Composite Functions

SCAN ME

REVISE THIS TOPIC

1 $f(x) = 3x + 4$ $g(x) = x + 10$ $h(x) = x^2$

(a) Work out fg(x)
Give your answer in the form $ax + b$ where a and b are integers

$$fg(x) = 3(x+10)+4$$
$$= 3x + 30 + 4$$
$$= 3x + 34$$

fg(x) = $3x + 34$
(2)

(b) Work out gf(x)
Give your answer in the form $ax + b$ where a and b are integers

$$gf(x) = 3x+4+10$$
$$= 3x+14$$

gf(x) = $3x + 14$
(2)

(a) Work out gh(x)

$$gh(x) = x^2 + 10$$

gh(x) = $x^2 + 10$
(1)

(Total for Question 1 is 5 marks)

2 $f(x) = x - 3$ $g(x) = x^2 + 1$ $h(x) = 10x$

(a) Work out $fg(x)$
Fully simplify your answer.

$$fg(x) = x^2 + 1 - 3$$
$$= x^2 - 2$$

$fg(x) =$ $x^2 - 2$
(2)

(b) Work out $hg(x)$
Fully simplify your answer.

$$hg(x) = 10(x^2 + 1)$$
$$= 10x^2 + 10$$

$hg(x) =$ $10x^2 + 10$
(2)

(c) Work out $gh(x)$
Fully simplify your answer.

$$gh(x) = (10x)^2 + 1$$
$$= 100x^2 + 1$$

$gh(x) =$ $100x^2 + 1$
(2)

(Total for Question 2 is 6 marks)

3 $f(x) = \frac{x}{4}$ $g(x) = 4x - 8$ $h(x) = \sqrt{x}$

(a) Work out fg(x)
Fully simplify your answer.

$$fg(x) = \frac{4x-8}{4}$$
$$= x - 2$$

fg(x) = $x - 2$
(2)

(b) Work out gf(x)
Fully simplify your answer.

$$gf(x) = 4\left(\frac{x}{4}\right) - 8$$
$$= x - 8$$

gf(x) = $x - 8$
(2)

(c) Work out hf(x)
Fully simplify your answer.

$$hf(x) = \sqrt{\frac{x}{4}}$$
$$= \frac{\sqrt{x}}{\sqrt{4}}$$

hf(x) = $\frac{\sqrt{x}}{2}$
(2)

(Total for Question 3 is 6 marks)

4 $f(x) = x - 5$ $g(x) = x^2 + 30$

(a) Work out fg(x)
Fully simplify your answer.

$$fg(x) = x^2 + 30 - 5$$
$$= x^2 + 25$$

fg(x) = $x^2 + 25$
(2)

(b) Work out fg(3)

$$fg(x) = x^2 + 25$$
$$fg(3) = 3^2 + 25$$
$$= 9 + 25$$

34
(2)

(c) Work out gf(x)
Give your answer in the form $ax^2 + bx + c$ where a, b and c are integers.

$$gf(x) = (x-5)^2 + 30$$
$$= (x-5)(x-5) + 30$$
$$= x^2 - 5x - 5x + 25 + 30$$
$$= x^2 - 10x + 55$$

gf(x) = $x^2 - 10x + 55$
(3)

(Total for Question 4 is 7 marks)

5 $f(x) = 2x + 1$ $g(x) = \sqrt{x + 3}$

(a) Work out g(13)

$g(13) = \sqrt{13+3}$
$= \sqrt{16}$

4 ....
(1)

(b) Work out fg(13)

$fg(13) = f(4)$
$= 2(4) + 1$

9 ....
(1)

(c) Work out gf(16)

$gf(x) = \sqrt{2x+1+3}$
$= \sqrt{2x+4}$

$gf(16) = \sqrt{2(16)+4}$
$= \sqrt{36}$

6 ....
(2)

(Total for Question 5 is 4 marks)

6 $f(x) = x + 2$ $g(x) = x^3$ $h(x) = \sqrt{x}$

(a) Work out gf(3)

$gf(x) = (x+2)^3$
$gf(3) = (3+2)^3$
$= 5^3$

125

(2)

(b) Work out gh(x)
Give your answer in the form x^k where k is a fraction.

$gh(x) = (\sqrt{x})^3$
$= (x^{1/2})^3$
$= x^{3/2}$

gh(x) = $x^{3/2}$

(2)

(c) Work out gf(x)
Give your answer in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are integers.

$gf(x) = (x+2)^3$
$= (x+2)(x+2)(x+2)$
$= (x^2 + 2x + 2x + 4)(x+2)$
$= (x^2 + 4x + 4)(x+2)$
$= x^3 + 2x^2 + 4x^2 + 8x + 4x + 8$

gf(x) = $x^3 + 6x^2 + 12x + 8$

(3)

(Total for Question 6 is 7 marks)

7 $f(x) = 2^x$ $g(x) = 1 - x$ $h(x) = 2 + x$

(a) Work out gf(-3)

$gf(x) = 1 - 2^x$

$gf(-3) = 1 - 2^{-3}$

$2^{-3} = \frac{1}{2^3}$

$1 - \frac{1}{8}$

$\frac{7}{8}$

(2)

$hg(x) - gh(x) = k$ where k is an integer.

(b) Find the value of k.

$hg(x) = 2 + (1 - x)$
$= 2 + 1 - x$
$= 3 - x$

$gh(x) = 1 - (2 + x)$
$= 1 - 2 - x$
$= -1 - x$

$hg(x) - gh(x) = (3 - x) - (-1 - x)$
$= 3 - x + 1 + x$

$k =$ 4

(4)

(c) Show that $\dfrac{fh(x)}{fg(x)} = 2^{ax+b}$ where a and b are integers.

$fh(x) = 2^{2+x}$

$fg(x) = 2^{1-x}$

$\dfrac{fh(x)}{fg(x)} = \dfrac{2^{2+x}}{2^{1-x}}$

$= 2^{(2+x)-(1-x)}$

$= 2^{2+x-1+x}$

$= 2^{2x+1}$

(3)

(Total for Question 7 is 9 marks)