Perpendicular Lines

SCAN ME

REVISE THIS TOPIC

1 The equation of line L_1 is $y = 5x + 1$
The equation of line L_2 is $5y + x = 20$

Show that these two lines are perpendicular. **[3 marks]**

$y = 5x + 1$

gradient = 5

$5y + x = 20$

$5y = 20 - x$

$y = 4 - \frac{1}{5}x$

gradient $= -\frac{1}{5}$

$5 \times -\frac{1}{5} = -1$ therefore L_1 and L_2 are perpendicular

2 The equation of line L_1 is $y = 8 - 3x$
The equation of line L_2 is $9y - 3x - 6 = 0$

Show that these two lines are perpendicular. **[3 marks]**

$y = 8 - 3x$

gradient $= -3$

$9y - 3x = 6$

$9y = 6 + 3x$

$y = \frac{6}{9} + \frac{3}{9}x$

gradient $= \frac{1}{3}$

$-3 \times \frac{1}{3} = -1$ therefore L_1 and L_2 are perpendicular

3 The equation of line L_1 is $2y = x + 10$
The equation of line L_2 is $4y + 8x = 16$

Show that these two lines are perpendicular. **[3 marks]**

$2y = x + 10$

$y = \frac{1}{2}x + 5$

gradient $= \frac{1}{2}$

$4y + 8x = 16$

$4y = 16 - 8x$

$y = 4 - 2x$

gradient $= -2$

$\frac{1}{2} \times -2 = -1$ therefore L_1 and L_2 are perpendicular

4 The equation of line L_1 is $y = \frac{3}{4}x + 1$

The equation of line L_2 is $6y + 8x = 30$

[3 marks]

Show that these two lines are perpendicular.

$y = \frac{3}{4}x + 1$

gradient $= \frac{3}{4}$

$6y + 8x = 30$

$6y = 30 - 8x$

$y = 5 - \frac{8}{6}x$

gradient $= -\frac{4}{3}$

$\frac{3}{4} \times -\frac{4}{3} = -1$ therefore L_1 and L_2 are perpendicular

5 The equation of line L_1 is $2y = 3x - 6$
The equation of line L_2 is $8y - 12x - 40 = 0$

Show that these two lines are **not** perpendicular. **[3 marks]**

$2y = 3x - 6$

$y = \frac{3}{2}x - 3$

gradient $= \frac{3}{2}$

$8y - 12x = 40$

$8y = 40 + 12x$

$y = 5 + \frac{12}{8}x$

gradient $= \frac{3}{2}$

Both gradients are the same so L_1 and L_2 are parallel not perpendicular

6 The equation of line L_1 is $y = kx + 4$
The equation of line L_2 is $2y + 4x = 10$

Lines L_1 and L_2 are perpendicular. Work out the value of k. **[3 marks]**

$y = kx + 4$ gradient $= k$

$2y + 4x = 10$

$2y = 10 - 4x$

$y = 5 - 2x$

gradient $= -2$

$-2 \times k = -1$

$k = \frac{-1}{-2}$

$k = \frac{1}{2}$

7 The equation of line L_1 is $2y = kx - 2$
The equation of line L_2 is $3y + x = 18$

Lines L_1 and L_2 are perpendicular. Work out the value of k. **[3 marks]**

$2y = kx - 2$

$y = \frac{k}{2}x - 1$

gradient $= \frac{k}{2}$

$3y + x = 18$

$3y = 18 - x$

$y = 18 - \frac{1}{3}x$

gradient $= -\frac{1}{3}$

$\frac{k}{2} \times -\frac{1}{3} = -1$

$-\frac{k}{6} = -1$

$-k = -6$

$k = 6$

$k = 6$

8 The equation of line L_1 is $y = 3 - \frac{2}{5}x$

The equation of line L_2 is $ky - 6x - 20 = 0$

Lines L_1 and L_2 are perpendicular. Work out the value of k. **[3 marks]**

$y = 3 - \frac{2}{5}x$

gradient $= -\frac{2}{5}$

$ky = 6x + 20$

$y = \frac{6}{k}x + \frac{20}{k}$

gradient $= \frac{6}{k}$

$-\frac{2}{5} \times \frac{6}{k} = -1$

$-\frac{12}{5k} = -1$

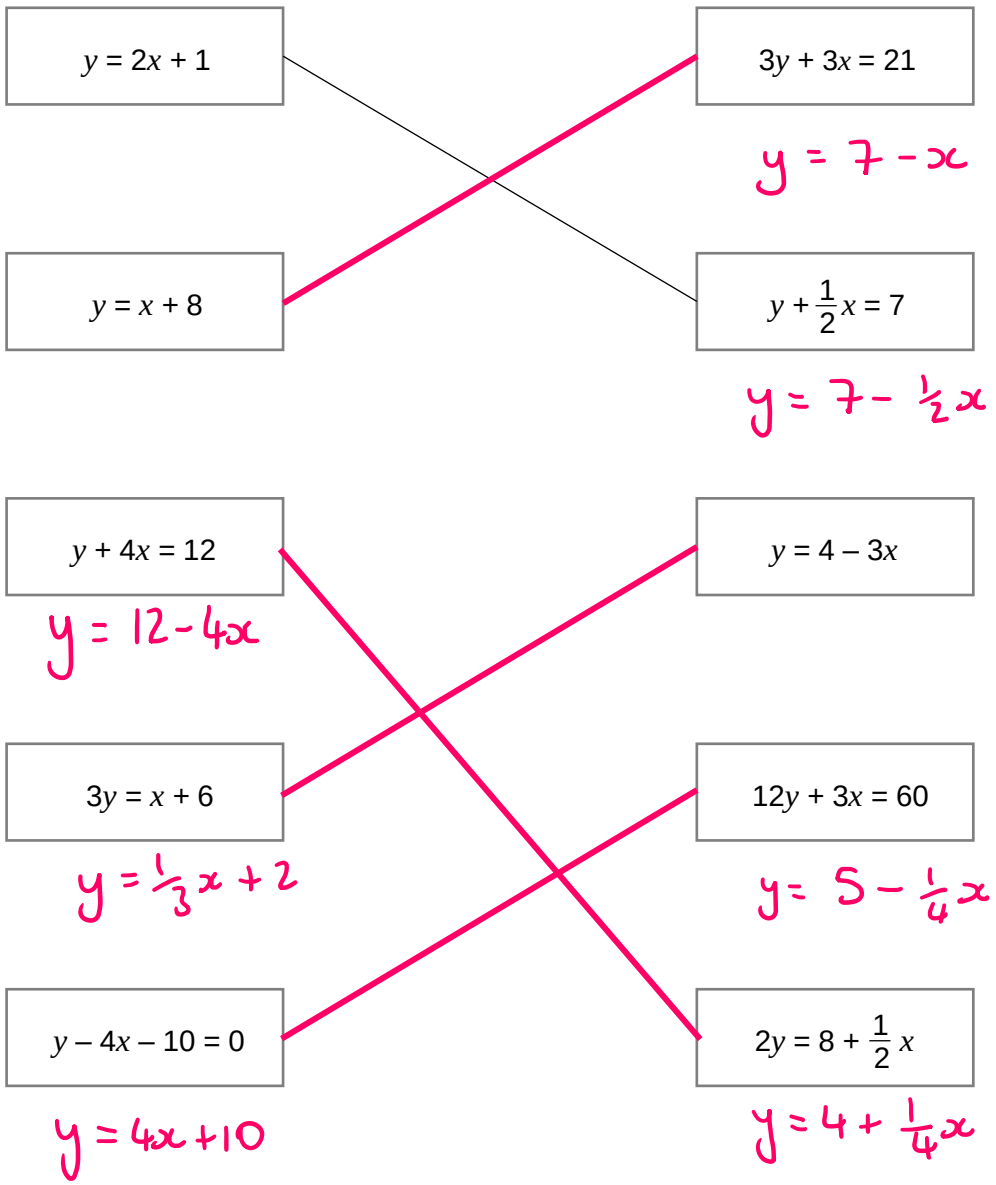
$-12 = -5k$

$k = \frac{12}{5}$

9 Here are some equations of straight lines.
Match each equation on the left with one on the right so that the lines with those two equations are perpendicular.

One has been done for you.

[3 marks]



10 The equation of line $\mathbf{L_1}$ is $y = 3x + 1$
Line $\mathbf{L_2}$ is

perpendicular to line $\mathbf{L_1}$

and

passes through the point (9, 4)

Work out an equation for line $\mathbf{L_2}$ **[3 marks]**

gradient of $L_1 = 3$

gradient of $L_2 = -\frac{1}{3}$

$y = -\frac{1}{3}x + c$

$4 = -\frac{1}{3}(9) + c$

$4 = -3 + c$

$c = 7$

Answer $y = -\frac{1}{3}x + 7$

11 The equation of line $\mathbf{L_1}$ is $y = 5 - 4x$
Line $\mathbf{L_2}$ is

perpendicular to line $\mathbf{L_1}$

and

passes through the point (4, 12)

Work out an equation for line $\mathbf{L_2}$ **[3 marks]**

gradient of $L_1 = -4$

gradient of $L_2 = \frac{1}{4}$

$y = \frac{1}{4}x + c$

$12 = \frac{1}{4}(4) + c$

$12 = 1 + c$

$c = 11$

Answer $y = \frac{1}{4}x + 11$

9

12 The equation of line $\mathbf{L_1}$ is $y = \frac{1}{2}x + 3$

Line $\mathbf{L_2}$ is

perpendicular to line $\mathbf{L_1}$

and

passes through the point (-3, 7)

Work out an equation for line $\mathbf{L_2}$ **[3 marks]**

gradient of $L_1 = \frac{1}{2}$

gradient of $L_2 = -2$

$y = -2x + c$

$7 = -2(-3) + c$

$7 = 6 + c$

$c = 1$

Answer $y = -2x + 1$

13 The equation of line $\mathbf{L_1}$ is $y = 2 - \frac{1}{6}x$

Line $\mathbf{L_2}$ is

perpendicular to line $\mathbf{L_1}$

and

passes through the point (2, 7)

Work out an equation for line $\mathbf{L_2}$ **[3 marks]**

gradient of $L_1 = -\frac{1}{6}$

gradient of $L_2 = 6$

$y = 6x + c$

$7 = 6(2) + c$

$7 = 12 + c$

$c = -5$

Answer $y = 6x - 5$

14 $A = (2, 6)$ $B = (1, 9)$ $C = (15, 2)$

Work out the equation of the line that
is perpendicular to line AB
and
passes through point C **[4 marks]**

gradient of AB $= \frac{9-6}{1-2}$

$= \frac{3}{-1}$

$= -3$

$y = \frac{1}{3}x + c$

$2 = \frac{1}{3}(15) + c$

$2 = 5 + c$

$c = -3$

Answer $y = \frac{1}{3}x - 3$

15 $A = (0, 6)$ $B = (3, 8)$ $C = (6, 6)$

Work out the equation of the line that
is perpendicular to line AB
and
passes through point C **[4 marks]**

gradient of AB $= \frac{8-6}{3-0}$

$= \frac{2}{3}$

$y = -\frac{3}{2}x + c$

$6 = -\frac{3}{2}(6) + c$

$6 = -9 + c$

$c = 15$

Answer $y = -\frac{3}{2}x + 15$

16 $A = (5, -3)$ $B = (3, 5)$ $C = (-5, 2)$

Work out the equation of the line that
is perpendicular to line AB
and
passes through point C **[4 marks]**

gradient of AB $= \frac{5-(-3)}{3-5}$

$= \frac{8}{-2}$

$= -4$

$y = \frac{1}{4}x + c$

$2 = \frac{1}{4}(-5) + c$

$2 = -\frac{5}{4} + c$

$c = 2 + \frac{5}{4}$

$c = \frac{8}{4} + \frac{5}{4}$

$c = \frac{13}{4}$

Answer $y = \frac{1}{4}x + \frac{13}{4}$

17 $A = (-4, 5)$ $B = (6, 1)$ $C = (-8, -9)$

Work out the equation of the line that
is perpendicular to line AB
and
passes through point C **[4 marks]**

gradient of AB $= \frac{1-5}{6-(-4)}$

$= \frac{-4}{10}$

$= -\frac{2}{5}$

$y = \frac{5}{2}x + c$

$-9 = \frac{5}{2}(-8) + c$

$-9 = -20 + c$

$c = 11$

Answer $y = \frac{5}{2}x + 11$

18 The equation of line $\mathbf{L_1}$ is $y = 3 - 2x$

Line $\mathbf{L_2}$ is

perpendicular to line $\mathbf{L_1}$

and

passes through the point (6, 2)

18 (a) Work out the coordinates of the point where line $\mathbf{L_2}$ intersects the x-axis.

[3 marks]

gradient of $L_1 = -2$

gradient of $L_2 = \frac{1}{2}$

$y = \frac{1}{2}x + c$

$2 = \frac{1}{2}(6) + c$

$2 = 3 + c$

$c = -1$

At x-axis $y = 0$

$y = \frac{1}{2}x - 1$

$0 = \frac{1}{2}x - 1$

$1 = \frac{1}{2}x$ $\times 2$

$2 = x$ $\times 2$

Answer (2 , 0)

18 (b) Work out the coordinates of the point where line $\mathbf{L_2}$ intersects the y-axis.

[2 marks]

At y-axis $x = 0$

$y = \frac{1}{2}(0) - 1$

$y = -1$

Answer (0 , −1)

13

19 The equation of line L_1 is $y = 2x + 2$
Line L_2 is

perpendicular to line L_1

and

passes through the point (-8, 11)

Lines L_1 and L_2 intersect at the point P.
Line L_1 intersects the x-axis at the point Q.
Line L_2 intersects the x-axis at the point R.

Work out the area of triangle PQR.

[6 marks]

gradient of $L_1 = 2$

gradient of $L_2 = -\frac{1}{2}$

$y = -\frac{1}{2}x + c$

$11 = -\frac{1}{2}(-8) + c$

$11 = 4 + c$

$c = 7$

L_2 is $y = -\frac{1}{2}x + 7$

L_1 and L_2 intersect when

$2x + 2 = -\frac{1}{2}x + 7$ (×2)

$4x + 4 = -x + 14$ (×2)

$5x = 10$

$x = 2$ $P = (2, 6)$

$y = 2(2) + 2$

$y = 6$

$2x + 2 = 0$

$2x = -2$

$x = -1$

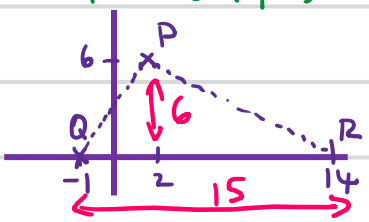
$Q = (-1, 0)$

$-\frac{1}{2}x + 7 = 0$

$7 = \frac{1}{2}x$

$x = 14$

$R = (14, 0)$



Area $= \frac{1}{2} \times 15 \times 6$

Answer 45 units^2

6