



SCAN ME

Equation of a Tangent

← REVISE THIS TOPIC

1 A circle has equation $x^2 + y^2 = 20$

The point P lies on the circle.
The coordinates of P are $(2, 4)$

The line L is tangent to the circle at point P .

Find an equation of L .
Give your answer in the form $y = mx + c$

gradient of $OP = \frac{4}{2}$
 $= 2$

gradient of
tangent $= -\frac{1}{2}$

$$y = -\frac{1}{2}x + c$$
$$4 = -\frac{1}{2}(2) + c$$
$$4 = -1 + c$$
$$c = 5$$

$$y = -\frac{1}{2}x + 5$$

(Total for Question 1 is 4 marks)



2 A circle has equation $x^2 + y^2 = 90$

The point P lies on the circle.

The coordinates of P are $(9, 3)$

The line L is tangent to the circle at point P .

Find an equation of L .

Give your answer in the form $y = mx + c$

$$\begin{aligned} \text{gradient of } OP &= \frac{3}{9} \\ &= \frac{1}{3} \end{aligned}$$

$$\text{gradient of tangent} = -3$$

$$\begin{aligned} y &= -3x + c \\ 3 &= -3(9) + c \\ 3 &= -27 + c \\ c &= 30 \end{aligned}$$

$$y = -3x + 30$$

(Total for Question 2 is 4 marks)



3 A circle has equation $x^2 + y^2 = 29$

The point P lies on the circle.

The coordinates of P are $(2, 5)$

The line L is tangent to the circle at point P .

Find an equation of L .

Give your answer in the form $y = mx + c$

$$\text{gradient of } OP = \frac{5}{2}$$

$$\text{gradient of tangent} = -\frac{2}{5}$$

$$y = -\frac{2}{5}x + c$$

$$5 = -\frac{2}{5}(2) + c$$

$$5 = -\frac{4}{5} + c$$

$$c = 5 + \frac{4}{5}$$

$$c = \frac{25}{5} + \frac{4}{5}$$

$$c = \frac{29}{5}$$

$$y = -\frac{2}{5}x + \frac{29}{5}$$

(Total for Question 3 is 4 marks)



4 A circle has equation $x^2 + y^2 = 17$

The point P lies on the circle.

The coordinates of P are $(1, -4)$

The line L is tangent to the circle at point P .

Find an equation of L .

Give your answer in the form $y = mx + c$

$$\begin{aligned} \text{gradient of } OP &= \frac{-4}{1} \\ &= -4 \end{aligned} \quad \begin{aligned} \text{gradient of} \\ \text{tangent} &= \frac{1}{4} \end{aligned}$$

$$y = \frac{1}{4}x + c$$

$$-4 = \frac{1}{4}(1) + c$$

$$-4 = \frac{1}{4} + c$$

$$c = -4 - \frac{1}{4}$$

$$c = -\frac{16}{4} - \frac{1}{4}$$

$$c = -\frac{17}{4}$$

$$y = \frac{1}{4}x - \frac{17}{4}$$

(Total for Question 4 is 4 marks)



5 A circle has equation $x^2 + y^2 = 34$

The point P lies on the circle.

The coordinates of P are $(-3, 5)$

The line L is tangent to the circle at point P .

Find an equation of L .

Give your answer in the form $y = mx + c$

$$\text{gradient of } OP = -\frac{5}{3}$$

$$\text{gradient of tangent} = \frac{3}{5}$$

$$y = \frac{3}{5}x + c$$

$$5 = \frac{3}{5}(-3) + c$$

$$5 = -\frac{9}{5} + c$$

$$c = 5 + \frac{9}{5}$$

$$c = \frac{25}{5} + \frac{9}{5}$$

$$c = \frac{34}{5}$$

$$y = \frac{3}{5}x + \frac{34}{5}$$

(Total for Question 5 is 4 marks)



6 A circle has equation $x^2 + y^2 = 65$

The point P lies on the circle.

The coordinates of P are $(7, k)$, where $k < 0$

The line L is tangent to the circle at point P .

Find an equation of L .

Give your answer in the form $y = mx + c$

$$7^2 + k^2 = 65$$

$$49 + k^2 = 65$$

$$k^2 = 16$$

$$k = \pm 4$$

$$\text{as } k < 0, \underline{\underline{k = -4}}$$

$$\begin{aligned} \text{gradient of } OP &= -\frac{4}{7} \\ \text{gradient of tangent} &= \frac{7}{4} \end{aligned}$$

$$y = \frac{7}{4}x + c$$

$$-4 = \frac{7}{4}(7) + c$$

$$-4 = \frac{49}{4} + c$$

$$c = -4 - \frac{49}{4}$$

$$c = \frac{-16}{4} - \frac{49}{4}$$

$$c = -\frac{65}{4}$$

$$y = \frac{7}{4}x - \frac{65}{4}$$

(Total for Question 6 is 5 marks)



7 A circle has equation $x^2 + y^2 = 117$

The point P lies on the circle.

The coordinates of P are $(9, k)$, where $k > 0$

The line L is tangent to the circle at point P .

Find an equation of L .

Give your answer in the form $y = mx + c$

$$9^2 + k^2 = 117$$

$$81 + k^2 = 117$$

$$k^2 = 36$$

$$k = \pm 6$$

$$\text{as } k > 0, \underline{\underline{k=6}}$$

$$\begin{aligned} \text{gradient of } OP &= \frac{6}{9} \\ &= \frac{2}{3} \\ \text{gradient of tangent} &= -\frac{3}{2} \end{aligned}$$

$$y = -\frac{3}{2}x + c$$

$$6 = -\frac{3}{2}(9) + c$$

$$6 = -\frac{27}{2} + c$$

$$c = 6 + \frac{27}{2}$$

$$c = \frac{12}{2} + \frac{27}{2}$$

$$c = \frac{39}{2}$$

$$y = -\frac{3}{2}x + \frac{39}{2}$$

(Total for Question 7 is 5 marks)



8 A circle has equation $x^2 + y^2 = 22.25$

The point P lies on the circle.

The coordinates of P are $(-4, k)$, where $k < 0$

The line L is tangent to the circle at point P .

Find an equation of L .

Give your answer in the form $ay + bx + c = 0$ where a, b and c are integers.

$$(-4)^2 + k^2 = 22.25$$

$$16 + k^2 = 22.25$$

$$k^2 = 6.25$$

$$k = \pm 2.5$$

$$\text{as } k < 0, \underline{\underline{k = -2.5}}$$

$$\text{gradient of } OP = \frac{-2.5}{-4}$$

$$= \frac{2.5}{4}$$

$$= \frac{5}{8}$$

$$\text{gradient of tangent} = -\frac{8}{5}$$

$$y = -\frac{8}{5}x + c$$

$$-\frac{5}{2} = -\frac{8}{5}(-4) + c$$

$$-\frac{5}{2} = \frac{32}{5} + c$$

$$c = -\frac{5}{2} - \frac{32}{5}$$

$$c = -\frac{25}{10} - \frac{64}{10}$$

$$c = -\frac{89}{10}$$

$$y = -\frac{8}{5}x - \frac{89}{10}$$

$$\times 10 \downarrow \quad \downarrow \times 10$$

$$10y = -16x - 89$$

$$10y + 16x + 89 = 0$$

$$10y + 16x + 89 = 0$$

(Total for Question 8 is 6 marks)



9 A circle has equation $x^2 + y^2 = 13$

The point P lies on the circle.

The coordinates of P are $(2, 3)$

The line L is tangent to the circle at point P .

The line L crosses the x -axis at the point Q .

Work out the coordinates of the point Q .

$$\text{gradient of } OP = \frac{3}{2}$$

$$\text{gradient of tangent} = -\frac{2}{3}$$

$$y = -\frac{2}{3}x + c$$

$$3 = -\frac{2}{3}(2) + c$$

$$3 = -\frac{4}{3} + c$$

$$c = 3 + \frac{4}{3}$$

$$c = \frac{9}{3} + \frac{4}{3}$$

$$c = \frac{13}{3}$$

$$y = -\frac{2}{3}x + \frac{13}{3}$$

$$\text{At } Q, y = 0$$

$$0 = -\frac{2}{3}x + \frac{13}{3}$$

$$\frac{2}{3}x = \frac{13}{3}$$

$$2x = 13$$

$$x = \frac{13}{2}$$

$$x = 6.5$$

(6.5 , 0)

(Total for Question 9 is 5 marks)



10 A circle has equation $x^2 + y^2 = 212$

The point P lies on the circle.

The coordinates of P are $(14, -4)$

The line L is tangent to the circle at point P .

The line L crosses the y -axis at the point A .

Work out the coordinates of the point A .

gradient of $OP = -\frac{4}{14} = -\frac{2}{7}$ gradient of tangent = $\frac{7}{2}$

$$y = \frac{7}{2}x + c$$

$$-4 = \frac{7}{2}(14) + c$$

$$-4 = 49 + c$$

$$c = -4 - 49$$

$$c = -53$$

$$y = \frac{7}{2}x - 53$$

At A , $x = 0$

$$y = \frac{7}{2}(0) - 53$$

$$y = -53$$

(.....,.....)

(Total for Question 10 is 5 marks)



11 A circle has equation $x^2 + y^2 = 90$

The point P lies on the circle.
The coordinates of P are $(3, 9)$

The line L is tangent to the circle at point P .
The line L crosses the y -axis at the point A and the x -axis at the point B .

Work out the area of triangle AOB .

$$\begin{aligned} \text{gradient of } OP &= \frac{9}{3} \\ &= 3 \end{aligned}$$

$$\text{gradient of tangent} = -\frac{1}{3}$$

$$y = -\frac{1}{3}x + c$$

$$9 = -\frac{1}{3}(3) + c$$

$$9 = -1 + c$$

$$c = 10$$

$$y = -\frac{1}{3}x + 10$$

$$\text{At } A, x = 0$$

$$y = -\frac{1}{3}(0) + 10$$

$$y = 10$$

$$A = (0, 10)$$

$$\text{At } B, y = 0$$

$$0 = -\frac{1}{3}x + 10$$

$$\frac{1}{3}x = 10$$

$$x = 30$$

$$B = (30, 0)$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 30 \times 10 \\ &= 150 \end{aligned}$$



12 A circle has equation $x^2 + y^2 = 320$

The point P lies on the circle.

The coordinates of P are $(-8, 16)$

The line L is tangent to the circle at point P .

The line L crosses the x -axis at the point A and the y -axis at the point B .

Work out the length of AB .

Give your answer in the form $a\sqrt{5}$ where a is an integer.

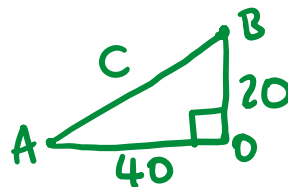
gradient of $OP = -\frac{16}{8}$
 $= -2$

gradient of
 tangent $= \frac{1}{2}$

$y = \frac{1}{2}x + c$
 $16 = \frac{1}{2}(-8) + c$
 $16 = -4 + c$
 $c = 16 + 4$
 $c = 20$
 $y = \frac{1}{2}x + 20$

At A , $y = 0$
 $0 = \frac{1}{2}x + 20$
 $-\frac{1}{2}x = 20$
 $x = -40$
 $A = (-40, 0)$

At B , $x = 0$
 $y = \frac{1}{2}(0) + 20$
 $y = 20$
 $B = (0, 20)$



$C^2 = 20^2 + 40^2$
 $C^2 = 2000$
 $C = \sqrt{2000}$
 $C = \sqrt{400 \times 5}$

$20\sqrt{5}$

.....units

(Total for Question 12 is 6 marks)



13 A circle has equation $x^2 + y^2 = 29$

The point P lies on the circle.
The coordinates of P are $(5, 2)$

The line L is tangent to the circle at point P .
The line L crosses the y -axis at the point A and the x -axis at the point B .

Work out the length of AB .
Give your answer to 4 significant figures.

$$\text{gradient of } OP = \frac{2}{5}$$

$$\text{gradient of tangent} = -\frac{5}{2}$$

$$y = -\frac{5}{2}x + C$$

$$2 = -\frac{5}{2}(5) + C$$

$$2 = -\frac{25}{2} + C$$

$$C = 2 + \frac{25}{2}$$

$$C = \frac{4}{2} + \frac{25}{2}$$

$$C = \frac{29}{2}$$

$$y = -\frac{5}{2}x + \frac{29}{2}$$

At A , $x = 0$

$$y = -\frac{5}{2}(0) + \frac{29}{2}$$

$$A = (0, \frac{29}{2})$$

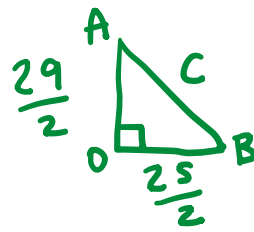
At B , $y = 0$

$$0 = -\frac{5}{2}x + \frac{29}{2}$$

$$\frac{5}{2}x = \frac{29}{2}$$

$$x = \frac{29}{5}$$

$$B = (\frac{29}{5}, 0)$$



$$C^2 = (\frac{29}{2})^2 + (\frac{29}{5})^2$$

$$C^2 = 243.89$$

$$C = \sqrt{243.89}$$

$$C = 15.6169\dots$$

$$15.62$$

.....units

(Total for Question 13 is 6 marks)



14 A circle has equation $x^2 + y^2 = 48$

The point P lies on the circle.

The coordinates of P are $(\sqrt{12}, 6)$

The line L is tangent to the circle at point P .

The line L crosses the y -axis at the point A .

Show that the length of AP is an integer.

gradient of $OP = \frac{6}{\sqrt{12}}$

gradient of tangent $= -\frac{\sqrt{12}}{6}$

$$y = -\frac{\sqrt{12}}{6}x + c$$

$$6 = -\frac{\sqrt{12}}{6}(\sqrt{12}) + c$$

$$6 = -\frac{12}{6} + c$$

$$6 = -2 + c$$

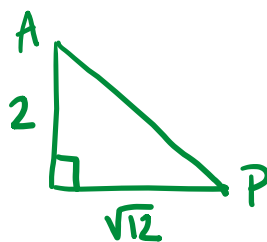
$$c = 8$$

$$y = \frac{\sqrt{12}}{6}x + 8$$

At A , $x = 0$

$$y = \frac{\sqrt{12}}{6}(0) + 8$$

$$A = (0, 8)$$



$$AP^2 = 2^2 + (\sqrt{12})^2$$

$$AP^2 = 4 + 12$$

$$AP^2 = 16$$

$$AP = \sqrt{16}$$

$$AP = 4$$

4 is an integer



15 A circle has equation $x^2 + y^2 = 25$

The point P lies on the circle.

The coordinates of P are $(\sqrt{5}, \sqrt{20})$

The line L is tangent to the circle at point P .

The line L crosses the x -axis at the point A .

Work out the area of triangle AOP .

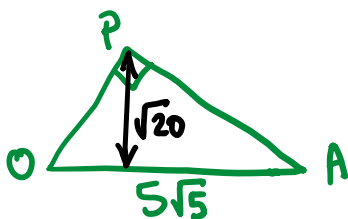
$$\begin{aligned}
 \text{gradient of } OP &= \frac{\sqrt{20}}{\sqrt{5}} \\
 &= \sqrt{4} \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 \text{gradient of} \\
 \text{tangent} &= -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 y &= -\frac{1}{2}x + C \\
 \sqrt{20} &= -\frac{1}{2}(\sqrt{5}) + C \\
 C &= \sqrt{20} + \frac{\sqrt{5}}{2} \\
 C &= 2\sqrt{5} + \frac{\sqrt{5}}{2} \\
 C &= \frac{5\sqrt{5}}{2} \\
 y &= -\frac{1}{2}x + \frac{5\sqrt{5}}{2}
 \end{aligned}$$

$$\text{At } A, y = 0$$

$$\begin{aligned}
 0 &= -\frac{1}{2}x + \frac{5\sqrt{5}}{2} \\
 \frac{1}{2}x &= \frac{5\sqrt{5}}{2} \\
 x &= 5\sqrt{5} \\
 A &= (5\sqrt{5}, 0)
 \end{aligned}$$



$$\begin{aligned}
 \text{Area} &= \frac{1}{2} \times 5\sqrt{5} \times \sqrt{20} \\
 &= \frac{1}{2} \times 5 \sqrt{100} \\
 &= \frac{1}{2} \times 5 \times 10 \\
 &= \frac{1}{2} \times 50
 \end{aligned}$$

25

.....units²
 (Total for Question 15 is 6 marks)

