC = 90 - x (a tangent meets a radius at 90°)

Angle BCO = 90 - x

Angle COB = 180 - (90 - x) - (90 - x)

= 180 - 90 + x - 90 + x

= 2x

(angles in a triangle add to 180°)

Angle CAB = x angle at the centre is twice the angle at the circumference

Angle CAB = Angle CBT = x

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